Title: On the gauge theory/Sasaki-Einstein correspondence

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Abstract:

Sasaki-Einstein spaces ⇔ 4d SCFTs

Sergio Benvenuti 2005

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Dario Martelli, James Sparks

- Branes at singularities and quivers
- AdS/CFT and Sasaki-Einstein spaces
- New classes of dual pairs
- Brane tilings and periodic quivers
- Checks of the duality:
  1. BPS baryons and SUSY 3-cycles
  2. BPS mesons and BPS geodesics
- Applications:
  1. Marginal deformations
  2. Semiclassical strings

CONICAL SINGULARITIES AND ADS/CFT

N D3-BRANES

IN A SMOOTH REGION

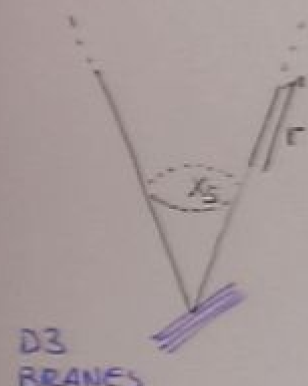
$AdS_5 \times S^5$

SUPERGRAVITY

$N=4$ SYM

SUPERCONFORMAL

SU(N) GAUGE THEORY



CONICAL SINGULARITY

$$ds_6^2 = dr^2 + r^2\, ds_5^2$$

$(AdS_5 \times X^5 \text{ SUGRA})$

$$\left.\begin{array}{l}\text{THE CONE IS CALABI-YAU} \\ \Updownarrow \\ X^5 \text{ IS SASAKI-EINSTEIN}\end{array}\right\} \begin{array}{l}N=1 \\ \text{SUPERCONFORM} \\ \text{SYMMETRY}\end{array}$$

EXAMPLE: THE CONIFOLD

- CALABI-YAU CONE

$x, y, z, w \in \mathbb{C}^4$

$\boxed{xy = zw}$

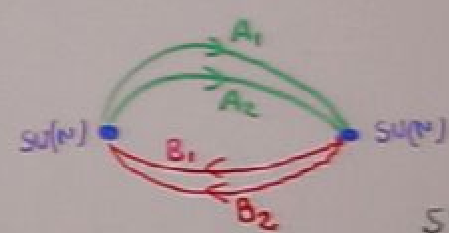
ALGEBRAIC DESCRIPTION

- GAUGE THEORY: $SU(N) \times SU(N)$

KLEBANOV WITTEN

	$SU(N)$	$SU(N)$
A_i	N	$\overline{N}$
B_i	$\overline{N}$	N

QUIVER DIAGRAM



SYMMETRY

$SU(2) \times SU(2) \times U(1)_R \times U(1)_B$

$$W = tr\left[A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1\right]$$

- SASAKI-EINSTEIN METRIC

$$ds_5^2 = \frac{1}{9}\left(d\psi + \cos(\theta_1)\, d\varphi_1 + \cos(\theta_2)\, d\varphi_2\right)^2 + \frac{1}{6}\left(d\theta_1^2 + \sin^2(\theta_1)\, d\varphi_1^2\right) + \frac{1}{6}\left(d\theta_2^2 + \sin^2(\theta_2)\, d\varphi_2^2\right)$$

TOPOLOGY

$S^2 \times S^3$

$(b_2 = b_3 = 1)$

NEW EXPLICIT SASAKI-EINSTEIN METRICS (!)

$Y^{p,q}$ GAUNTLETT et al

$L^{p,q|r}$ CVETIC et al

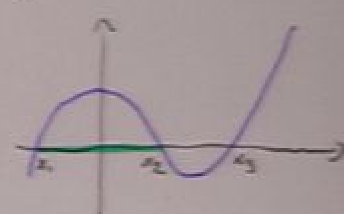
$$\boxed{ds^2_{[5]} = \left(\frac{d\psi_R}{3} + \sigma_\phi d\phi + \sigma_\psi d\psi\right)^2 + \frac{1}{4}\, ds^2_{[4]}}$$

