Abstract: This is an introduction to background independent quantum theories of gravity, with a focus on loop quantum gravity and related approaches.

Basic texts:

- Quantum Gravity, by Carlo Rovelli, Cambridge University Press 2005
- Quantum gravity with a positive cosmological constant, Lee Smolin, hep-th/0209079
- Invitation to loop quantum gravity, Lee Smolin, hep-th/0408048
- Gauge fields, knots and gravity, JC Baez, JP Muniain

Prerequisites:

- undergraduate quantum mechanics
- basics of classical gauge field theories
- basic general relativity
- hamiltonian and lagrangian mechanics
- basics of lie algebras
- Chern-Simons Theory
- Back to BF
- General Relativity 4/5/6 in 0261079
• Chern-Simons Theory
• Back to BF
• General Relativity 4/5/6 in 0201077
3-manifold compact $M \quad A \in G \rightarrow SU(2)$
3-manifold compact $M$ $A_a \in G \rightarrow SU(2)$

$F_\mu^\nu = \partial \! \! \! / A_\mu + [A_\mu, A_\nu]_{\text{gf}}$
3-manifold compact $M$

$A_\alpha \in G \to SU(2)$

$F_{\mu \nu} \propto \partial A_\mu + A_\mu A_\nu$
3-manifold compact $M$

general Lie Algebra $\gamma^a$

$\left[ \gamma^a, \gamma^b \right] = i f^{abc} \gamma^c$

$A_a \in \mathfrak{g} \rightarrow \mathfrak{su}(2)$

$F_{ab} = dA^a + A^a \wedge A^b$
3-manifold compact $M$

general Lie Algebra $\mathfrak{g}$

$[\mathfrak{g}, \mathfrak{g}] = \mathfrak{g}$

$A_a \in G \rightarrow SU(2)$

$F_{\mu\nu} = dA^\mu + A^\mu A^\nu$

$A_\mu = A^I_\mu \gamma^I$
3-manifold compact $\mathcal{M}$

general Lie Algebra $\mathfrak{g}$

$[\gamma_i, \gamma_j] = \delta_{ij} \gamma_k$

$A_a \in G \rightarrow SU(2)$

$F_{ab} = \partial A^a + A^a A^b$

$s =$

$A_a = A_i^I \gamma_I$
3-manifold compact $M$

Lie Algebra $\mathfrak{g}$

$[\gamma^i, \gamma^j] = \delta^{ij} \gamma^k$

$A_a \in G \Rightarrow su(2)$

$L_{a\mu} = \partial A^\mu + A^\nu \gamma^\mu A_{\nu}$

$A_a = \Lambda^I a \gamma_I$

$S = \int \frac{3}{2} \epsilon_{abc} \omega_a \delta_{bc}$
s-manifold compact $M$ $A_\alpha \in \mathfrak{g} \Rightarrow \mathfrak{su}(2)$

$[I^I, I^J] = f_{IJ}^K I^K$ $A_{a \alpha} = A^I_{\alpha} I^I$ $A_\alpha = A^I_{\alpha} I^I$

$s = \sum_{s=0} Y_{s\alpha} = \sum_{s=0} Y_{s\alpha} = A^I_{\alpha} \left( A^I_{\alpha} + \frac{2}{3} s \sigma \right) A_{n\alpha} \lambda_\beta n_\beta$
3-manifold compact $\mathcal{M}$

general Lie Algebra $\gamma$

\[
[\gamma^I, \gamma^J] = \frac{2i}{g} \epsilon^{IJK} \gamma^K
\]

$S = \sum s_{\text{con}} = \sum Y_{CS}$

$Y_{CS} = A^I_{\alpha\beta} (A^J_{\gamma\delta} + \frac{2}{3} g^{IJK} \epsilon_{\alpha\beta\gamma\delta}) A^\alpha A^\beta A^\gamma A^\delta$
3-manifold compact $M$

\[ A_a \in G \rightarrow SU(2) \]

\[ \gamma^I \]

\[ [\gamma^I, \gamma^J] = \frac{1}{2} \epsilon^{IJK} \gamma^K \]

\[ A^a = A^I_a \gamma^I \]

\[ S = \int_{\text{3-conn}} = \int_Y \alpha \]

\[ \chi_v = A^I_a \left( A^I_a + \frac{2}{3} \epsilon^{IJK} A^J_a A^K_a \right) \]

\[ \frac{\delta S}{\delta A^I_a} \]
$\Gamma$-manifold compact $\mathcal{M}$

\[
\Gamma \Gamma = \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma \gamma 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$s$-manifold compact $M$ \( A_a \in G \to SU(2) \)

\[ F_{\mu\nu} = \partial A^\mu + A^\nu A^\mu \]

\[ \left[ \gamma^I, \gamma^J \right] = \delta^{IJ} \gamma^k \]

\[ A^I = A^I_\alpha \gamma^\alpha \]

\[ \text{Chern-Simons} \]

\[ S = \int_{\Sigma} \left( A^\mu \wedge \left( \partial_\mu A^\nu + \frac{2}{3} F^{\nu\rho\sigma} A_\rho A_\sigma \right) + \frac{1}{2} \ast F^{\mu\nu} \right) \]

\[ \frac{\delta S}{\delta A^\mu_\alpha} = \partial_\mu F_{\alpha c} = 0 \quad \therefore F_{\mu\nu} = 0 \]
$\text{3-manifold compact } M$

$A_a \in G \rightarrow SU(2)$

$F_{\mu\nu} = dA^\mu + A^\mu A^\nu_{\text{Lie}}$

$[\gamma^I, \gamma^J] = \delta^{IJ} \gamma^K$

$A_\mu = \Lambda^a \gamma^a \gamma^\mu$

$S = \int_{t=0}^\infty dt \, \text{exp} \left( \int Y_{\text{CS}} \right) \quad Y_{\text{CS}} = \Lambda^a \Lambda^a + \frac{2}{3} \int \frac{1}{2} \epsilon^{\mu\nu\rho} A_\mu A_\nu F_{\rho\sigma} F_{\mu\nu} = C$

$F_{\mu\nu} - \frac{2}{3} \Lambda^a F_{a\mu} \Lambda^a \gamma^\mu = 0$

=$ \Lambda^a F^a + \frac{1}{3} \epsilon^{\mu\nu\rho} A_\mu A_\nu A_\rho$

$\frac{\delta S}{\delta A^a_\mu} = \epsilon^{\mu\nu\rho} F_{\nu\rho} = C$
3-manifold compact $M$

general Lie algebra $\gamma_i$

$[\gamma_i, \gamma_j] = \delta_{ij} \gamma_k$

$A_4 = A_9 \gamma_1$

$S = \sum_{i=1}^{3} Y_{C_5} = A_5 A_5 + \frac{2}{3} s_{ij} = A_5 A_5 A_5$

$F_{\mu \nu} = 0$

$F_{\mu \nu} = \frac{1}{3} F_{\mu \nu} A_5 A_5 A_5$

$A_5 \in G \rightarrow SU(2)$

$F_{\mu \nu} = dA_5 + \frac{2}{3} A_5 A_5 A_5$
$S$-manifold compact $M$

$A_a \in G \to SU(2)$

$[T^i, T^j] = f^{ijk} T^k$

$A_\mu^I = A^I_{\mu I}$

$S = S Y_{cs} = A^I_{\mu I}(A^I_{\mu I} + \frac{2}{3} g f^{IJK} A^J_{\mu K} A^K_{\mu I})$

$\sum A^c_{\mu c} = \sum F_{\mu c}^I = 0$

$A^I_{\mu I} F^I_{\mu I} = 3$
$\text{3-manifold compact } M$

\[ A_\alpha \in G \rightarrow \mathfrak{g}(2) \]

\[ F_{\mu \nu} = d A^\mu + A^\rho A^\mu A_{\rho \nu} \]

\[ [\gamma^I, \gamma^J] = \frac{1}{2} i \epsilon^{IJK} \gamma^K \]

\[ A_4 = A_4 \gamma^I \]

\[ S = \sum_\text{conf} = \sum \gamma_{Cs} \quad \gamma_{Cs} = A^I \mathcal{A}_{I} + \frac{2}{3} \mathcal{F}_{IJ} \mathcal{F}^{IJ} = A_1 A_2 A_3 A_4 \]

\[ \frac{\delta S}{\delta A^I_4} = \sum_\text{conf} F^I_{\mu \nu} = 0 \]

\[ \mathcal{F}^{I} = \mathcal{A}^{I}_{12} \mathcal{A}_{13} \mathcal{A}_{14} \]
3-manifold compact $\mathcal{M}$ with general Lie algebra $\mathfrak{g}$

$$[\mathfrak{g}, \mathfrak{g}] = \mathfrak{g}$$

$$F_\mu^\nu = dA^\mu + A^\nu A^\mu$$

$$A^\mu = A^\nu_i \gamma_i$$

$$S = S_{\text{3-geom}} = S_{\text{YM}}$$

$$\text{YM} = A^\nu_i \frac{1}{3} F_{\mu\nu}^i 3 = 0$$

$$\frac{\delta S}{\delta A^\nu_i} = A^\nu_i F_{\mu\nu}^i$$

$$S_{\text{YM}} = \frac{1}{2} \nabla^2 F_{\mu\nu}^i$$

$$\text{YM} = \frac{1}{2} \nabla^2 A^\nu_i$$
\[ z_{i_1 i_2 \ldots i_n} F_{a_1 a_2} \]
$3$-manifold compact $\mathcal{M}$

general Lie $\text{Alg}_{\mathfrak{su}(2)}$

$[\mathfrak{g}, \mathfrak{g}] = \mathfrak{g}$

$F_{\mu \nu} = \partial \mathfrak{A} + \mathfrak{A} \wedge \mathfrak{A}$

$A_0 = \mathfrak{A} \wedge \mathfrak{I}$

$S = \int \text{vol} = \int \text{Y}_{\text{cs}}$

$\mathfrak{Y}_{\text{cs}} = \mathfrak{A} \wedge \mathfrak{A} + \frac{2}{3} \mathfrak{F} \wedge \mathfrak{F} = \mathfrak{A} \wedge \mathfrak{A} \wedge \mathfrak{A}$

