Title: Introduction to quantum gravity

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Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

- Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005
- Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079
- Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048
- Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

- undergraduate quantum mechanics
- basics of classical gauge field theories
- basic general relativity
- hamiltonian and lagrangian mechanics
- basics of lie algebras
The basic structure of QCD...
\langle GR \Rightarrow QR \rangle
The basic structure of LQG

Intro GR as a gauge theory
The basic structure of LQG

Intro GR as a gauge theory

hep-th/0201079
I  The basic structure of LQG -
hep-th/0408048

II  Intro GR as a gauge theory
hep-th/0201079
Sections 2.3  6
       (today)
Fix Spatial Topology $S^3 = \Sigma$
Fix Spatial Topology $S^3 = \mathbb{E}$
Fix spatial topology $S^3 - \Sigma$

Graph
Fix spatial Topology $S^3 = \mathbb{E}$

All graphs $P = \{ \}$
Fix Spatial Topology \( S^3 = \mathbb{E} \)

All Graphs \( \mathcal{G} = \{ \text{all graphs} \} \)
Fix spatial topology

\[ S^3 = \mathbb{E} \]

All graphs \( P \)

\[ G = \{ \text{all graphs} \} \]

Finite or infinite loops
Fix Spatial Topology \[ S^3 = \mathbb{E} \]

All graphs \[ G \]
Fix spatial topology $S^3 = \mathbb{E}$

All graphs $P = \{ \text{all graphs} \}$

$G = \{ \text{all graphs} \}$

Finite set of nodes and links
Fix spatial topology

\[ S^3 = \Omega \]

all graphs \( P = \{ \text{all graphs} \} \)

finite \( G \triangleleft \{ \text{all graphs} \} \)

\( \{ G \} = \{ \text{all embeddings of } G \text{ into } \Sigma \text{ up to topology} \} \)
Fix spatial topology $S^3 = \Sigma$

All graphs $P = \{ G \}$

Embeddings of $G$ into $\Sigma$ up to topology countable set
Fix Spatial Topology

All Graphs \( \mathcal{P} = \{ \text{all graphs} \} \)

\( G = \{ \text{all graphs} \} \) is finite set of \( \text{num}_3 \) inputs

\( \{ G \} = \{ \text{all } G \} \rightarrow \) subspace of \( \Sigma \) up to topology

Countable set

\( S^3 = \mathbb{E} \)
$\phi_3 \sim$ orthogonal basis
Toposy

\[ S^3 = \mathbb{S} \]

\(\{g\} \mapsto \text{fill \ embeddings \ to \ get \ homeomorphism} \)

\(\text{into } \mathcal{F} \text{ wrt topology} \)

Orthogonal basis
Topology

\[ S^3 = \mathbb{C} \]

\[ \Phi \times = \text{orthonormal basis} \]
$\mathcal{H}_3 = \text{orthonormal basis} \quad 1^a >$

$1^a > = \sum_a^3$
\[ x_3 = \text{orthonormal basis} \quad |\phi\rangle \]
\[ |\psi\rangle = \sum c_i |\phi_i\rangle \quad \sum |c_i|^2 = 1 \]
\( \| \beta_3 \| = \text{orthonormal basis} \)  
\( |\beta_3 > = \sum a_\beta | \beta > \)  
\( \sum_{\beta} |a_\beta|^2 = 1 \)  
\( \langle \beta_1 | \beta_3 > = 5 \beta_3 \)
$\hat{H}_\psi = \text{orthonormal basis} \quad |\psi\rangle \quad \text{Separable}$

$|\psi\rangle = \sum c_n |\hat{n}\rangle \quad \sum |c_n|^2 = 1$

$\langle \psi | \psi \rangle = \delta_{\psi \bar{\psi}}$
\( \phi_3 = \text{orthonormal basis} \)

\[ | \phi \rangle = \sum \alpha \phi \]

\[ | \alpha \rangle^2 = 1 \]

\[ \langle \phi \phi | = \delta \phi \phi \]
$\mathcal{H}_3 = \text{orthonormal basis } |x\rangle \quad \text{Separable}$

$|x\rangle = \sum \alpha_n |n\rangle \quad \sum |\alpha_n|^2 = 1$

$\langle \phi | \psi \rangle = \delta_{\phi \psi}$

$\triangledown \text{Volume operator }$
\[ x = \text{orthonormal basis} \]

\[ |x > = \sum a_n |p_n > \quad \leq \quad l a_n^2 = 1 \]

\[ \langle p_m | p_n > = \delta_{mn} \]