LOCAL KÄLER-EINSTEIN

$$\begin{aligned}
\sigma_\phi &= \frac{(\alpha - x)(1-y)}{2\alpha} - \frac{1}{3} \\
\sigma_\psi &= \frac{(\beta - x)(1+y)}{2\beta} - \frac{1}{3} \\
ds^2_{[4]} &= \frac{\rho^2}{f(x)}dx^2 + \frac{f(x)}{\rho^2}\left(\frac{(1-y)}{\alpha}d\phi + \frac{(1+y)}{\beta}d\psi\right)^2 + \\
&\quad \frac{\rho^2}{g(y)}dy^2 + \frac{g(y)}{\rho^2}\left(\frac{(\alpha-x)}{\alpha}d\phi - \frac{(\beta-x)}{\beta}d\psi\right)^2
\end{aligned}$$

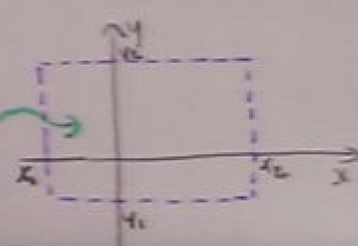
$$\left.\begin{aligned}
f(x) &= x(\alpha - x)(\beta - x) - \mu \\
g(y) &= \frac{1}{2}(\alpha + \beta - y(\alpha - \beta))(1 - y^2)
\end{aligned}\right\}\ \text{CUBIC FUNCTIONS IN ONE VARIABLE}$$

$$\rho^2 = \frac{1}{2}(\alpha + \beta - y(\alpha - \beta) - 2x)$$



CAN BE SEEN AS
$U(1)^3$ FIBRATION
OVER $\{x, y\}$ BASE

$U(1)^3$ "TORIC" ISOMETRY $\Leftarrow \{\psi_R, \varphi, \psi\}$ T^3



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$$\sigma_\phi = \frac{(\alpha - x)(1-y)}{2\alpha} - \frac{1}{3}$$
$$\sigma_\psi = \frac{(\beta - x)(1+y)}{2\beta} - \frac{1}{3}$$

$$ds^2_{[4]} = \frac{\rho^2}{f(x)}dx^2 + \frac{f(x)}{\rho^2}\left(\frac{(1-y)}{\alpha}d\phi + \frac{(1+y)}{\beta}d\psi\right)^2 + \frac{\rho^2}{g(y)}dy^2 + \frac{g(y)}{\rho^2}\left(\frac{(\alpha - x)}{\alpha}d\phi - \frac{(\beta - x)}{\beta}d\psi\right)^2$$

$$\left.\begin{aligned} f(x) &= x(\alpha - x)(\beta - x) - \mu \\ g(y) &= \frac{1}{2}(\alpha + \beta - y(\alpha - \beta))(1 - y^2) \end{aligned}\right\} \text{ CUBIC FUNCTIONS IN ONE VARIABLE}$$

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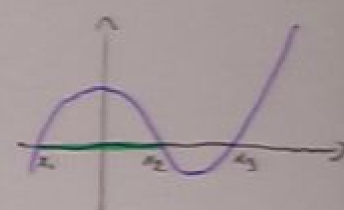
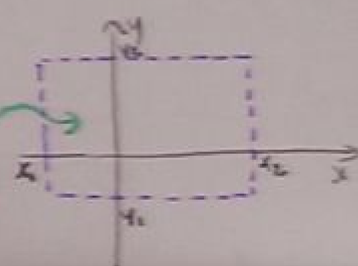


CAN BE SEEN AS $U(1)^3$ FIBRATION OVER $\{x, y\}$ BASE



$U(1)^3$ "TORIC" ISOMETRY $\Leftarrow \{\psi_R, \varphi, \psi\}$ T^3

3 POINTS OF VIEW

DIFFERENTIAL GEOMETRY
(METRICS, VOLUMES...)

QUIVER GAUGE THEORIES

ALGEBRAIC GEOMETRY
(SETS OF COMPLEX EQUATIONS)

IN THE TORIC CASE

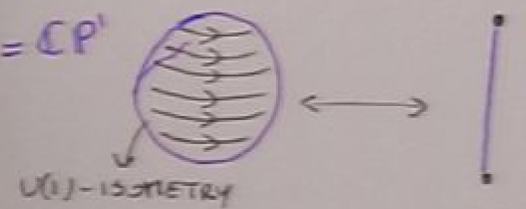
"TORIC" METRICS
($U(1)^3$ - INVARIANCE)