$S_\mathfrak{A} = 0$

small jet $\mathfrak{Y}_{\mathfrak{A}_0} = \mathfrak{A}_0 \wedge \mathfrak{I}$

$\mathfrak{F}_{\mu \nu} = 2 \mathfrak{F} \wedge \mathfrak{F}$

$S \mathfrak{Y}_{\mathfrak{cs}} = d(2\text{-form})$, check $S \mathfrak{Y}_{\mathfrak{cs}} = 0$
Small gaugino

\[ \delta N_i^c = \alpha c X^c \]

\[ \delta E_i^c = \lambda^c \lambda_i^c \epsilon_i^c \]

\[ \delta Y_{iij} = d(2 \text{-form}) \]

CHECK

\[ \delta \sum Y_{iij} = 0 \]

\[ SU(2) \]

\[ M = 5^2 \]
Small gauge terms $\delta A^a_i = D^i_a X$  
$\delta F_{ij} = 3 \epsilon^{ijk} \lambda_m F_{sk}$

$\delta Y_{id} = d(2\text{-form})$ CHECK $\delta \delta Y_{id} = 0$

SU(2)  
M = $\sum^2$
Small insertions: $\delta A_4^a = D_a X^i$
$\delta F_{ac} = \epsilon^{ijk} A_{ji} F_{ak}$
$\delta Y_{ab} = d(2\text{-form})$ CHECK $\delta \delta Y_{ab} = 0$

$SU(2) \cong S^3$ $M = S^2$
Small jactrons \( S A_i^c = D x^I \)

\( \delta F_{a i} = 2 j m \lambda_{\alpha} F_{i \alpha} \)

\( \delta \chi_{ab} = d(2\text{-form}) \text{ CHECK } \delta \delta \chi_{ab} = 0 \)

\[ SU(2) \cong S^3 \quad M = S^3 \]
Small gauge terms

\[ S_{\text{A}} = \mathcal{L} \mathcal{X} \]
\[ S_{\text{F}} = \mathcal{L}_{\mathcal{X}} \mathcal{A} \]

\[ \delta Y_{\text{A}} = d(2\text{-form}) \text{ CHECK } \delta S_{Y_{\text{A}}} = 0 \]

\[ \text{su}(2) \cong S^3 \]
\[ M = S^3 \]

LARGE GAUGE TRANS.
Small symmetries: $\delta A_i = D_i^X$  
$\delta F_{ij} = \epsilon^{ijk} \lambda_m F_{mk}$  
$\delta Y_{i5} = d(2\text{-form})$  
Check: $\delta \delta Y_{i5} = 0$

$SU(2) \simeq \mathbb{S}^3$  
$M = S^2$

Large gauge truncation $g(x)$
\[ \text{Small symmetry:} \quad \delta A_a = 2 \alpha X^a \quad \delta F_{ab} = 2 \epsilon_{mnp} A_m \wedge F_{np} \]
\[ \delta Y_{ab} = d(2\text{-form}) \quad \text{CHECK} \quad \delta S Y_{ab} = 0 \]

\[ \text{Grav.} \quad SU(2) \times S^3 \quad M = S^2 \]

Large gauge trans. \( g(x) \): \( A_a \to g'(iA+A)g \)
\[ \begin{align*}
  & \text{small junctions } S\mathcal{A}_k = Q \chi^T \\
  & \delta F_\perp = \epsilon^{\mu \nu} \chi A_\mu F_\perp \\
  & \delta Y_{k_s} = \mu(2-\text{form}) \text{ check } \delta S Y_{k_s} = 0
\end{align*} \]

\[ \text{ SU}(2) \times S^3 \quad M = S^3 \]

Large gauge trans. \( g(x) \ni A_\mu \rightarrow \frac{g}{2} (\mu + A) g \]
\begin{align*}
\text{Small gauge term: } & S A^I = \{a \chi^I \} \\
\text{Check: } & \delta S Y_{\alpha_5} = 0 \\
\text{Lattice gauge term: } & \mathcal{g}(x) \colon A_a \to \mathcal{g}(1 + A)_a \mathcal{g}
\end{align*}
small spectra, $S\Delta^2 = L_i \chi^T$

$\delta Y_{ab} = \delta (2-\text{form})$

Check $\delta SY_{15} = 0$

$\text{SU}(2) \times S^5$

$M = S^3$

Large gauge transit $g(x) : A \rightarrow g'i(A)g$

$W \rightarrow W_{\text{fin}}$
Small junctions $SA^a = x d^a T$

$\delta F^a_{\mu} = e^{i \mu} \lambda_{\mu} F^a_{\mu}$

$\delta Y_{\mathbf{1}} = d(2 \text{- form})$ CHECK $\delta \delta Y_{\mathbf{1}} = 0$

$SU(2) \times S^3 \quad M = S^3$

LARGE GAUGE TRANS $g(u) : A_\mu \rightarrow g^{-1}(u + A_\mu) g$

$w$
\[ S = \int \frac{1}{Z} e^{-S} \]

\[ S_A = 0 \]

\[ \frac{\delta S}{\delta A^a} = \epsilon^{abc} F_{bc} \]

\[ S_Y = \lambda^2 A^a \]

\[ \delta F^2 = \epsilon_{abc} A^a \]

\[ \delta S_Y = \lambda^2 A^a \]

\[ \delta S_Y = 0 \]

\[ W = \text{Large gauge transformation} \]

\[ A_a = \tilde{g} (a + A_a) g \]
small symmetric \[ S^A = 2A^X \]

\[ \delta Y_{\lambda} = d(2-form) \quad \text{CHECK} \quad \sum \delta Y_{\lambda} = 0 \]

\[ \text{SU}(2) \times S^5 \quad M = S^3 \]

Large gauge transit \[ g(x) : A_\mu \rightarrow g^{-1}(\mu + 1)g \]

\[ S_{15} \rightarrow S_{15s} \]

\[ W = \text{Wimper} \rightarrow S^3 \]
Small gauged \( S^3 \): \( S_{\mu} = 2\lambda X^\mu \)
\( F_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} A^\rho F^{\sigma} \)
\( \delta Y_{\alpha} = d(2\text{-form}) \text{ CHECK } \int \delta Y_{\alpha} = 0 \)

\[ SU(2) \times S^3 \quad M = S^3 \]

Large gauge trans. \( g(x) \) \( A_\mu \rightarrow g^{-1}(\mu + \lambda) g \)

\( S_{T_3} \rightarrow S_{T_3} + 8\pi^2 W \)

\[ W = \text{wilpsychism?} \]
Small gauge theory, \( SA_n = \mathcal{X} \)

\[ \delta Y_{15} = \mu(2-\text{form}) \quad \text{CHECK} \quad \delta \sum Y_{15} = 0 \]

\( SU(2) \otimes S^3 \quad M = S^3 \)

Large gauge transformation \( g(\nu) \); \( A_n \rightarrow \vec{g}(1+A), g \)

\[ \vec{
abla} \rightarrow \vec{\nabla} + 8\pi^2 W \]

* ODD Parity \( x \rightarrow -x \)
Small gauge transformation, \( S \phi_i = D \phi^x \)

\[ \delta Y_{\alpha} = \alpha (2 \text{- form}) \text{ CHECK: } \delta \sum Y_{\alpha} = 0 \]

\( SU(2) \times S^3 \quad M = S^3 \)

Large gauge transformation, \( \Omega(x) \):

\[ A_{\alpha} \rightarrow \tilde{g}^{-1}(u + A) \tilde{g} \]

\[ S R_{\alpha} \rightarrow S R_{\alpha} + 8 \pi^2 W \]

**ODD Parity**

\[ x \rightarrow -x \]

\[ A_{\alpha} \rightarrow -A_{\alpha} \]
Small symmetries $SA_4^L = \mathbb{Z}_2 \times \mathbb{Z}_2$

$\delta Y_{15} = d(2-\text{term})$  **CHECK** $\int \delta Y_{15} = 0$

$\text{grd: } SU(2) \times S^3 \quad M = S^3$

Large gauge trans. $g(x): A_\mu \rightarrow g^{-1}(\mu + A_\mu)g$

$\begin{aligned}
\sum_{\text{odd parity}} & \rightarrow - \sum_{\text{odd parity}} \\
A_\mu & \rightarrow -A_\mu \\
S_{\text{tr}} & \rightarrow -S_{\text{tr}}
\end{aligned}$
\[ \sum \text{ something} \]
\[ 8 \gamma_{ss} = d(2 \text{-form}) \quad \text{CHECK} \quad \oint \gamma_{ss} = 0 \]

\[ \text{SU}(2) \times S^3 \quad M = S^3 \]