Volume operator in \( \mathbb{H}^3 \)
\( \mathbf{H}_2 \) = orthonormal basis \( |\varphi\rangle \)  

\[ |\psi\rangle = \sum \alpha_n |\varphi_n\rangle \leq \sum |\alpha_n|^2 = 1 \]

\[ \langle \varphi_1 | \varphi_2 \rangle = \delta_{\varphi_1 \varphi_2} \]

Volume operator in \( \mathbf{H}_3 \)  

\[ \langle \psi | \varphi_2 \rangle = |\varphi_2\rangle \varphi \]

\[ V = \frac{1}{n!} \text{ number of 4 or higher valent nodes} \]
Spatial Topology  $S^3 = \Sigma$

14$\rho > 0 \sum_{i} a_i$

$< F I R' > = 5$

Volume operator in $\Sigma^3$
Spatial Topology

$S^3 = \Xi$

$\{ \}$ fill embedded set with $G$?

$\Xi$ null topology

$\phi_k = \text{orthonormal basis}$

$14 \Rightarrow \Xi \leq \alpha_p$

$\text{Volume operation in } \Xi^3$
\[ h = \text{orthonormal basis} \quad 1 \rightarrow \text{Separable} \]

\[ |\Psi\rangle = \sum_{q} \langle \Psi_{q} | \Psi \rangle \quad \frac{1}{2} |\psi_{1}|^2 = 1 \]

\[ \langle \Psi_{1} | \Psi_{2} \rangle = \delta_{12} \]

Volume operator in \( H_{3} \)

\[ \langle \Psi_{1} | \Psi_{2} \rangle = \langle \Psi_{1} | \Psi_{2} \rangle \]

\[ V_{a} = \frac{r^{3}}{R} \quad \text{number of 4 or higher valent nuclei} \]
$\tilde{\mathbf{Z}} = \text{orthonormal basis}$

$|\rangle = \sum a_n |\tilde{\mathbf{n}}\rangle$

$\langle \tilde{\mathbf{n}} | = \frac{1}{\sqrt{\Omega}}$

$|\mathbf{\tilde{r}}\rangle = \mathbf{\tilde{r}}$

Volume operator in $\mathbb{H}^3$

$|\mathbf{\tilde{r}}\rangle - |\mathbf{\tilde{r}}\rangle$

$V_A = \frac{\Omega}{4\pi} \text{ number of short holes}$
$\mathbf{e}_3 = \text{orthonormal basis} \quad \langle \mathbf{e}_3 | \mathbf{e}_3 \rangle = \frac{\epsilon}{\hbar} \quad \frac{\sqrt{3}}{2} \leq \frac{\epsilon}{\hbar} \leq \frac{\sqrt{3}}{3} \quad |\epsilon| \hbar^2 = 1$

$\langle \phi | \phi \rangle = \sum_{i=1}^{\infty} \phi_i \phi_i^*$

Volume operator in $H_3$:

$\langle \phi | \phi \rangle - \langle \phi | \phi \rangle V_n$

$V_n = \frac{3}{h} \varepsilon$ number of 4-orbital valence hole
\[ z = \text{boundary} \quad D^3 \quad \partial z = s^2 \]
\[ S = \text{boundary} \quad D^3 \quad \partial S = S^2 \]

Area of boundary
All Graphs $\mathcal{G} = \{\text{all graphs}\}$

Finite $\iff$ no boundary

$E < 3$ for boundary

{$G$} = $\{\text{all embeddings into 3-up topology}\}$

Counter set
All graphs \( \mathcal{G} = \{ G \} \) where \( G = \sigma \{ \text{all graphs} \} \).

If \( \exists \) has boundary

Graphs can have edges and on boundary

\( G \) is embeddings into \( \exists \) up topology countable set
All graphs \( \mathcal{P} = \{ G \mid \text{all graphs} \} \)

Finite set of \( \mathcal{P}_3 \)

1 if \( \exists \) has boundary

\[ \{ G \} = \{ \text{all embeddings into 3 up to topology count the set} \} \]

Graphs can have edges end on boundary
All graphs $P = \{ G \}$ 
where $G = \{ \text{all graphs} \}$

Finite set of node-set lists $1 \leq \ell \leq \text{has boundary}$

Graphs can have edges end on boundary
\[ V_\mathfrak{f} = \mathfrak{f}^3 \text{ number of 4 or higher valent nodes} \]

\[ \Sigma = \text{boundary} \]

\[ D^3 \quad \Sigma = S^2 \]

Area of boundary \[ \mathcal{A} \geq \mathcal{L} \]

\[ \left| \mathcal{L} \right|^2 = \left| \mathcal{L} \right|^2 \mathfrak{f} \]
$S = \text{boundary } D^3 \rightarrow \varepsilon = S^2$