TORIC QUIVERS:
- PERIODIC QUIVER
- BRANE TILING AND DIMERS
- FOLDED QUIVER

TORIC DIAGRAM

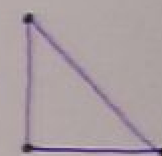
TORIC GEOMETRY

$S^2 = \mathbb{C}P^1$

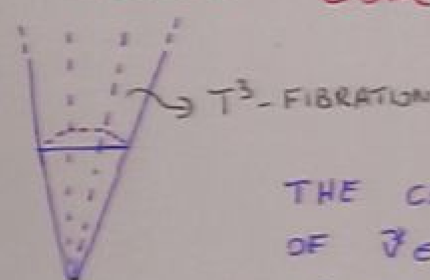


$\mathbb{P}^1 \times \mathbb{P}^1$

$\mathbb{P}^2$



TORIC CONES

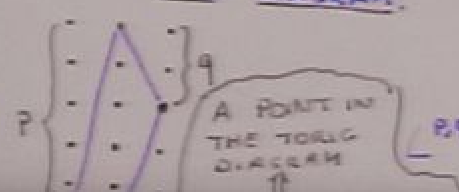


THE CONE IS THE SET OF $\vec{v} \in \mathbb{R}^3$:

$$(\vec{v}, \vec{v}_a) \geq 0$$

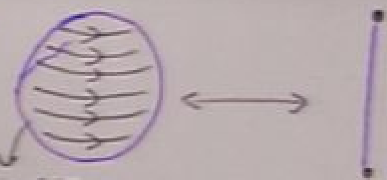
FOR A SET OF INTEGRAL VECTORS $\vec{v}_a$ IN A BASE $\vec{v}_a = (1, v_x, v_y)$, THE 2-VECTORS (v_a^x, v_a^y) DEFINE THE <u>TORIC</u> <u>DIAGRAM</u>.

$Y^{p,q}$



$L^{p,q|r}$: THE MOST GENERAL TORIC DIAGRAM WITH FOUR CORNERS

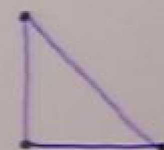
$S^2 = \mathbb{CP}^1$



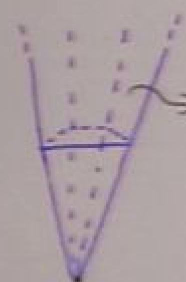
U(1)-ISOMETRY

$\mathbb{P}^1 \times \mathbb{P}^1$

$\mathbb{P}^2$



TORIC CONES



T^3- FIBRATION

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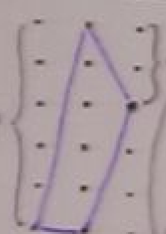
$Y^{p,q}$ p



q

A POINT IN THE TORIC DIAGRAM

$\Updownarrow$

A FACE IN THE CONE

$p,q|r$: THE MOST GENERAL TORIC DIAGRAM WITH FOUR CORNERS

The recursive construction of the $\mathcal{Y}^{p,q}$ quivers

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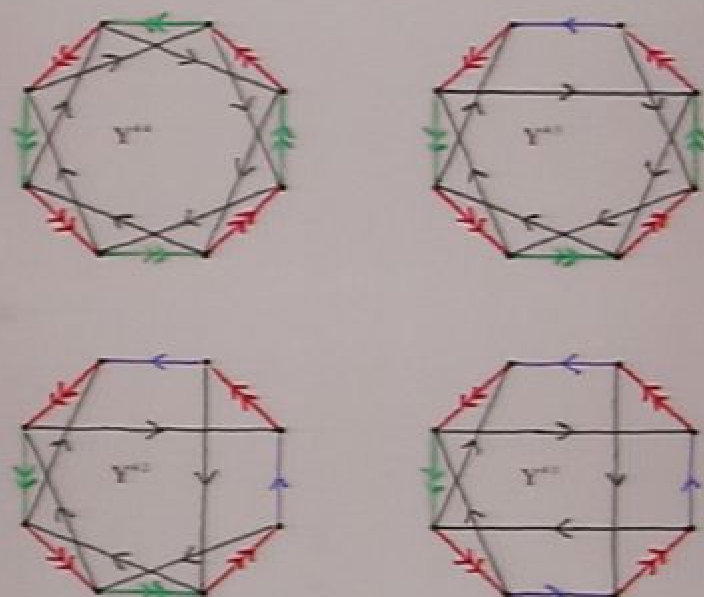
USING SU(2) SYMMETRY