Large gauge tran. \( \text{g.c.} \): \quad \Lambda_n \rightarrow \overline{\Lambda}'(\Lambda + A), \quad g \]

\[ \gamma_{ss} \rightarrow \gamma_{ss} + 8 \pi^2 \text{w} \]

DD parity \quad x \rightarrow -x \quad \Lambda_n \rightarrow -\Lambda_n \quad S_f \rightarrow -S_f \]

\[ W = \text{Wilson line} \]

\[ S^3 \]
\[ \sum_{a_1 \cdots a_n} b_{a_1 \cdots a_n} F_{a_1, \cdots, a_n} \]

\[ T_L \quad \cdots \quad T_R \]
\[ <T[\theta]> = \sum_{\text{dim}(A)} e^{\frac{i KS^{CS}}{2 \pi}} \]
\[ T_L \quad \quad \quad T_R \]

\[ \langle T[A] \rangle = \sum_{\text{all } A} e^{\frac{ikS_c}{2\pi}} T[0,A] \]
\[ T_L \quad \quad \quad \quad \quad T_R \]

\[ \langle T[A] \rangle = \sum_{a_1 \ldots a_n} F_{a_1 \ldots a_n} \quad e^{\frac{ik \cdot S \cdot c_5}{2 \pi}} \quad T[O_{1,1}] \quad \text{with} \]

\[ \text{Infer related equations or expressions.} \]
\[ T_L \quad T_R \]

\[ \langle T[0] \rangle = \sum_{a_1, \ldots, a_n} T_{a_1, \ldots, a_n} F_{a_1, \ldots, a_n} \]

\[ e^{\frac{\pm 1}{2} \pi} \]

\[ T[0, H] \leftrightarrow (\psi)_{W[y]} \]
\[ T_L \bigcirc \bigcirc \quad T_R \bigcirc \bigcirc \]

\[ \langle \tau \rangle = \sum_{\text{dim}(A)} e^{\frac{i \kappa S c_5}{2 \pi}} T[\vartheta, \bar{h}] = (\text{K} \times \text{K}) \text{W}[Y] \]
\[ \text{manifold compact } M \quad A_0 \in C^* SU(2) \]
\[ F_{\alpha}^2 = \partial A_{\alpha} + \lambda A_{\alpha} \]

\[ S = \sum \text{ terms} = \sum Y_{cs} \quad Y_{cs} = A_{\alpha} A_{\alpha} + \frac{2}{3} F_{\alpha}^2 \quad \text{max} \quad \text{terms} \]

\[ \frac{dS}{dA_{\alpha}} = 2 F_{\alpha} \quad F_{\alpha} = 0 \]

\[ \text{small argument } S A_{\alpha} = \sum A_{\alpha} \quad g F_{\alpha} = \sum \lambda_{\alpha} F_{\alpha} \]

\[ S Y_{cs} = \frac{1}{2} \text{ (terms) } \quad \text{check } \quad \sum S Y_{cs} = 0 \]

\[ \text{large gauge terms } g(v) = A_0 \quad g(v) = g(1 + A) \quad g \]

\[ S Y_{cs} \rightarrow S Y_{cs} + 8 \pi^2 W \]

\[ W \rightarrow \text{ wavy line} \]
$T_L \bigcirc \bigcirc T_R \bigcirc \bigcirc$

\[
\langle T[\phi] \rangle = \sum_{\Delta(A)} e^{i S_{\phi}} T[O_{\phi}]^{\text{inv}} \omega[Y]
\]
\[ M = \sum x^2 \]
\[ M = \sum_{i}^{(E)} R \quad \exists \text{compact} \]

[Diagram of a compact set]
\[ M = \mathbb{S}^2 \times \mathbb{R} \]

\[ S = \text{Sdt} \, \text{Sin} \]

\[ S = \text{Sdt} \, \text{Sin} \]

\[ \exists \text{cont} \]

\[ \exists \]
\[ \mathcal{M} = \mathcal{L}^2 \times \mathbb{R} \; \; \mathcal{S} \; \text{compact} \]

\[ \mathcal{S} = \text{Set} \; \mathcal{S} \]

\[ n \in \mathcal{S} \]
\( \text{M} = \sum \alpha^2 \times R \)

\( S = \sum \text{area} \times \text{height} \)

\( \alpha = 0.112 \)

\( \theta \)
\[ M = \sum_i^2 S \times \mathcal{R} \exists \text{compact} \]

\[ S = \sum_{i=1}^{3} 4 \]
$M = \sum^{2n} x \in \mathbb{R} \setminus \mathbb{Z}$

$s = \sum_{i=1}^{n} \prod_{j=1}^{m} [A_i \cdot A_j]$
\[ M = \sum_{\mathcal{E}} \mathcal{V} \times R \quad \exists \text{ compact} \]

\[ S = \sum \mathcal{S} \mathcal{S} \psi \mathcal{V} \mathcal{V} [A, \phi, A] \]
\[ M = \sum_{i \in \mathbb{R}} \mathbb{E} \text{ compact} \]

\[ S = \text{set} \sum_{i \in \mathbb{R}} \mathbb{E} \left[ \mathbb{A}_1 \mathbb{A}_2 + \mathbb{A}_3 \right] \]
\[ M = \sum_{i} \epsilon_i e_i \otimes R \quad \exists \text{ compact} \]

\[ S = \sum_{\mu} S_{\mu} \epsilon_{\mu} \left[ A_{\mu}^1 \otimes A_{\mu}^2 + A_{\mu}^2 \otimes \epsilon_{\mu} \right] \]

\[ a = 0, 1, 2 \]

\[ q_{0, 1, 2} \]
\[ M = \sum_{i} \varepsilon_{i} R \]

\[ S = \int d\tau \sum_{i} \varepsilon_{i} \left[ A_{i} \phi A_{i}^{\dagger} + A_{i}^{\dagger} F_{ij} A_{j} \right] \]

\[ a = 0, 1, 2 \]

\[ \tau = 0, 1, 2 \]
\[ M = \sum_{i=1}^{\infty} x_i R \]

\[ S = \sum_{x} S_{ax} \exp \left[ \frac{\pi^2}{A_0^2} A_0^2 + A_0^2 \right] 
+ \exp \left[ \frac{\pi^2}{A_0} A_0 \right] \]

\[ a = 0, 1, 2 \]

\[ n = 0, 1, 2 \]
\[ M = \sum_x R \ \exists \ \text{compact} \]

\[ S = \int dt S \bar{A} \cdot \bar{A}^* \left[ A_0 \cdot A_0 + A_0 \cdot F \cdot \frac{1}{2} + \frac{1}{2} A_0 \cdot A_0 \cdot A_0 \cdot A_0 \right] \]

Moment:

\[ \Pi_\frac{T}{\tau} = \frac{\delta S}{\delta A_0} = \oint \bar{A} \cdot \bar{A} \]

\[ a = 0, 1/2, 0, \frac{1}{2} \]
\[ \mathcal{M} = \sum_{\ell} \chi_{\ell} \times \mathbb{R} \quad \exists \text{ compact} \]

\[ S = \int dx \int dy \sum_{\ell} \left[ A_0^2 + \frac{A_0^2}{R_0^2} \right] \]

\[ \pi_1 = \frac{\delta S}{\delta A_0} = \sum_{\ell} A_0^2 \]

\[ \{ A_0(x,y), A_0(x',y') \} = \frac{1}{\pi} \int S^2(x,y) \]

\[ a = 0, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots \]
\[ M = \sum_i x_i R \quad \exists \text{compact} \]

\[ S = S dt \sum y_i \varepsilon_i \varepsilon_i \left[ A_{x_i}^2 A_{y_i}^2 + A_{x_i}^2 F_{x_i}^2 + \frac{\varepsilon_i}{\varepsilon_i} A_0 + \frac{\varepsilon_i}{\varepsilon_i} \chi \right] \]

\[
\Pi_{i} = \frac{\delta S}{\delta A_{x_i}^2} = \varepsilon_i A_{x_i}^2 \\
\Pi_0 = 0 \Rightarrow C_{i} = F_{12} = 0
\]
\[ \mathcal{M} = \sum_{i}^{(2)} x R \leq \text{comp} + \]

\[ S = \text{Sum} S_{\text{in}} \varepsilon \delta \left[ A_{i}^{2} A_{i}^{2} + A_{0}^{2} F_{\mu}^{\mu} + \varepsilon^{2} A_{0} A_{0} A_{\mu} A_{\mu} \right] \]

\[ \text{Moment} \]

\[ \Pi_{i}^{2} = \frac{S_{\text{in}}}{S_{\text{out}}} - S^{2} A_{i}^{2} \]

\[ \Pi^{0} = 0 \Rightarrow C^{2} = F_{\mu}^{\mu} = 0 \]
\[ M = \sum_{i=1}^{n} x_i \]

\[ S = S_{\text{det}} S_{\text{vol}} \epsilon \alpha \beta \gamma \left[ A_{\alpha}^\beta A_{\beta}^\gamma + A_{\alpha}^\gamma A_{\gamma}^\beta \right] + x_{\beta}^\alpha A_{\alpha}^\beta \delta_{\beta}^\gamma A_{\gamma}^\beta \]