Area of boundary $A[\varepsilon] = |\mathbf{r} \rangle \langle \mathbf{r}|$

$\mathcal{A} = \varepsilon \# \text{ of punctures}$
All Graphs $P = \{ \text{all graphs} \}$

$G = \{ \text{all graphs} \}$

$\mathcal{G}$ has boundary

$G \times \{ \text{all embeddings} \}$

Into $\mathcal{G}$ up to topology (countable set)

Graphs can have edges and an boundary
\[ V_f = \prod \] number of \( f \) or higher rotation holes

\[ s = \text{boundary} \quad D^3 \quad d \Xi = s^2 \]

Area of boundary \[ A[\Xi] \quad m \geq 1 \]

\[ \eta^2 = \text{the } \# \text{ of punctures} \]
$z = \text{boundary}$
$D^3 \partial z = S^2$

Area of boundary $\hat{A}[z] | \psi > = | \hat{p} > \varphi$

$\frac{\partial}{\partial t} = 2p \rho \text{ # of punctures}$
\[ V = \text{number of 4 and higher valent holes} \]

\[ z = \text{boundary} \]

\[ D^3 \quad d \varepsilon = s^2 \]

\[ \text{Area of boundary} \quad A[\varepsilon] \quad \langle \varepsilon \rangle = 1 \langle \varepsilon \rangle \quad q_{n} \]

\[ q_{p} = \text{Re} \quad \# \text{of punctures} \]
\[ V_{\beta} = \frac{\beta^5}{\pi} \text{ number of } 4\text{-or higher} \text{ valued nodes} \]

\[ \Sigma = \text{boundary} \quad D^3 = \Sigma = S^2 \]

Area of boundary \[ \mathcal{A}[\Sigma] \quad \mathcal{A}[\Sigma] = \mathcal{A}[\beta] \]

History \[ \mathcal{P}_H = \frac{\ell^2}{\pi} \text{ # of punctures} \]

Local dynamics
All Graphs $\mathcal{P} = \{ \text{all graphs} \}$.

Finite set of non-singular
$G \subseteq \mathcal{P}$ has boundary
$\mathcal{B} = \{ G \}$

Graphs can have edges and a boundary

$\Gamma \subseteq \mathcal{P}$ has topology
$\mathcal{\Gamma} = \{ \Gamma \}$

Embedding $\mathcal{\Gamma}$ into a topological
compact set

$G \in \mathcal{\Gamma}$
Local dynamics

History

Area of boundary $A_{\partial \Omega} = 10\pi A_p$

$\Phi = \frac{1}{2} \frac{\partial^2 \phi}{\partial t^2} + U \phi$

$\nabla^2 \phi = \rho$

$\nabla \cdot \phi = 0$

$D_{\Omega} = \frac{\partial}{\partial t}$

$S_{a,b} = 0$

$3 = \sum_{n=0}^{\infty} n^2$

$V_n = \frac{e^{im\theta}}{R}$ number of $4\pi$-flux through faces
\( z = \text{boundary} \quad D^3 \quad d^2 z = S^2 \quad \sum A, \nabla = 0 \)

Area of boundary \( \hat{A} \mid \Gamma \rangle = \hat{1} \langle \hat{\rho} \rangle \)

\( \hat{\rho} = \hat{p} \hat{e} \) of punctures

History

Local dynamics

meas local moves
\[ \nu_{\text{P}} = \nu_{\text{R}} \quad \text{number of 4 or higher valent nodes} \]

\[ \mathcal{S} = \text{boundary} \quad D^3 \quad \partial \mathcal{S} = \mathcal{S}^2 \]

\[ \text{Area of boundary} \quad \hat{A}[\mathcal{S}] |_{\mathcal{P}} = \langle \mathcal{P} | \hat{a}_{\mathcal{P}} \rangle \]

\[ \hat{a}_{\mathcal{P}} = \nu_{\text{P}} \quad \text{at punctures} \]

\[ \mathcal{P} = \mathcal{P}_{\text{R}} \quad \text{history} \]

\[ 3 \quad \text{valent nodes} \]

\[ \text{Local dynamics} \quad \text{moves local} \]

\[ \text{means local} \]

\[ \text{moves} \]
\[ V = \sum \frac{1}{2} \text{number of 4- or higher valent nodes} \]

\[ \mathbb{Z} = \text{boundary} \]

\[ \Delta^3 \cdot \Delta^2 = \mathbf{S}^2 \]

\[ \mathbf{A}[\mathbf{E}] \mid \Gamma \rangle = \mid \mathbf{P} \rangle \Psi_P \]