Gauge group: $SU(N)^{2p}$

$\mathcal{W}$ is cubic or quartic with $SU(2)$ symmetry

$Y^{44} = \mathbb{C}^3/\mathbb{Z}_8$ 

$Y^{40} = \frac{\text{CONIFOLD}}{\mathbb{Z}}$ 

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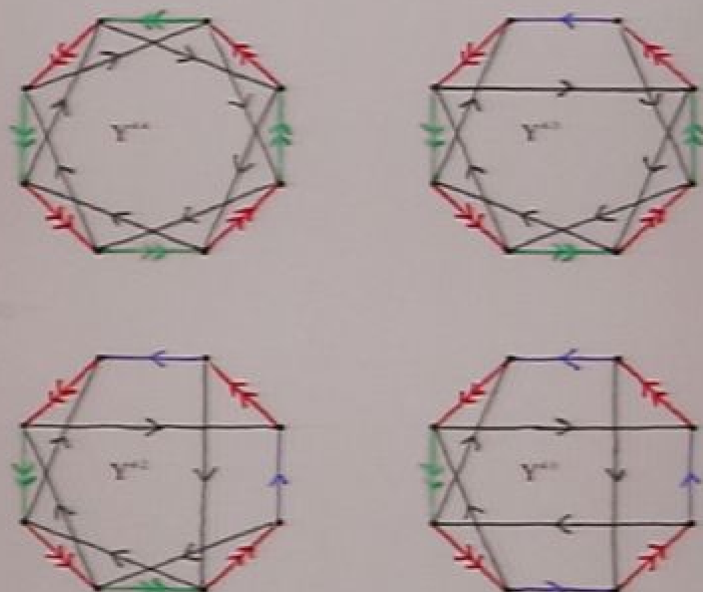

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CHECKS OF THE DUALITY

a-MAXIMIZATION $\Longrightarrow$ SCALING DIMENSION OF THE BIFUNDAMENTAL FIELDS

$$a \propto tr(R^3) \propto \frac{N^2}{\text{VOLUME}(X^5)}$$

R-SYMMETRY

TIPICALLY IRRATIONAL NUMBERS

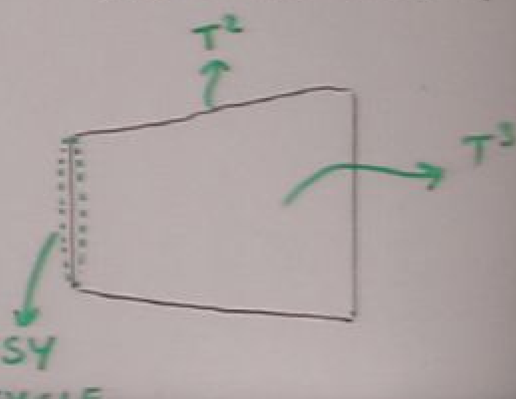
BARYONIC OPERATORS

$$\mathcal{B} = \varepsilon^{\alpha_1 \dots \alpha_N} X^{\beta_1}_{\alpha_1} X^{\beta_2}_{\alpha_2} \cdots X^{\beta_N}_{\alpha_N} \varepsilon_{\beta_1 \dots \beta_N}$$

$$\dim[\mathcal{B}] = N \cdot \dim[X] \propto N \frac{\text{VOLUME}(3\text{-CYCLE})}{\text{VOLUME}(X^5)}$$