Moment

\[ \Pi_i = \sum_{j=1}^{n} x_{\beta}^j A_{\beta}^i \]

\[ \Pi^0 = 0 \Rightarrow C^i = F_{12} = 0 \]

\[ H = \int \chi \xi \]
\[ M = \sum_{i=1}^{n} x_i \] 3 \text{ compact}

\[ S = S \text{ect} S \text{ect} \sum_{\alpha} [A_{\alpha} \cdot A_{\alpha} + A_{\alpha} \cdot \bar{A}_{\alpha} + A_{\alpha} \cdot E_{\alpha}] + [\Delta A_{\alpha} A_{\alpha} A_{\alpha} A_{\alpha}] \]

\[ \Pi^i = \frac{\delta S}{\delta A_i} - \delta S A_i \]

\[ \Pi^0 = 0 \Rightarrow C^i \]

\[ H = \sum_{\alpha} \chi^i C_i \]
\[ M = \left( \mathbb{E} \times R \right)^3 \text{ compact} \]

\[ S = \sum \int \sum x \int \left[ A^1_x A^2_y + A^3_x A^4_y \right] \]

\[ \text{moment} \]

\[ \Pi^1_x = \frac{\dd \Sigma}{\dd A^1_x} = 3 \lambda \gamma A^1_x \]

\[ \Pi^0 = 0 \Rightarrow C^1 = F_{12} = 0 \]

\[ H = \sum \lambda \gamma \epsilon \]
\[ M = \sum_{i=1}^{N} \chi_i R \]

\[ S = \sum_i S_i \rho \chi_i + \sum_i A_i^\pm F_i^\pm \]

Moment:

\[ P_i^\pm = \frac{\delta S}{\delta A_i^\pm} = \phi^{\mp} A_i^\pm \]

\[ \Pi^0 = 0 \Rightarrow C^0 = F_i^0 \]

\[ H = \sum \chi_i \rho \]

\[ q = 0, \pm \frac{1}{2} \]

\[ \phi \]

\[ (x, y) \]
\[ M = \sum_{i=1}^{n} x_R \Rightarrow \text{compact} \]

\[ S = \int dA S \int d^{\infty - 4} \left[ A_0^2 \hat{A}_0^2 + A_e^2 \hat{A}_e^2 + \ldots \right] \]

\[ S A_l_{\text{on}} = 0 \]

\[ \Pi^1 = \frac{\delta S}{\delta A_l^1} - \delta \hat{A}_l^1 \]

\[ \Pi^0 = 0 \Rightarrow C^2 = F_{12} = 0 \]

\[ H = \int d^4x \hat{C} \]
\[ M = \sum_{i}^{(2)} x R \]

\[ S = \sum_{i \in I} \sum_{k} \varepsilon_{ijk} \left[ A_{i}^{a} A_{j}^{b} A_{k}^{c} + A_{j}^{i} F_{i}^{b} \right] + \{ A_{i}^{a}, \lambda_{j}^{b} \} = \sum_{i}^{(2)} S(x, y) \]

\[ \Pi^{i} = \frac{\delta S}{\delta A_{i}^{a}} = \varepsilon_{ijk} A_{j}^{b} \]

\[ \Pi^{0} = 0 \Rightarrow C^{0} = F_{0}^{0} = 0 \]

\[ H = \sum_{i \in I} x \xi_{i} \]
\[ 4D \quad M = \Sigma^3 \times \mathbb{R} \quad \Leftrightarrow \text{compact} \]
$4D \ M = S^3 \times \mathbb{R} \ \tilde{S}^3$ compact \ $\theta \ S^3 = 0$
4D $\mathcal{M} = \mathbb{S}^3 \times \mathbb{R}$ \text{ compact $\partial \mathcal{M} = 0$}

$A_0$, gauge field in $G = SU(2)$

$F$
$4D \quad M = \Sigma^3 \times R \quad \Sigma^3 \text{ compact } \partial \Sigma = 0$

$A_\mu \quad \text{gauge field } \mu \quad (G = SU(2))$

$F_{\mu \nu}$
4D $\mathcal{M} = \mathbb{R}^3 \times \mathbb{R}$ compact $\partial \mathcal{M} = 0$

$A_a$ gauge field $4$ ($G = SU(2)$) $a = 0, 1, 2, 3$

$F_{\mu\nu}$
4D \[ M = \Sigma^3 \times R \]

\[ \Sigma^2 \text{ compact } \partial \Sigma = 0 \]

\[ a = \frac{0.128}{4} \]

\[ B_{95} = 2 \text{ km } g/\text{s} \]

\[ A_r \text{ gaseous } \]

\[ F_{96} \]
4D \quad \mathcal{M} = \mathbb{S}^3 \times \mathbb{R} \quad \mathbb{S}^3 \text{ compact } \delta \varepsilon = 0 \quad a = 0.125 \\
B_{\Phi} \quad 2 \text{ from } g/30 \quad \text{gauge field } \Phi \quad G = \text{SU}(2) \\
\text{valuing } G \quad F_{\Phi} \\
S = S
4D $M = \Sigma^3 \times R$ $\Phi^2$ compact $\partial \Sigma = 0$

$\alpha = 0, 1, 2, \ldots$

$B_{\mu \nu} - 2 \varepsilon_{\mu \nu \rho \sigma} \frac{\partial A_\rho}{\partial x^\sigma} = F_{\mu \nu}$

$S = S + \text{terms} = S$
$M = \Sigma^3 \times \mathbb{R}$

3-dimensional compact $\Sigma^3$ with $\varepsilon = 0$

$A_2$ gauge field in $G = SU(2)$

$B_{45} \sim 2$ form $g / 8 \pi$

$S = S^4 + \text{form} = S$
4D $M = \Sigma^3 \times \mathbb{R}$ \hspace{1cm} \Sigma^3\text{ compact} \hspace{1cm} \partial \Sigma = 0 \hspace{1cm} a = 0.625$

$B_{\alpha\beta} = \text{some expression} \hspace{1cm} A_\alpha \hspace{1cm} g_\text{gravitational} \hspace{1cm} C = \text{something} \hspace{1cm} \eta = \text{something}$

$S = S + \text{something} = S$
\[ M = \sum_{x} x R \] for compact

\[ S = \oint \sum_{x} e^{x} [A_{i}^{x} A_{j}^{x} + A_{i}^{x} F_{i}^{x}] + \oint A_{i}^{x} A_{i}^{x} A_{i}^{x} \] and \[ \mathcal{A}_{x, y} = 0 \]

momentum

\[ \Pi_{i}^{x} = \frac{\delta S}{\delta A_{i}^{x}} - \delta_{x} A_{i}^{x} \]

\[ \Pi_{i}^{x} = 0 \Rightarrow C_{x}^{x} = F_{i}^{x} = 0 \]

\[ H = \int \sum x^{2} c^{x} \]
4D $M = \mathbb{S}^3 \times \mathbb{R}$, $\mathbb{S}^3$ compact, $a = 0, \frac{1}{2}$

$B_{\alpha \beta}^{\perp}$ form 2-form, $B_{\alpha \beta} = \nabla_{\alpha \beta} A_{\gamma}$, gauge field in $G = SU(2)$

$F_{\lambda \mu}$

$\delta B^{\perp}_{\alpha \beta} = -\frac{i}{2} \epsilon^{\perp}_{\alpha \beta \gamma \delta} \lambda_{\gamma} B_{\delta \tau}$

$S = S_{\text{form}} = S$
$4D \quad M = \mathbb{S}^3 \times R$ \quad $\mathbb{S}^3$ compact $\Rightarrow \epsilon = 0$ $a = 0.425$

$B_{45}^1 \quad \text{mod.} \quad \text{gauge field} \quad G = \text{Surf}$

$S = S^{4\text{th.}} = S(B_{4A}^I F^I)$
\[ M = \Sigma^3 \times \mathbb{R} \quad \text{compact} \quad \mathcal{E} = 0 \quad \mathcal{A} = 0.1234 \]

\[ B_{45} \quad \text{2.6mm, q/20} \quad \text{vol/vol in} \quad \mathcal{G} = \text{SU(2)} \]

\[ A_2 \quad \text{gauge field in} \quad \mathcal{G} = \text{SU(2)} \]

\[ F_{45} \quad \delta B^5 = -2 \pi \lambda \delta \theta B_{45} \]

\[ S = S_{\text{4-form}} = S(B_4^I F^I) - F_4^I F^2 \]
\[ \mathbf{M} = \mathbf{\Sigma}^3 \times \mathbf{R} \]

\[ \mathbf{\Sigma}^3 \text{ compact \& \& } \mathbf{\delta} \mathbf{\Sigma} = 0 \]

\[ \mathbf{a} = 0.625 \]

\[ \mathbf{B}_{+5} - \frac{1}{6} \mathbf{m} \frac{1}{2} \mathbf{e} \]

\[ \text{vol}_{\mathbf{mg}} \quad \mathbf{A}_{\mathbf{e}} \]

\[ \mathbf{F}_{+5} \]

\[ \delta \mathbf{B}_{+5} = - \mathbf{i} \frac{1}{2} \mathbf{A} \mathbf{B} \]

\[ \mathbf{S} = \mathbf{S} + \mathbf{h}_{+5} = \mathbf{S}[\mathbf{B}_{+5} \mathbf{F}_{+5} - 1 \mathbf{B}_{+5} \mathbf{B}_{+5} - \mathbf{F}_{+5} \mathbf{F}_{+5}] \]
$4D \quad M = \mathbb{S}^3 \times \mathbb{R} \quad \mathbb{S}^3$ compact $\Theta \varepsilon = 0 \quad \alpha = 0, 12\bar{5}$