Area of boundary \[ \mathbf{A}[\mathbf{E}] \mid \Gamma \rangle = \mid \mathbf{P} \rangle \Psi_P \]

\[ G_P = \begin{array}{c} \text{History} \\
\text{3-valent nodes} \\
\text{3-valent nodes} \\
\text{Local dynamics}
\end{array} \]

\[ \text{Local dynamics means local moves} \]
\( z = \text{boundary} \)

\( D^3 \partial z = S^2 \)

\[ \hat{A}[\chi] \left| \Gamma \right> = \hat{R} \left| \varphi \right> \]

\[ \hat{R} \varphi = \hat{R} \varphi \text{ at punctures} \]

3-valent nodes

History

Local dynamics means local moves
\[ V_p = \frac{d^3}{d
u^3} \text{ number of 4 or higher valent nodes} \]

\[ \Sigma \Lambda, V^\gamma = 0 \]

Area of boundary \[ \hat{A} \left[ \Omega \right] | \Gamma > = | \Gamma > \psi \]

\[ \psi_p = \frac{d^2}{d
u^2} \text{ # of punctures} \]

History

3-valent nodes

Local dynamics means local moves

\[ \text{expansion} \]
\[ V = \pi^2 \text{ number of torus holes} \]

\[ z = \text{boundary} \]

\[ D^3 \partial z = S^2 \]

\[ \sum [A, V] = 0 \]

Area of boundary \( \hat{A} [\partial z] | \Gamma > = 1 \hat{\rho} [\partial z] \)

History \( \mathbb{R}^2 \) hot punctures

Local dynamics means local moves

3-vector moves

\[ \hat{\phi} = \mathbb{R}^3 \]
\[ V_F = \ell^3 \text{ number of 4 or higher valent nodes} \]

\[ S = \text{boundary}, \quad D^3 \quad dS = \frac{S^2}{2} \]

\[ \text{Area of boundary} \quad \hat{A}[\epsilon \delta] \gamma = 1^3 \quad \nu_F \]

\[ \nu_F = \ell^2 \frac{\Gamma}{4\pi} \quad \text{of punctures} \]

\[ \nu_F = \ell R \quad \text{of punctures} \]

History

Local dynamics means local moves

3-valent nodes

\[ \Rightarrow \text{expansion} \]
4-valent nodes
4. Valen nodes

Diagram showing two geometric shapes connected by an arrow.
\[ V^n = \mathbb{R}^3 \quad \text{number of 4 or higher valent nodes} \]

\[ 2 = \text{boundary of } D^3 \]
\[ d^2 = S^2 \]
\[ \sum A, \nabla S = 0 \]

Area of boundary \[ \hat{A}^{[3]}|\Gamma\rangle = |\hat{\rho}\rangle \varphi \]

History

Local dynamics

3-valent nodes

\( \text{contraction} \)

\( \text{expansion} \)
All graphs \( P = \{ \text{all graphs} \} \)

\( G = \{ \text{all graphs} \} \)

Finite, if \( \exists \text{ boundary} \)

\[ G \] is embedded into \( \exists \text{ up to topology} \)

compact set

Graphs can have edges, and on boundary
\[ V_\beta = \ell_{P}^{2} \text{ number of } \ell \text{ or higher valent holes} \]

\[ S = \text{boundary} \quad D^3 = 3^3 = S^2 \]

Area of boundary \[ A[E] = |\gamma > = |\ell > \gamma' \]

\[ \gamma' = \ell_{P}^{2} \text{ # of punctures} \]

History

3-valent nodes

Local dynamics means local moves

Contract

Expansion

Not frutal
4-valent nodes

Measurable Ergodic:
If you can go from any input graph to any output graph in a finite number of moves.
4-valent nodes

Markov Ergodic:
In you can go from any input graph to any output graph in a finite hot move.
4-valent nodes

Maximally Ergodic:
If you can go from any input graph to any output graph in a finite # of moves.
\[ V_{P^2} = \frac{1}{P_1} \text{ number of 4ier hyperl valent nodes} \]

\[ S = \text{boundary} \quad D^3 \quad \partial D^3 = S^2 \quad \partial S = 0 \]

Area of boundary \[ \hat{A}[\varepsilon] \mid \gamma \rangle = | \hat{\gamma} \rangle \phi \]

\[ \Phi = \frac{L^2}{P_2} \text{ # of punctures} \]

History

Local dynamics

Contraction

Expansion

Not physical
\[ \mathcal{V}_P = \mathfrak{P}_P \text{ number of 4 or higher valent nodes} \]

\[ \mathcal{S} = \text{boundary} \quad \mathcal{D}^3 \quad \partial \mathcal{E} = \mathcal{S}^2 \]