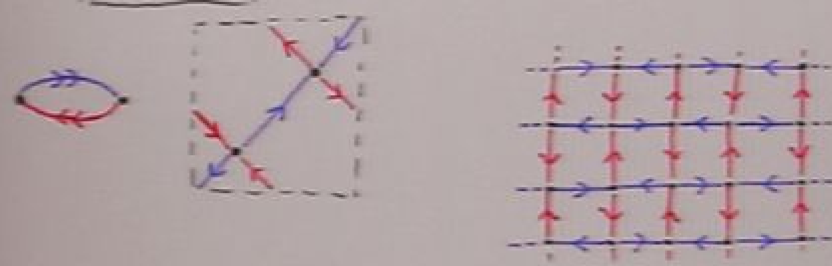
THE DUAL STATE IS A D3-BRANE WRAPPED ON A SUPERSYMMETRIC 3-CYCLE

BASE OF THE GEOMETRY

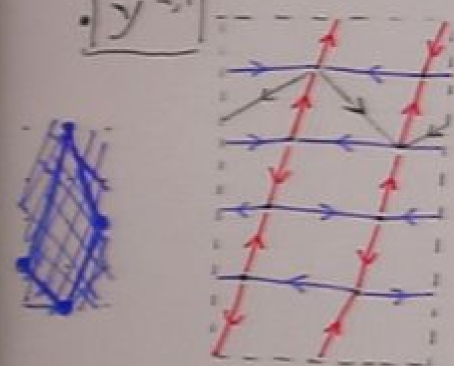


BRANE DIMERS AND PERIODIC QUIVERS

• $\boxed{\text{CONIFOLD}}$



• $\boxed{Y^{2,1}}$



IMPORTANT FACTS:

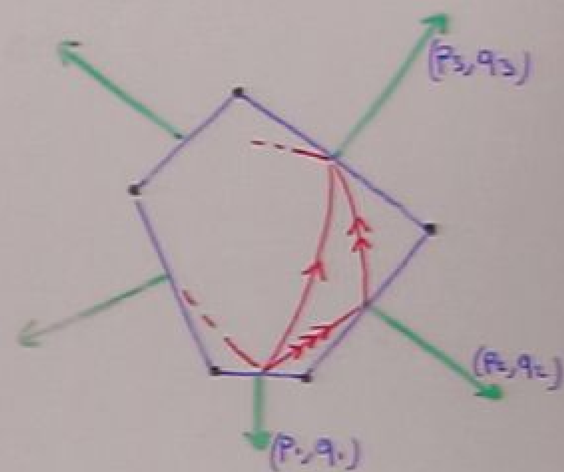
- THE SUPERPOTENTIAL IS INCLUDED IN THE DIAGRAM (ONE TERM FOR EVERY FACE)
- THE DUAL GRAPH REPRESENTS INTERSECTION OF 5-BRANES DESCRIBING THE SINGULARITY
- CONNECTION TO THE "CALABI-YAU CRISTALS"

FROM TORIC DIAGRAMS TO GAUGE THEORIES

"FOLDED QUIVER"

FORMULA
FOR THE NUMBER
OF BIFUNDAMENTAL
FIELDS

$$(\#)_{ij} = p_i q_j - p_j q_i$$



ALL THE FIELDS
HAVE THE SAME
GLOBAL SYMMETRY
QUANTUM NUMBERS

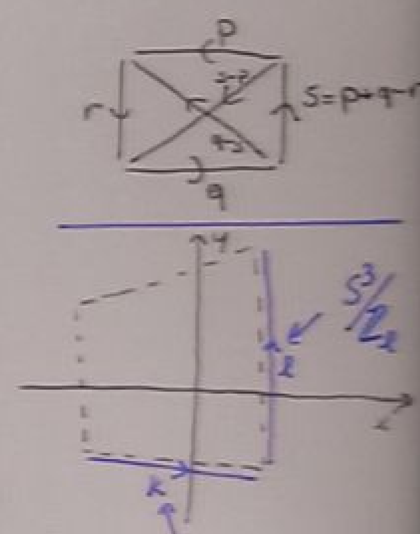
$$U(1)_R \times U(1)_F^2 \times U(1)^b$$

R-SYMMETRY

TORIC FLAVOR SYMMETRY

BARYONIC SYMMETRY

EXAMPLE $L^{pq|r}$



THIS PICTURE EASILY GIVES:

- SCALING DIMENSION OF BARYONS
- CUBIC ANOMALIES (e.g. $tr(R^3)$)
- TOPOLOGY OF THE SUSY 3-CYCLES ($S^3/\mathbb{Z}_k$)

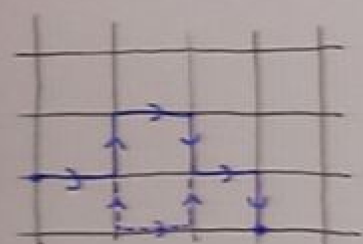
BPS MESONS

CHIRAL RING = $\dfrac{\text{HOLOMORPHIC OPERATORS}}{\mathcal{F}\text{-TERM RELATIONS}}$