$B_{a\bar{b}} \equiv \text{norm } q / c o \quad \text{vector } \mu \in G \quad G = \text{SU(2)}$

$A_{\mu} \quad \text{gauge field } \mu \quad G = \text{SU(2)}$

$F_{\mu\nu} \quad \delta B_{\mu} = -i T^{a c} \lambda_{c} B_{\mu}^{a}$

$S = S^{\text{formal}} = S \left[ B_{a}^{\mu} F_{\mu}^{\nu} - \frac{1}{2} B_{a}^{\mu} B_{\bar{a}}^{\bar{\mu}} - \frac{1}{2} F_{\mu}^{\nu} F_{\mu}^{\nu} \right]$
\[ 4D \quad M = \Sigma^3 \times \mathbb{R} \quad \Sigma^3 \text{ compact and } \partial \Sigma = 0 \quad a = 0.625 \]

\[ B^{i} - 2 \text{ fermionic fields } A_{\mu} \quad A_{\mu} \text{ gauge field in } G = SU(2) \quad \delta B^{i}_{\mu} = -i \gamma^{\nu} \kappa \lambda_{T} B^{\nu}_{\mu} \]

\[ S = S^{\text{YM}} = S \left( B^{i}_{\mu} F^{i}_{\mu} - \frac{1}{2} B^{i}_{\mu} B^{i}_{\mu} - \kappa F^{+} F^{-} \right) \]
\[ B_{\alpha \beta} = \text{form} \text{ of } \mathbf{E} \text{ in } G \]

Field equations:

\[ S = S^{\text{total}} = S \left[ B_{\alpha}^\mu F_{\alpha}^\mu - \frac{1}{2} B_{\alpha}^\imath B_{\alpha}^\imath - \frac{1}{4} \mathbf{F}^2 \right] \]
\[ S = S^{\text{topology}} = S\left[B_\alpha^\tau F^{\tau} - \frac{1}{2} B_\alpha^\tau B_\alpha^\tau - \Psi F_{\alpha} F_{\alpha} F^{\alpha} F^{\alpha}\right] \]

Equations:
\[ \frac{\delta S}{\delta B_{\alpha}^\tau} \]
\[ F - \Lambda B = 0 \]
\[ S = \sum_{\text{fields}} S [B_{A B}^\perp F_{A B} - \frac{1}{2} A^\perp B_{A B}^\perp - \mathbf{F} \cdot \mathbf{F}^\perp] \]

Field equations:

\[ \frac{\delta S}{\delta A} = 0 \]

\[ S_{B A}^\perp - 2 \sum_{\text{vol of } A} F_{A B} \]

Base field in \( G = SU(2) \)
\[ S = \sum_{\text{terms}} = \sum \left[ (B^+ \neq f(r,m) \text{ or } \text{value in } G \cdot F_{\mu} \cdot \delta B^+ = -\frac{1}{2} \eta_{\lambda \gamma} B^+ \cdot B^+ \cdot \gamma \right] \]

Field equation:

\[ F - \lambda B = 0 \quad \frac{\delta S}{\delta A} = \]
\[ S = S_{\text{form}} = S \left( B_A^\tau - \lambda B_A B^\tau - \frac{1}{2} F^\tau F^\tau \right) \]

Field equation:

\[ F - \lambda B = 0 \quad \delta S \delta A = \]
\[ B_{\text{vac}}^{1/2} \text{ eV/m} \]

\[ S = S + \text{terms} = S \left( B^a_{\mu} + \frac{1}{2} A^a_{\mu} \right) - \frac{1}{2} F_{\mu \nu} F^{\mu \nu} \]

Field equations:

\[ \frac{\delta S}{\delta B_{\mu}^{\mu}} = F - \Lambda B = 0 \]

\[ \frac{\delta S}{\delta A_{\mu}} = \partial B + \Lambda B = 0 \]

\[ \delta B_{a}^{\mu} = \frac{1}{2} \eta_{a b} F_{b}^{\mu} \]

\[ \delta B_{a}^{\mu} = \frac{1}{2} \eta_{a b} F_{b}^{\mu} \]

\[ \delta B_{a}^{\mu} = \frac{1}{2} \eta_{a b} F_{b}^{\mu} \]
\[ S = S + \text{terms} = S \left[ B^1_{\alpha} - \frac{1}{2} (A^\beta B^\beta_{\alpha}) - \varepsilon_{\alpha \beta \gamma} A^\beta A^\gamma \right] \]

Field equations:
\[
\frac{\delta S}{\delta B^1_{\alpha}} = F - \Lambda B = 0 \\
\frac{\delta S}{\delta A} = \frac{d}{dt} B + \Lambda a B = 0 \\
\frac{\delta}{\delta \Lambda B} = 0
\]
\[ S = S_{\text{fourth}} = S \left( B_n^\perp, \frac{1}{2} \nabla B_n^\perp B_n^\perp - \epsilon F \right) \]

Field equations:
\[ \frac{\delta S}{\delta B_n^\perp} = 0 \]
\[ \frac{\delta S}{\delta A} = dB + \Lambda B = 0 \]
\[ \frac{\partial \Lambda B}{\partial A} = 0 \]
\[ B_{ab} = -2 \text{ form } g_{\mu \nu} \]
\[ S_{\mathcal{B}_{ab}} = \int d^4x \sqrt{g} \left( \frac{1}{2} F_{ab}^\mu F^{\mu \nu} - \frac{1}{4} g^{\alpha \beta} F_{\alpha \beta} F_{\mu \nu} \right) \]

\[ S = S_{\text{form}} = S \left( B_{ab}^\mu \frac{F_{\alpha \beta}}{6n} - \frac{1}{2} B_{ab}^\mu B_{ab}^\nu - \frac{1}{2} F_{\alpha \beta} F_{\mu \nu} \right) \]

Field equations:
\[ \frac{\delta S}{\delta B_{ab}^\mu} = 0 \]
\[ \frac{\delta S}{\delta A} = \partial B + A \Lambda B = 0 \]
\[ \partial B = 0 \]

Branch cuts at infinity \[ \partial F = 0 \]
\[ T_\text{L} \quad \text{TR} \quad = \sum_{\sigma_1, \sigma_2, \ldots, \sigma_n} F_{\sigma_1 \sigma_2 \ldots \sigma_n} \]

\[ \langle T^{[\omega]} \rangle = \sum_{\text{dim}(A)} e^{\frac{i\phi}{2}} S^c S^c T^{[\omega, R]} = (\phi_{W}[Y]) \]
\[ M = \sum^{(2)} \times R \]

\[ S = \sum_{\alpha} \Sigma_{\beta} \epsilon^{\alpha \beta \gamma} \left[ A_{\beta}^{a_1} A_{\gamma}^{a_2} + \Lambda_{\beta}^{a_1} \Lambda_{\gamma}^{a_2} - \tau_{\beta,\gamma} A_{a_1}^{a_2} A_{a_2}^{a_3} A_{a_3}^{a_4} \right] \]

\[ \text{Momentum} \]

\[ \Pi^I = \frac{\sum S}{S A_{I}} - \epsilon^{\alpha \beta \gamma} A_{I}^{\alpha} \]

\[ \Pi^0 = 0 \Rightarrow C^I = F_{I}^{I} = 0 \]

\[ H = \sum x^i \epsilon^{i} \]
Hamiltonians
Hamiltonian:

\[ S = S_1 + S_3 \sum_3 \epsilon_3 \nu \]

\[ \nu_3 \sum_3 \nu \]
Hamiltonian: \[ S = S_0 + \sum_{x} \left[ \epsilon^* \left( B_x (\partial_x - \partial_0^*) \right) \right] \]

\[ \psi_n = \psi_3 \]
Hamiltonian:

$$S = S_0 + \sum_{i} \left[ \sum_{\nu} \left( B_{\nu} \left( \omega_{\nu} + \nu \right) - D_{\nu} \right) + B_{\nu} F_{\nu} \right]$$

$$v_{\nu} k^3 \left( \sum_{\nu} \right)$$
Hamiltonian:

\[ S = S_1 + S_2 \sum_x \left[ e^{i\psi} (B_x (\psi_d \Delta - \psi_d \Delta^* ) + B_x F_x \right] \\
\]

\[ + \frac{1}{2} B_{xx} B_{xx} \]
Hamiltonian: \[ S = \sum_{a=1}^{N} \sum_{x=1}^{3} \left[ \sum_{\nu=1}^{n} \left( B_{1\nu} (D_{a\nu} - D_{a\nu}^*) + B_{2\nu} F_{a\nu} \right) \right] \]
Hamiltonian:
\[ S = S_{\text{kin}} + S_{\text{pot}} + \sum_{i} \left[ \epsilon_{i} n_{i}^{*} \right] B_{i} \left( \rho_{i} - \rho_{i}^{*} \right) - B_{0} F_{x} \]

\[ \Pi_{i} = \frac{\delta S}{\delta a_{i}} = \epsilon_{i} n_{i}^{*} B_{i}^{\pm} + \pm i B_{0} \left| B_{i} \right|^{2} \]
Hamiltonian: \[ S = S_0 + \sum_{x} \left[ 3^\nu \left( B_x (Q_{ax} - Q_{ax}^*) + B_0 \frac{F_{kl}}{2} \right) \right] \]

\[ \Pi^a = \frac{\delta S}{\delta A^a} = 3^\nu B^\pm_{,a} \]

\[ \Pi^0 = 0 \]

\[ P = \frac{\delta S}{\delta B} = 0 \]
Hamiltonian:

$$S = S_d + S_{d3} + \sum_{i}^{3} x^k \left[ \delta^0 \left( B_{i1} (d_0 A_k - F \cdot A) + B_{i2} F \cdot A \right) \right]$$

$$\Pi^i = \frac{\delta S}{\delta A_k} = \delta^0 x^k B_{i} \pm \frac{1}{2} \delta^0 x^k B_{i}$$