Area of boundary \[ \mathcal{A} \left[ \mathcal{E} \right] \mid \mathcal{E} \rangle = \mathcal{P} \langle \mathcal{P} \] \[ \mathcal{K} \mathcal{P} = \mathcal{P}_3 \text{ # of punctures} \]

History

Local dynamics means local moves

3-valent nodes

\[ \text{contract} \quad \Rightarrow \quad \text{expansion} \]

\text{Not physical}
$z = \text{boundary} \quad D^3 \quad \partial \Sigma = S^2 \quad \Sigma \partial A, \Sigma \partial B = 0$

Area of boundary $A[\Sigma] \mid \gamma > = | \Phi > \varphi$

$\varphi_H = \frac{L^2}{\ell_p^2} \# \text{of punctures}$

History

3-valent nodes

$\leftrightarrow$ contraction

$\leftrightarrow$ expansion

Local dynamics means local moves

Not physical
\( V_p = \sum \) number of 4+ higher valent nodes

\[ d^3 \; \partial = \partial^2 \]

Area of boundary
\[ A [ \partial ] \mid \Gamma > = | \Gamma > q_p \]

\[ q_p = \frac{r_p^2}{r_p} \text{ # of punctures} \]

History
3-valent nodes

Local dynamics
means local moves

Nex pronic
\[ V_p = k_p \]  
number of 4\textsuperscript{th} order valent nodes

\[ S = \text{boundary}, \quad D^3 \partial D^2 = S^2 \]
\[ \text{Area of boundary} \quad A[\partial \Sigma] | r \rangle = | r \rangle \langle r | \]

\[ g_F = \frac{r^2}{4} \text{ # of punctures} \]

History

3-valent nodes

\[ \xrightarrow{\text{contract}} \]

Local dynamics

\[ \xrightarrow{\text{expansion}} \]

\[ \text{Not analytic} \]

Exchanges
4-valent nodes

Morse Exodice: Let $\gamma$ be a closed curve.
If you can go from any input graph to any output graph in a finite number of moves.
history (3-valent)
\[ \pi_0 \rightarrow \pi_1 \rightarrow \ldots \rightarrow \pi_n \]
each move is
$z = \text{boundary}$

$D^3 \partial z = S^2$

$[A^3 \partial] | \Gamma > = | \Gamma > \psi$

Area of boundary

$\sum \frac{\partial x}{\partial \tau} = 0$

History

$B$-valent nodes

Local dynamics means local moves

Not preserved

$\mathcal{G} \psi = \exp \# \text{of punctures}$

Exchanges
history \( C_3 \)-valent

\( \Phi_0 \rightarrow \Phi_1 \rightarrow \ldots \rightarrow \Phi_n \)

each move is local move
4-valent nodes

Maximal Ergodicity:

If you can go from any input graph to any output graph in a finite number of moves.
4-valent nodes

Non-ergodic:

Is there a graph such that you can go from any input graph to any output graph in a finite set of moves?

Ergodic:

In a finite set of moves...
4. Valant Nodes

Meaning Ergodic: If you can go from any input graph to any output graph in a finite number of moves.
4-valent nodes

Morse Ergodic:
Is you can go from any input graph to any output graph in a finite nodal move...
history (3-valent) \rightarrow \text{foam.}

\Gamma_0 \rightarrow \Gamma_1 \rightarrow \ldots \rightarrow \Gamma_n

each move is local move
history (3-valent) \rightarrow \text{fourth}

Γ_0 \rightarrow \mathcal{A} \rightarrow \ldots \rightarrow \mathcal{F}_n$

each move is local move
history (3-valent) \[ \eta \rightarrow \hat{\eta} \rightarrow \cdots \rightarrow \hat{\eta}_n \]

each move is local move

Dynamics, Absorb, Annihilate, Exchange
history (3-valent) \[ \downarrow \]
\[ \Gamma_0 \Rightarrow \Gamma_1 \Rightarrow \ldots \Rightarrow \Gamma_n \]
each move is local move

Dynamics close, count, actuate

history has an amplitude \[ A(\mathcal{F}) = \prod_{\text{move}} A(\text{move}) \]
4-valent nodes

If you can go from any input graph to any output graph in a finite number of moves.
4-valent nodes

If you can go from any node graph to any other graph in a finite number of moves.