- FOR QUIVERS: CLOSED ORIENTED PATHS

EXAMPLE: $\mathrm{tr}\left(A_1 B_1 A_1 B_2 A_1 B_2\right) + \ldots$

- PERIODIC QUIVERS



ALL THE PATHS WITH THE SAME END POINTS ARE EQUAL IN THE CHIRAL RING

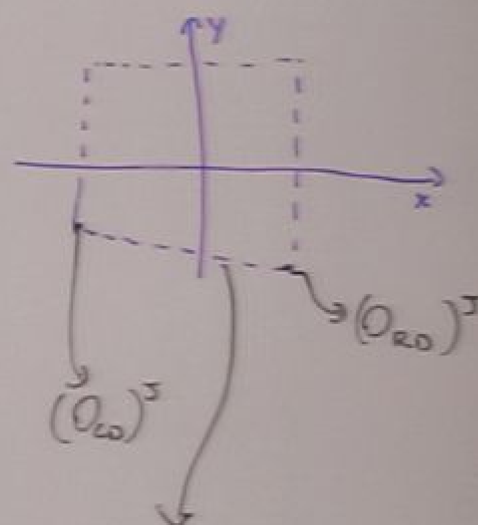
THE $U(1) \times U(1)$ TORIC FLAVOR CHARGES ARE COUNTED BY THE HORIZONTAL AND VERTICAL SHIFTS

BPS GEODESICS

POINT-LIKE STRINGS IN $AdS_5 \times X^5$
MOVING ALONG THE R-SYMMETRY DIRECTION (ψ_R)

$$ds^2 = \left(d\psi_R + A_1(x,y)\,d\varphi_1 + A_2(x,y)\,d\varphi_2\right)^2 + ds_4^2$$

$$\begin{cases} p_{\varphi_1}/p_R = A_1(x,y) \\ p_{\varphi_2}/p_R = A_2(x,y) \end{cases}$$



DUAL CFT OPERATORS:

LONG BPS MESONS

IN A COHERENT BASE

COHERENT SUPERPOSITION
OF O_{LD} AND O_{RD}:

$$\prod_{i=1}^{J}\left(\lambda e^{i\alpha} O_{LD} + \eta e^{-i\alpha} O_{RD}\right)$$

LONG "STRAIGHT" PATHS

THE DIRECTION OF THE PATH
IN THE PERIODIC QUIVER

$\Updownarrow$

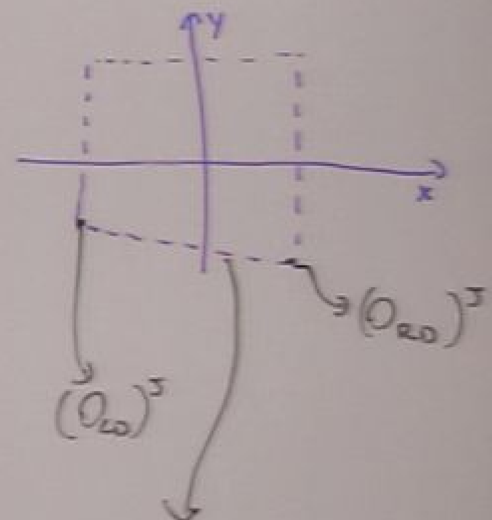
POSITION OF THE STRING IN
THE (x,y) BASE

BPS GEODESICS

POINT-LIKE STRINGS IN $AdS_5 \times X^5$
MOVING ALONG THE R-SYMMETRY DIRECTION ($\sim \psi_R$)

$$ds^2 = \left(d\psi_R + A_1(x,y)\,d\varphi_1 + A_2(x,y)\,d\varphi_2\right)^2 + ds_4^2$$

$$\begin{cases} P_{\varphi_1}/P_R = A_1(x,y) \\ P_{\varphi_2}/P_R = A_2(x,y) \end{cases}$$



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LONG BPS MESONS

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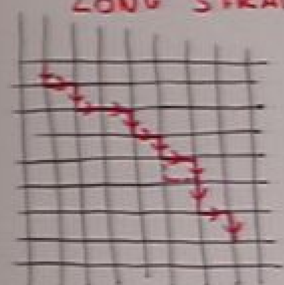
LONG "STRAIGHT" PATHS