Constraints (check) just in 2+1
Hamiltonian:

\[ S = S_{lat} + \sum_{x} \left[ \tilde{\sigma} \cdot \mathbf{v} \left[ B_{i7} (\partial \chi - \partial A) + \mathbf{B} \cdot \mathbf{F} \right] + \Delta B_{ii} \right] \]

\[ \Pi^{i} = \sum_{J_{n}} B_{i7} = \tilde{\sigma} \cdot \mathbf{v} \]

\[ \Pi^{0} = 0 \]

\[ P = \sum_{J_{n}} = 0 \]
Hamiltonian:
\[ S = \sum_{i} S_{i+} S_{i-} \left[ 3 \epsilon_{ijk} \left( \text{B}_{ij} \left( \text{B}_{jk} - \text{D}_{ij} \right) + \text{B}_{ik} \right) \right] + \text{B}_{0} F_{x} \]

Constraints (Jeffreys) Justin 2+1

\[ \Pi^x = \frac{\delta S}{\delta A_{x}} = \epsilon \frac{\delta S}{\delta \alpha} \]
\[ \Pi^0 = 0 \]
\[ P = \frac{\delta S}{\delta B} = 0 \]
Hamiltonian: 
\[ S = S_0 + S_0 \sum x \left[ 3 \nu x \left( B_{1y} \left( \delta \partial x - \partial y \right) + B_{0x} F_{2x} \right) \right. \]

\[ \Pi^x = \frac{\delta S}{\delta A_y} = 3^{\nu x} B_{2x}^{\pm} \]
\[ \Pi^0 = 0 \]
\[ P = \frac{\delta S}{\delta B} = 0 \]

\text{Constraint:} (check) \text{ justin 2+1}
Hamiltonian: \[ S = S_{dx} S_{dx} \left[ \bar{3} \epsilon_{km} \left[ \bar{B}_{ij} \left( \partial_{t} \bar{A}_{k} - \bar{A}_{t} \partial_{k} \right) \right] + B_{dx} F_{dx} \right] \]

\[ \Pi^{a} = \frac{\delta S}{\delta A_{t}} = 3 \epsilon^{a}_{jk} B_{jk} \]

Constraints (check) justin 2+1

\[ D_{a} \Pi^{a} = 0 \]

\[ \Pi^{0} = 0 \]

\[ P = \frac{\delta S}{\delta B} = 0 \]
Hamiltonian:

\[ S = S_1 + S_2 + \sum_{x} \left[ \xi^u v^k \left( B_i \left( \partial_0 x^k - \partial_k x^0 \right) \right) + B_{ax} F_{x} \right] \]

\[ \Pi^a_i = \frac{\partial S}{\partial \dot{A}_a^i} = \sum_{a} B_{a}^{\pi} \]

Constraints (check) justified 2 + 1

\[ G^\pi = \partial_a \Pi^a_i = 0 \]
Hamiltonian: \( S = S_A + S_B \sum_{k} \left[ e^{\theta k} \left( B_{17}(B_{A_k} - B_{A_k}) + B_{27} \right) \right] \)

\[ \Pi^I = \frac{\delta S}{\delta A_k} = \sum_{\mu} B_{\mu A_k} \quad \text{Constraints (check)} \quad \text{justify } 2+1 \]

\[ \Pi^0 = 0 \]

\[ P = \frac{\delta S}{\delta B} = 0 \]

\[ \dot{\Pi}^I = F^I - \lambda B^I = 0 \]
Hamiltonian:

\[ S = S_\text{int} + \sum_x \left[ \varepsilon \psi^* \left( B_x \left( 2 \Delta_k - 2 \Delta_x \right) \right) + B_x F_{2x} \right] \]

\[ \Pi^i = \frac{\delta S}{\delta A_k} = \varepsilon \psi^* \beta_{xk}^\pm + \frac{1}{2} B_{2x} (B_{3x}^*) \]

Constraints (check): justin 2+1

\[ \text{Constraints (check)} \]

\[ \left\{ \begin{array}{c}
G^i = D_x \Pi^i = 0 \\
F^i = F^i - \Lambda \beta^i = 0
\end{array} \right. \]
Hamiltonian: $S = S_{a+} S_{a-} \left[ 3 \mu^x \left[ B_{a+} (D_{a+} a_a - D_{a-} a_a) + B_{a-} a_{a-} \right] \right]$

\[ \Pi^a = \frac{\delta S}{\delta a_a} = 3 \mu^x B^\pm_{a+} + 4 \alpha B_{a+} B_{a-} \]

Constraints (check) just hold 2+1

\[ \gamma^I = D_a \Pi^I = 0 \]

\[ \gamma^I = \Pi^I - \nabla \cdot \Phi = 0 \]
Hamiltonian:

\[ S = S_A + \sum_i \left[ \sum_k \left( \xi_i^* \mathcal{B}_{ik} \mathbf{a}^\dagger - \xi_i \mathcal{B}_{ik} \right) + B_{0i} \mathbf{F}_{iA} \right] \]

\[ \Pi^i = \frac{\partial S}{\partial A^i} = \xi_i^* \mathcal{B}_{ik}^\dagger \]

\[ \Pi^0 = 0 \]

\[ \mathbf{P} = \frac{\partial S}{\partial \mathbf{B}} \]

Constraints (check):

\[ G^I = D_4 \Pi^I = 0 \]

\[ \mathcal{G}^I = \mathcal{T}^I - \sum_k \xi_i \mathcal{B}_{ik}^\dagger \Pi^I = 0 \]

\[ \mathcal{A}_k \]
Hamiltonian: \( S = S_0 + S_{\Delta x} \sum_{i} \left[ \mathbf{E}_i \cdot (\mathbf{v}_i - \mathbf{u}_i) + B_{i} F_{Ii} \right] \)

\[ \Pi^i = \frac{\delta S}{\delta A_h} = \sum_{i} e_i B_i^\pm, \quad \Pi^b = 0 \]

\[ P = \frac{\delta S}{\delta B} = 0 \]

Constraints (check): \( G^I = D_a \Pi^I = 0 \)

\[ \Phi^I = F^I - \Lambda_6 \Pi^I = 0 \]
Hamiltonian:

\[ S = S_0 + \sum_{x} \left[ \epsilon_x \left( \langle a_x | (c_0 \delta a_x - c_0 \delta a_x) + B_0 F_x \right) \right] \]

\[ \Pi^a = \frac{\partial S}{\partial \dot{x}^a} = \sum_{x} \epsilon_x B_{x}^a \]

Constraints (check) just in 2+1

\[ G^I = D_0 \Pi^{I} = 0 \]

\[ \mathcal{F}^I = F^I - \lambda \sum_{x} \Pi^I_x = 0 \]

\[ \{ \mathcal{A}^I(x), \mathcal{F}^I(y) \} = S^I \delta(x-y) \]

\[ \Pi^0 = 0 \]

\[ P = \frac{\partial S}{\partial B^0} = 0 \]
- Chern-Simons Theory
- Back to BF
- General Relativity

\[ H = \frac{1}{2m} P^2 + V(X) \]
Chern-Simons Theory

Back to BF

General Relativity

\[ H = \frac{1}{2m} p^2 + V(x) \quad P_x = \partial_x S \]

\[ S(x) \]
- Chern-Simons Theory
- Back to BF
- General Relativity

\[ H = \frac{1}{2m} P^2 + V(x) = E \]
\[ P_a = \frac{\partial}{\partial x^a} S \]
\[ E = s \]
\[ S = \frac{1}{2m} (\partial S)^2 + V \]
Chern-Simons Theory

Back to BFV

General Relativity

\[ H = \frac{1}{2m} P^2 + V(x) = E \]
\[ P_x = \partial_x S \]
\[ E = \frac{\partial S}{\partial x} \]

\[ S = \frac{1}{2m} (\partial S)^2 + V(x) \]
\[ \Pi^i = \frac{SS}{SA} = 3^{\text{check}} B_{3k}^i \]

\[ \Pi^0 = 0 \]

\[ P = \frac{SS}{SB} = 0 \]

Constraints (check)

\[ G^I = \mathcal{O}, \Pi^I = 0 \]

\[ \xi^I = F^I - \lambda_\nu \mu \Pi^I = 0 \]

\[ \text{and} \quad \exists \]
\[ \Pi^I = \frac{\mathcal{L} L}{\delta A_k} = \sum_{\mu} B_{\mu}^I \]

\[ \Pi^0 = 0 \]

\[ P = \frac{\mathcal{L}}{\delta B} = 0 \]

Constraints (check)

\[ G^I = \partial_\mu \Pi^I = 0 \]

\[ F^I = F_{\gamma} - \lambda \varepsilon_{\mu\kappa\lambda} \Pi^\kappa = 0 \]

\[ SCA \]
\[ \Pi_i^a = \frac{SS}{SA_i} = \varepsilon^i \pi B_{A_i} \]

\[ \Pi^0 = 0 \]

\[ P = \frac{SS}{SB} = 0 \]

\[ \Pi_i^N = \frac{SS}{SA_i} \]