Dynamics:\begin{align*}
    \pi_0 &\to \pi_1 &\to \cdots &\to \pi_n \\
    \text{every move is local move}
\end{align*}

History: \( n \)-valent \( H \) is given by

\[ a(H) = \prod_{\text{vertex}} a(\text{out}) \]
**4-valent nodes**

If you can go from any input graph to any output graph in a finite set moves.

**history (4-valent)**

\[ \xi_0 \to \xi_1 \to \ldots \to \xi_n \]

each move is localwine

**Dynamics**

\[ \text{Above, Actn, Actn} \]

\[ \text{history basis amplitudes} \]

\[ a(\xi) = \prod a(\xi_{\text{actn}}) \]

**Solution amplitude**

\[ \xi \to \xi' \]

\[ a_{\xi \to \xi'} = \sum_{\xi''} a(\xi) \]

\[ \xi'' \to \xi' \]
Consider the moves $P, Q$. 
Consider the moves $P, Q$

$P > Q$ if $P$ acts on a subgraph altered by $Q$
4-valent nodes

Morera-Euclidean: Is there a configuration
if you can go from any input graph to any output graph
in a finite # of moves.
4-valent nodes

Metric Euclidean: but requires a fundamental group

If you can go from any input graph to any output graph in a finite number of moves.
Consider two moves $P, Q$.

If $P$ acts on a subgraph altered by $Q$, then $P > Q$. 

If $P$ acts on a subgraph altered by $Q$.
Consider the moves $P$, $Q$.

$P \rightarrow Q$ if $P$ occurs on a subgraph adjacent by $Q$.

Par if not "cause!"
4. Valant Nodes

Morse-Engel: There exists an 
if you can go from any input 
graph to any output graph 
in a finite number of moves.
4-valent nodes

If you can go from any input graph to any output graph in a finite number of steps.

Example diagrams:
- $\text{Input}$ to $\text{Output}$
- $\text{Intermediate steps}$
4. Valant Nodes

Markov Ergodic: In a finite number of moves, you can go from any input graph to any output graph.
Consider two moves $P, Q$

$P \geq Q$, if $P$ acts on a subgraph altered by $Q$

Par, if not "cause" moves are a Partially ordered set
Consider the moves $P, Q$.

$P \succ Q$ if $P$ acts on a subgraph altered by $Q$.

$P \parallel R$ if not "causal".

Moves are a partially ordered set "causal set".
history (re-entrant) \[ \mathcal{H} \]

\[ r_0 \rightarrow r_1 \rightarrow \ldots \rightarrow r_{\text{end}} \]

each move is local

Dynamics: q, q, q, q, q, q, q, q

\[ \text{E}(\mathcal{A}, \mathcal{B}) = \prod_{\text{move}} \text{E}(\mathcal{A}, \mathcal{B}) \]

\begin{align*}
\text{Evolution amplitude} & \quad r \rightarrow r' = E(r, r') \\
& = \sum_{\alpha} a(\alpha) \\
& \quad |r \rightarrow r'|
\end{align*}

\[ E(r, r', \mathcal{A}) = \sum_{\alpha} a(\alpha(\mathcal{A})) \]
Spatial Topology

$S^3 = \{G\}$

$G = \{\text{all graphs\}}$

$\exists \varepsilon$ has boundary

$\{G\} = \{\text{all embeddings of } G \text{ into } \Sigma \text{ up to topology}\}$

Graphs can have edges and an boundary
more nodes... more moves
more nodes... more moves
more nodes... more moves
more nodes... more moves
more nodes... more moves
spins $SU(2)$ rep $j = \frac{1}{2}, \frac{3}{2}, \ldots$
spins $SU(2)$ rep \( j = \frac{1}{2}, \frac{3}{2}, \cdots \)

\[ r_j = \sqrt{2j+1} \]
spins \quad SU(2) \ \text{rep} \quad j = \frac{1}{2}, \frac{3}{2}, \ldots

don\text{ition} \quad r_j \oplus r_k = j \oplus k \quad r_j = V_{2j+1}

multiplying

complex \quad \text{vector spin}
spins  $SU(2)$ rep  \( j = \frac{1}{2}, \frac{3}{2}, \ldots \)

addition  \( r_j \oplus r_k = j \oplus k \)  \( r_j = V_{2j + 1} \)

multiplying  \( j \otimes k \)

complex vector spin
Spins $SU(2)$ rep $j = \frac{1}{2}, \frac{3}{2}, \cdots$

Addition $j_1 \otimes j_2 = \sum_k j_k$ $r_j = \sqrt{2j+1}$

Multiplication $j \otimes k = \sum_l \begin{pmatrix} l \end{pmatrix}_l$ $l \leq |j-k|, l \leq j+k$
spins \quad SU(2) \quad rep \quad j = \frac{1}{2}, 1, \frac{3}{2}, \ldots

addition \quad r_j \oplus r_k = \sum \mathbb{C} \quad r_j = \sqrt{2j + 1}

multiplication \quad j \otimes k = \sum_l \mathbb{C} \quad \text{commutative}

complex vector spin
Spins $SU(2)$ rep $j = \frac{1}{2}, \frac{3}{2}, \ldots$ complex vector spin