THE DIRECTION OF THE PATH
IN THE PERIODIC QUIVER

$\Updownarrow$

POSITION OF THE STRING IN
THE (x,y) BASE

FAST STRINGS IN SASAKI-EINSTEIN SPACES

$$ds_5^2 = \frac{1}{9}\left(d\psi + A^\mu dx_\mu\right)^2 + g_{[4]}^{\mu\nu} dx_\mu dx_\nu$$

x^μ. COORDINATES ON THE KÄLER-EINSTEIN BASE

STRINGS ON $\mathbb{R}_t \times X^5$ MOVING FAST ALONG ψ

FIX $t = \kappa\tau$

(σ, τ) WORLD SHEET COORDINATES

AND TAKE THE LIMIT

$$\begin{cases} \frac{\partial}{\partial\tau}(\psi - 3t) \to 0 \\ \frac{\partial}{\partial\tau} x^\mu \to 0 \\ \kappa \to \infty \end{cases}$$

$\kappa(\partial_\tau x)$ CONSTANT

THESE STRINGS ARE DESCRIBED BY A NON-RELATIVISTIC EFFECTIVE ACTION

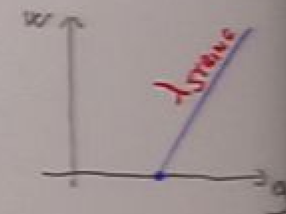
$$S_{NR} = \int \frac{\kappa}{3}\left(\frac{d\psi}{d\tau} + A^\mu \frac{dx_\mu}{d\tau}\right) - \frac{1}{2} g^{\mu\nu} \frac{dx_\mu}{d\sigma}\frac{dx_\nu}{d\sigma}$$

ALL THE INFORMATION ABOUT THE EXPLICIT SASAKI-EINSTEIN METRIC IS CONTAINED IN S_{NR}

EXTENDED STRINGS IN GAUGE THEORIES

WHAT IS THE GAUGE THEORY
DUAL OF $\lambda_{\text{STRING}} \to 0$?

- IN $\mathcal{N}=4$ SYM. FREE THEORY
- IN THE CONIFOLD: $\begin{cases} g_1, g_2 \neq 0 \\ \text{SUPERPOTENTIAL} = 0 \end{cases}$



- GENERIC QUIVER: SOME SUPERPOTENTIAL TERMS VANISH

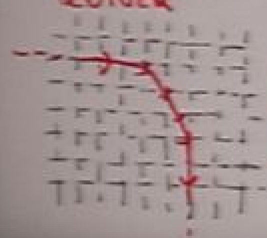
$\Downarrow$

THE CHIRAL RING IS ENHANCED

$\Downarrow$

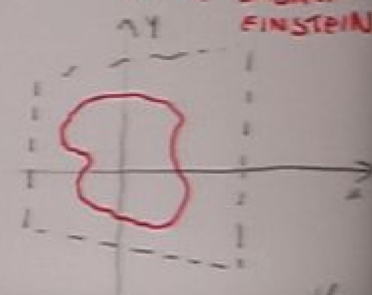
THERE ARE LONG BPS MESONS
DUAL TO EXTENDED HOLOMORPHIC
STRINGS MOVING IN THE FULL
SASAKI-EINSTEIN GEOMETRY

PERIODIC QUIVER



$\text{DIRECTION OF PATHS} \Longleftrightarrow \text{POSITION OF CLASSICAL STRINGS}$

TORIC BASE OF THE SASAKI-EINSTEIN



SUMMARY

- NEW ADS/CFT EXAMPLES WITH EXPLICIT SASAKI-EINSTEIN METRICS

- QUANTITATIVE CHECKS INSIDE THE "PROTECTED SECTOR"
  - BPS BARYONS
  - BPS MESONS
  - T' HOOFT ANOMALIES

- FIRST STEPS BEYOND THE BPS SECTOR

OPEN PROBLEMS

- COMPLETE THE DICTIONARY INSIDE THE PROTECTED SECTOR

- FAST HOLOMORPHIC STRINGS FROM GAUGE THEORY AT $\lambda_S > 0$ (SPIN CHAIN HAMILTONIAN)

- BREAK CONFORMAL INVARIANCE WITH FRACTIONAL BRANES AND FIND A KLEBANOV-STRASSLER SOLUTION