Constraints (check)

\[ G^I = \partial_\nu \Pi_i^I = 0 \]

\[ \delta^I = F_{\nu I} - \lambda \delta_{i k} \Pi_i^k = 0 \]

\[ C = \Pi_i^I \text{ on } \Sigma \]

\[ \text{SCA} \]
\[ H = \sum_{x} \lambda_x G_x + \mu_x \Xi_x \]

\[ G^1(s) = \mathcal{D}_i \frac{\delta s}{\delta x_i} = 0 \]
\[ H = \sum_{\xi} \lambda_{\xi} G^{\xi} + \mu_{\xi} F^{\xi} \]

\[
G^{1}(s) = D_{i} \frac{\delta S}{\delta A_{ii}} = 0
\]

\[
\lambda_{i} \frac{\delta S}{\delta A_{ii}} = F_{ij}
\]

\[
S^{H-J} = k S^{CS}
\]
\[ H = \sum_{\xi} \lambda_{\xi} G^{\xi} + \mu_{\xi} \Xi^{\xi} \]

\[ G^{1}(s) = D_i \frac{\delta S}{\delta \Lambda_i^0} = 0 \]

\[ \Lambda_{i/k} \frac{\delta S}{\delta \Lambda_{i/k}} = F \]

\[ S^{H-J} = k S^{cs} \]

\[ k = \lambda \]
\[
H = 3 \chi e G + \mu^2 \frac{\partial}{\partial A}
\]

\[
G^2(s) = D_i \frac{S_s}{S_{n^2}} = 0 \quad \Rightarrow \quad S^H \cdot J = K \cdot S^C, \quad K = \frac{1}{4}
\]

\[
\Lambda \sum_1^{N} \frac{\partial S}{\partial A_{\mu \nu}} = F_{ij}
\]
\[ g^1(s) = D_i \frac{\delta S}{\delta A^i} = 0 \]
\[ \Lambda \sum_{i,k} \frac{\delta S}{\delta A_{ik}} = F_{ik} \]
\[ S^{H-I} = k S^{cS} \quad k = \frac{1}{A} \]
\[ A \wedge \Pi \wedge \frac{\delta S}{\delta A} \]
\[ H = \sum_{\xi} \lambda_{\xi} G^{\xi} + m_{\xi} \Phi^{\xi} \]

\[ G^{\xi}(s) = D_{\xi} \frac{\partial S}{\partial A_{\xi}} = 0 \]

\[ S^{H-J} = k S^{cs} \]

\[ k = \lambda \]

\[ \prod \frac{\partial S}{\partial A} \]

\[ t, Y \]

\[ A_t(Y) = A_t(Y) \text{ simulated by } t = S^{cs} \]

\[ A_\phi = \text{anything you like} \]
\[ M = \sum_{(2)}^{(2)} x R \quad \exists \quad \text{compact} \]

\[ S = \sum_{\text{dvol}} \sum_{\text{dir}} \sum_{\text{spin}} \left[ A_{\mu}^I \pi_{\mu}^I + \Lambda_0 \frac{F_{\mu\nu}^I}{\sqrt{2}} \right] + \left\{ A_{\mu}^r A_{\nu}^l A_{\sigma}^k A_{\tau}^s \right\} = \frac{\sqrt{2}}{2} \sum_{\text{spin}} \left( \sum_{\text{dir}} \sum_{\text{dvol}} \right) \]

\[ n^\mu = \frac{\delta S}{\delta A_{\mu}^I} = -\frac{\delta S}{\delta A_{\mu}^I} \]

\[ n^0 = 0 \Rightarrow c^I = F_{\mu\nu}^I = 0 \]

\[ H = \int x^I \]
$11_i = S A_{1i}$

$H = \sum_{x} \lambda_{x} G^{x} + \mu_{i} F^{i}$

$G^{i}(s) = D_{i} \frac{d s}{d A_{1i}} = 0$

$\nabla \Sigma_{i} \frac{d s}{d A_{1i}} = F_{ij}$

$S^H - J = k S^C \frac{d s}{d A}$

$A \sim \Pi \sim \frac{d s}{d A}$

$A_{i}(y_j) = A_{i}(y) $ included by $t = S f_{i}$

$A_{0} = $ anything you like
Boundary \( \Sigma^3 \) =
Boundary $\partial \Sigma^3 = \mathbb{R}$
\[ \partial S^2 \neq 0 \]

\[ S^2 = \sum_{B \in \mathcal{A}} \]
\[ G^2(s) = \frac{D_i \sum s}{\sum A_i} = 0 \quad \text{and} \quad \sum_{j,k} \frac{SS}{SA_{jk}} = F_{ij} \]

\[ S_{H-I} = K S_{CS} \quad K = \frac{1}{A_{in}} \]

Hamiltonian:
\[ S = S_d + S_{3^{3/2}} \left[ \varepsilon_{\omega k} B_{3k}^{\pm} \right] + (\omega k - D_\omega A_\omega) + B_{\omega x} F_{\omega x} \]

\[ \Pi^i = \frac{\sum s}{\sum A_i} = \varepsilon_{\omega k} B_{3k}^{\pm} \]

\[ \Pi^0 = 0 \]

\[ P = \frac{\sum s}{\sum B} = 0 \]

\[ S = S(N) \quad SCA(i) \]

\[ G = \frac{x_i}{x_i} \]

\[ x_i - S_i, \quad \delta S_i \]

\[ SCA(i), \delta S_i \]

\[ \varepsilon = \frac{FA_{ik}}{\delta S} \]
Hamiltonian:

\[ S = S_A + \sum_x \left[ \sum_y \left[ B_{xy} \left( \phi_{xy} - \alpha_x \phi_y \right) \right] + B_{xx} F_{xx} \right] \]

\[ \Pi^x = \sum_x \left[ \frac{S_s}{A_x} \right] = \sum_y B_{yx} \]

\[ F_{xx} \]

Constraints (check):

\[ G_{ii} = D_{ii} \Pi^i = 0 \]

\[ \Pi^i = F_{iy} - \lambda_{iy} \Pi^i = 0 \]

\[ H = \sum_x \lambda_x G_{xx} + \mu_x F_{xx} \]
\[ \partial S \neq 0 \]

\[ S = \sum_{\Sigma} A \delta S = \sum_{\Sigma} \delta S \]
\[ \delta S = \int_{\partial \Sigma} \beta \, dS + \int_{\Sigma} \nabla \cdot \varepsilon \, dS \]
\[ \delta S = \sum_{x \in \mathbb{R}} B^I_a \partial S^a = \sum_{x \in \mathbb{R}} B^I_a S^a - S_{x \in \mathbb{R}} (\partial B)^I \cdot \delta A \]
\[ \delta S = \sum_{x \in R} B^a \, \delta A^a = \sum_{x \in R} B^a \, S^a - \sum_{x \in R} (\delta B) \cdot SA \]
\[ \delta S = \sum_{x \in \mathbb{R}} B^I x \, \delta A^I = \sum_{\omega \times R} B^I \delta A^I - \int_{\partial \Sigma} (\omega \cdot \mathbf{B}) \, \delta A \]

3 ways to deal with boundary term
\[ \delta S = \int_{\Sigma \times \mathbb{R}} B^I \, \delta A^I = \int_{\Sigma \times \mathbb{R}} B^I A^I \, \delta A^I - \int_{\Sigma \times \mathbb{R}} (\partial B^I) A^I \delta A \]

3 ways to deal w/ boundary term:

1) \( \delta A \big|_{\partial \Sigma} = 0 \)
\[ \delta S = \sum_{\Sigma} B^i \, \delta A^i = \sum_{\partial \Sigma} B^i \, A^i - S_{\Sigma} \delta A \]

3 ways to deal with boundary term:
1. \( \delta A |_{\partial \Sigma} = 0 \)
2. \( \partial R |_{\partial \Sigma} = 0 \)
\[
\delta S = \int_{\Sigma^2 \times \mathbb{R}} B^i A^i - \int_{\partial \Sigma^2 \times \mathbb{R}} (\partial B) \cdot A
\]

3 ways to deal with boundary term

1) \(\delta A \big|_{\partial \Sigma^2} = 0\)
2) \(B \big|_{\partial \Sigma^2} = 0\)
3) add a boundary action \(S \rightarrow S + S^0\)

\(S^0 = \int_{\partial \Sigma^2} \)
\[ \delta S = \int B^2 \, d\delta A = \int \vec{B} \cdot \delta \vec{A} \quad \text{3 ways to deal w/ boundary term} \]

1) \[ \delta A |_{\partial} \neq 0 \]
2) \[ B|_{\partial} = 0 \]
3) \[ \text{add a boundary field} \]

\[ \int S + \int S^0 \quad S^0 = \int_{\partial} \]
\[ \delta S = \int_{\Sigma} \left( \sum_{\mathbb{R}} B_{A} \delta A^{A} - \sum_{\mathbb{R}} (D B). A \right) \delta \Sigma \]