Addition: $r_j \oplus r_k = j \oplus k$, $j' = \sqrt{2j + 1}$

Multiplication: $j \otimes k = \sum \binom{j + k}{j-k} l^2 l \sim$ commutative

Is $j \otimes k \otimes l \in \mathfrak{so}$ rep
Spins \text{ } SU(2) \text{ rep } j = \frac{1}{2}, \frac{3}{2}, \ldots

Addition: \quad r_j \oplus r_k = r_{j+k} \quad r_j = \sqrt{2j+1}

Multiplication: \quad r_j \otimes r_k = \sum_{l=j-k}^{j+k} r_l \quad \text{commutative}

r_j \otimes r_k \oplus r_j \otimes r_k \text{ is O rep}

Facts about SU(2): Yes - unique

Triangle relation: \quad r \leq j+k

No

Local dynamics

Hist
spins
$SU(2)$ rep $j = \frac{1}{2}, \frac{3}{2}, \ldots$

addition $r_j \oplus r_k = j \oplus k$

multiplication $r_j \otimes r_k = \pi \otimes \lambda$ commutative

$\pi \otimes \lambda \otimes \lambda \\ \otimes r_k$

facts about $SU(2)$ $\otimes \lambda \otimes \lambda$

no $\pi \otimes \lambda \otimes \lambda$

$\lambda \otimes r_k$

Diagram: $\lambda \otimes r_k$
addition \( r \odot r, \otimes k \Rightarrow r^* = \sqrt{2k+1} \)

multiplication \( \psi \otimes k = \sum l \) commutative

Is \( \psi \otimes k \otimes l \Rightarrow \text{rep} \not\Rightarrow \mu k + l \text{ even} \)

Facts about \( SU(2) \) \text{ is unique} triangle relation \( e3 \leq 1 + k \)

spin network (Penrose 67)
Addition: $r_1 \otimes r_2 \cdot k \cdot Y_r = V_{2\ell + 1}$

Multiplication: $Y \otimes k = \sum l \otimes Y_{l \otimes k}$

Commutative

Is $\otimes K \otimes l \Rightarrow$ rep $\otimes l \otimes k$ even

Facts about $SU(\ell)$

Yes - unique triangle relation

No

Good triplet

Spin networks (Penrose 67)

Graph $I$ where edges $I \leftrightarrow \bar{I}$ $Y_{I}$ all good
addition \( r \otimes k \cdot j \otimes k \) \( y \cdot y = \sqrt{y(y+1)} \)

multiplication \( j \otimes k = \sum \varepsilon \ell \left( \begin{array}{c} \ell \varepsilon(\ell+1) \\ell+1 \varepsilon \end{array} \right) \) commuting

Is \( j \otimes k \otimes \ell \) a rep \( \chi \) even \( \chi k + k + k \) even

Facts about \( SU(2) \) Yes - unique triangle relation \( \varepsilon \)

Good triplet \( k \leq \ell \)

spin network (Penrose 67) (3 velocity)

graph \( G \) where edges \( \rightarrow \) \( \chi \) \( \chi \) all good

Spinnet \( \rightarrow \) \( \chi \) all good
$|\psi\rangle = \sum a_n |\psi_n\rangle \quad \sum_{n} |a_n|^2 = 1$

$\langle \psi|\psi\rangle = \sum_{n,m} c_{nm} |\psi_n\rangle \langle \psi_m| = 1$

Volume operator in $\mathbb{R}^3$: $\mathbf{V}_\mathbf{r} = |\mathbf{r}|^2$ number of 4th barycentric nodes
spin-network (Penrose 62) (3-valent)

graph $\Gamma$ whose edges $\Gamma_{ij}$ yield all good

spinnet embedding $\tilde{\Gamma}$ into $\mathbb{P}^3$
spin-network (Penrose '62) (3-valent)

graph \( \Gamma \) whose edges \( 1 \rightarrow 2 \)

spinnet embedding

\[ \text{Inv(\text{Penrose})} \rightarrow 0 \]
spin-network (Penrose '67) (3-valent)