3 ways to deal w boundary term

1) \( \delta A_{\Sigma} = 0 \)
2) \( B_{\Sigma} = 0 \)
3) add a boundary action \( S \rightarrow S + S^{0} \quad S^{0} = \int_{\partial \Sigma} \psi \)

\[ S^{0} \]
\[ \delta S = \int_{\mathbb{R}^3} \mathbf{B} \cdot \delta \mathbf{A} \, d\mathbf{r} = \int_{\mathbb{R}^3} \mathbf{B} \cdot \delta \mathbf{A} \, d\mathbf{r} - \int_{\partial \mathbf{R}} \mathbf{B} \cdot \delta \mathbf{A} \, d\mathbf{r} \]

3 ways to deal with boundary term:

1) \( \delta \mathbf{A} |_{\partial \mathbf{R}} = 0 \)
2) \( \mathbf{B} |_{\partial \mathbf{R}} = 0 \)
3) add a boundary action \( S \to S + \mathcal{S} \quad \mathcal{S} = \int_{\partial \mathbf{R}} \)

\[ \delta S^{\text{tot}} = \int_{\mathbb{R}^3} \delta \mathbf{A} [\mathbf{B} + \frac{\delta \mathbf{B}}{\delta \mathbf{A}}] \, d\mathbf{r} \]
\[ ds = S \sum_{x \in \mathbb{R}} B^x A^x \delta A^x = \int_{\mathbb{R}} B^x A^x \delta A^x - \int_{\mathbb{R}^2} (B \cdot \delta A) \cdot A \]

3 ways to deal with boundary term:

1) \( \delta A |_{x=0} = 0 \)
2) \( B |_{x=0} = 0 \)
3) add a boundary action \( S \rightarrow S + S^0 \)

\[ S^{\text{eff}} = \int_{\mathbb{R}} S \delta A [B + \frac{\delta B}{\delta A}] + \frac{\delta B}{\delta B} \delta B \]
\[ S = \int_{\mathbb{R}} B^2 \, \delta \, A^2 = \int_{\mathbb{R}} B^2 \, \delta A^2 - \int_{\mathbb{R}} (\partial B) \cdot \delta A \]

3 ways to deal with boundary term:
1) \( \delta A|_{\partial \Omega} = 0 \)
2) \( B|_{\partial \Omega} = 0 \)
3) add a boundary action \( S \rightarrow S + S^0 \)

\[ S^{\text{new}} = \int_{\mathbb{R}} \delta A \left[ B + \frac{\delta \mathcal{L}}{\delta A} \right] + \delta B \frac{\delta \mathcal{L}}{\delta B} \]
\[ S = S_{\mathbb{R}^n} A^2 - S_{\partial A} \]

3 ways to deal with boundary term:

1) \( \delta A |_{\partial A} = 0 \)
2) \( \delta B |_{\partial A} = 0 \)
3) Add a boundary action \( S \rightarrow S + S' \)

\[ S' = \int_{\partial A} \delta A [B + \delta f] \]
\[ \mathcal{S} = S_{\mathbb{R}^2}^{\mathbb{R}^2} \]
\[ DS = \sum_{x \in \mathbb{R}} B_x^2 \, dS = \sum_{x \in \mathbb{R}} B_x^2 \, dA^2 - \sum_{x \in \mathbb{R}} (\partial_x B_x) \, dA \]

3 ways to deal w boundary term

1) \[ \partial A |_{x = 0} = 0 \]
2) \[ B |_{x = 0} = 0 \]
3) add a boundary action \[ S \rightarrow S + S^0 \]

\[ S_{\text{New}} = \sum_{x \in \mathbb{R}} \delta A \left[ B + \frac{\delta B}{\delta A} \right] \]
\[ S = \sum_{x \in \mathbb{R}} B^x \delta A^x = \sum_{x \in \mathbb{R}} B^x \delta A^x - \sum_{x \in \mathbb{R}} \delta (\partial B) \cdot \delta A \]

3 ways to deal w. boundary term:
1) \( \delta A |_{x_0=0} = 0 \)
2) \( \mathbf{B} |_{x_0=0} = 0 \)
3) add a boundary action \( S \Rightarrow S + S' \)
   \[ S' = \sum_{x \in \mathbb{R}} \delta A \left[ B + \frac{\delta \mathcal{L}}{\delta A} \right] = \sum_{x \in \mathbb{R}} \delta A \left[ B - \frac{\mathbf{F}}{\partial_n} \right] \]
\[ ds = S_{\mathbb{R}} - \int_{\partial R} S_{\mathbb{R}} \]  

3 ways to deal with boundary term:  
1) $\delta A|_{\partial \mathbb{R}} = 0$  
2) $B|_{\partial \mathbb{R}} = 0$  
3) Add a boundary action $S \rightarrow S + S^0$  
\[ S^0 = \int_{\partial \mathbb{R}} Y_{\mathbb{R}}(A) \frac{k}{4\pi} \]  
\[ S^{\text{new}} = S_{\mathbb{R}}[B + \frac{\delta P}{\delta A}] = S_{\mathbb{R}}[B - \frac{k}{2\pi} F] \]
\[ \delta S = \int_{\mathbb{R}^2} \nabla A \cdot \nabla A^2 = \int_{\mathbb{R}^2} B \cdot A - \int_{\mathbb{R}^2} (\partial \cdot B) \cdot A \]

3 ways to deal w boundary term

1) \[ \delta A |_{\partial \mathcal{A}} = 0 \]
2) \[ B |_{\partial \mathcal{A}} = 0 \]
3) add a boundary action \[ S \rightarrow S + S^0 \]

\[ S_{\text{new}}^{\alpha} = \int_{\mathbb{R}^2} \delta A \left[ B + \frac{\delta \phi}{\delta A} \right] = \int_{\mathbb{R}^2} \delta A \left[ B - \frac{\delta F}{\delta A} \right] \]
\[ dS = S_{\varepsilon} B^\varepsilon A^2 = \left( B^\alpha S^2 \right)_{\varepsilon} = S_{\varepsilon, \alpha} \]

3 ways to deal w. boundary term:
1) \( B\mid_{\varepsilon=0} = 0 \)
2) \( B \mid_{\varepsilon} = 0 \)
3) add a boundary action \( S \rightarrow S + S^b \)
\[ S^{\text{add}} = S_{\varepsilon} A \left[ B + \frac{\delta P}{\delta A} \right] = S_{\varepsilon} A \left[ B - \frac{\delta F}{\delta A} \right] \]
\[ \delta S = \int_{\mathbb{R}} \delta A \cdot \delta A^2 = \int_{\mathbb{R}} \delta A \cdot \delta A^2 - \int_{\mathbb{R}} (\delta B \cdot \delta A^2) \]

3 ways to deal with boundary term:

1) \[ \delta A |_{\partial \mathbb{R}} = 0 \]
2) \[ \delta B |_{\partial \mathbb{R}} = 0 \]
3) Add a boundary action \[ S \rightarrow S + S^0 \]

\[ S^0 = \sum_{m} \int \left[ F - \frac{\kappa}{4\pi} B \right] \]

\[ \delta S^0 = \int_{\mathbb{R}} \delta A \left[ \delta B + \frac{\delta F}{\delta A} \right] = \int_{\mathbb{R}} \delta A \left[ \delta B - \frac{\kappa}{4\pi} F \right] \]
\[ \delta S = \int_{\mathbb{R}} B^B \delta A^a = \int_{\mathbb{R}} \frac{\partial B^B}{\partial x^a} \delta A^a - \int_{\partial \mathcal{R}} (\partial B^B) \delta A\]

3 ways to deal with boundary term:
1) \( \delta A^a \big|_{\partial \mathcal{R}} = 0 \)
2) \( B \big|_{\partial \mathcal{R}} = 0 \)
3) add a boundary action \( S \to S + S^\circ \)

\[
S^{\circ} = \int_{\mathbb{R}} \delta A^a \left[ B + \frac{\partial S}{\partial A^a} \right] = \int_{\mathbb{R}} \delta A^a [B]
\]
\[ DS = \int_{2xR} B \cdot \delta A - \int_{\partial xR} \frac{\delta A}{\partial x} = \int_{\partial R} \delta B \cdot \delta A - \int_{\partial R} \delta A \cdot \delta B \]

3 ways to deal with boundary term

1) \[ \delta A |_{\partial R} = 0 \]
2) \[ \delta B |_{\partial R} = 0 \]

3) add a boundary action \[ S \rightarrow S + S^0 \]

\[ S^0 = \int_{\partial R} \frac{\delta B}{\delta A} \]

\[ S^0_1 = \int_{\partial R} \delta A \left[ B + \frac{\partial B}{\partial R} \right] = \int_{\partial R} \delta A \left[ B - \frac{1}{2} F \right] \]

\[ \left( F - \frac{1}{2} B \right) \cdot \frac{\delta B}{\delta A} = 0 \]
\[ \delta S = \int_{\partial \Sigma} \delta A^a \varepsilon \hat{A} A^2 = \int_{\partial \Sigma} \left( B^a A^2 - S_{\text{int}} \right) \delta A^a \varepsilon \hat{A} \]

3 ways to deal with boundary term

1) \( \delta A^a \varepsilon \partial_{\Sigma} = 0 \)
2) \( \delta B \varepsilon \partial_{\Sigma} = 0 \)
3) add a boundary term \( S \to S + S^\beta \)

\[ S^{\beta} = \int_{\partial \Sigma} \delta A^a \left[ B + \frac{\delta B}{\delta A} \right] = \int_{\partial \Sigma} \delta A^a \left[ B - \frac{\delta B}{\delta A} \right] \]