graph $\Gamma$ whose edges $i\rightarrow j$ $Y_k$ all good

Spinnet embedding $\tilde{\Gamma}$ $\Rightarrow$

$\text{Inv}(\text{graph}) \Rightarrow \text{graph} \Rightarrow 0$
spin-network (Penrose 62) (3-valent)

graph \( \Gamma \) whose edges \( \gamma \) and \( \gamma' \) all good

spinnet embedding \( \hat{\Gamma} \)

\[ \text{Inv}(\mathfrak{so}(3,\mathbb{C}) \otimes \mathfrak{so}(2m)) \rightarrow \text{defines} \]
\[ \mathcal{H} = \text{ov.lational basis} \]

\[ | \Psi \rangle = \sum_{\alpha \beta} c_{\alpha \beta} | \alpha \rangle | \beta \rangle \]

\[ \langle \Psi | \Psi \rangle = \sum_{\alpha \beta} \langle \alpha | \beta \rangle \langle \beta | \alpha \rangle = 1 \]

\[ \langle \Psi | \Psi \rangle = \sum_{\alpha \beta} c_{\alpha \beta}^* c_{\beta \alpha} \]

A general spin network graph \( \Gamma \)

\[ \chi \in \mathcal{H} \]
\[ |\Psi\rangle^2 = \sum_{m,n} |m\rangle \langle n| \sum_{m,n} |m\rangle \langle n| = 1 \]

\[ \langle \Psi | \Psi \rangle = \sum_{m,n} |m\rangle \langle n| \sum_{m,n} |m\rangle \langle n| = 1 \]

A general spinnet graph

\[ X \in |\Psi\rangle \]
A general spinnet graph $\Gamma$

$X \Rightarrow Y$

$X \Rightarrow Y$

History

Local dynamics

Bivalent nodes

 contraction

expansion

Not proved

Exchanges
A general spinnet graph \( \Gamma \).

\[
\begin{align*}
\langle \partial \partial \partial \partial \rangle &= \mathbb{S} \mathbb{S} \mathbb{S} \\
\text{not O-dim}
\end{align*}
\]

\[
X = X \quad 1 \circ 1 \rightarrow 0 \\
1 \circ 1 \rightarrow 1 \circ 1
\]

History

Local dynamics

Bivalent graphs

contraction

expansion

Not Fusion

Exchanges
morally
moral \texttt{embedd} \\ \texttt{topological embedding} \\ \texttt{diffeomorphism embedding}
morally $orall$

topological embedding

by proposition

diffeomorphic embedding

quantum GR in 3+1 in $\mathbb{S}$

LOST worm
spin-network (Penrose 62) (3-valent)
graph $\Gamma$ whose edges $1 \rightarrow i$ $\forall y_k$ all good
spinnet embedding $\tilde{\Gamma} \rightarrow \mathbb{D}^3$ defines vector space
$\mathrm{Inv}(\mathbb{D}^3)$
morally

topological embedding

by precise

diffeomorphic
embedding

quantum GR in 3+1 in \( \Xi \)

LOST (worm)
History (3-vertex) \( \mathcal{F} \)

\[ \mathcal{F}_0 \rightarrow \mathcal{F}_1 \rightarrow \ldots \rightarrow \mathcal{F}_{n-1} \]

Each move is local

**Dynamics**

\[ a_{\text{interaction}}, a_{\text{source}}, a_{\text{scatter}} \]

History has an amplitude

\[ a(\mathcal{F}) = \prod_{\text{move}} a(\text{move}) \]

Evolution amplitude

\[ \mathcal{F} \rightarrow \mathcal{F}' = E(\mathcal{F}, \mathcal{F}') \]

\[ = \sum_{\text{final}} a(\mathcal{F}) \]

\[ E(\mathcal{F}, \mathcal{F}') = \sum_{\text{final}} \frac{a(\mathcal{F})}{a(\mathcal{F}')} \]
spin-network (Penrose 62) (3 valent)

graph $\Gamma$ whose edges $1 \rightarrow \gamma$ \( \gamma \in \text{all good} \)

Spin-net embedding $\hat{\Gamma} \rightarrow$ defines vector space

\[ \text{Inv}(\mathfrak{g}) \rightarrow 0 \]
history (3-valent) \neq 
\hat{r}_0 \rightarrow \hat{r}_1 \rightarrow \ldots \hat{r}_{n-1}

each move is local

Dynamics \quad a(\text{move}) = a_{\text{move}}, a_{\text{move}}, a_{\text{move}}

\text{history has an amplitude} \quad a(E) = \prod_{\text{moves}} a(\text{move})

\text{Evolution amplitude} \quad \hat{r} \rightarrow \hat{r'} = E(\hat{r}, \hat{r'}) = \sum_{\phi} a(E)