Title: Strings/Quantum Gravity 7

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Abstract:

Quasinormal spectrum
and
the black hole membrane paradigm

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- A system perturbed from a (stable) equilibrium oscillates with characteristic frequencies (normal modes)

- If energy dissipation is involved, these modes are damped, and the frequencies have nonzero imaginary parts. (The corresponding boundary value problem is non-Hermitiar These are the quasinormal modes.

- Perturbations of gravitational backgrounds with horizons are of this type.

- In the last 30 years or so, quasinormal spectra of black hole have been computed (mostly numerically) for many backgrounds in various dimensions

Example: Schwarzschild black hole in 4d

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega_2^2$$

Fluctuations:

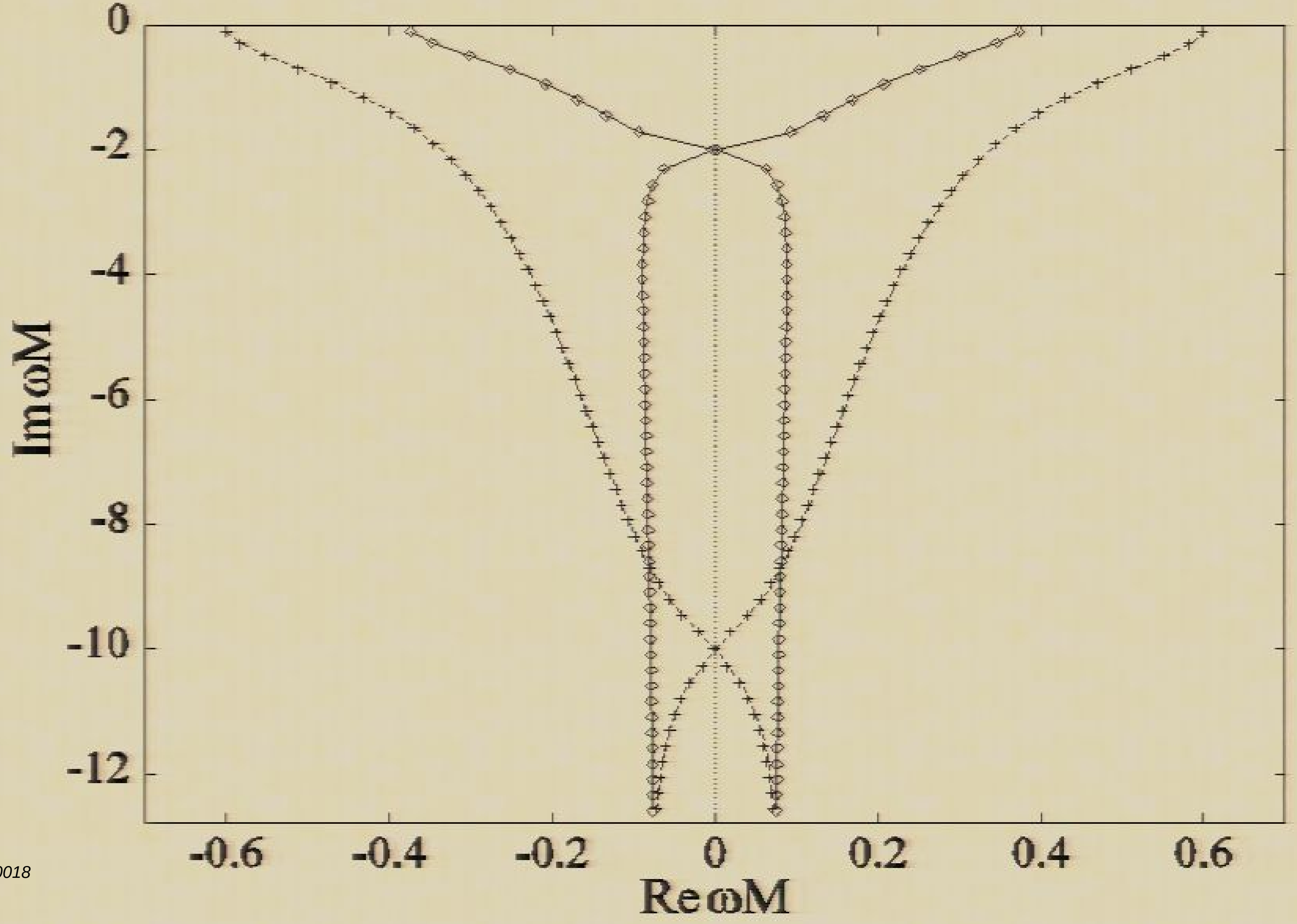
- gravitational $\quad g^{(0)}_{\mu\nu} + h_{\mu\nu}(r,t,\theta,\phi)$
- vector $\quad A_\mu(r,t,\theta,\phi)$
- scalar $\quad \phi(r,t,\theta,\phi)$

With $\quad \phi \sim e^{-i\omega t}\varphi(r,\theta,\phi)\,,$

Einstein's equations + boundary conditions at the horizon and spatial infinit

give the spectrum $\quad \omega_n = \omega_n(M)\,, \qquad n = 0, 1, 2, \ldots$

Quasinormal spectrum for a 4-dim Schwarzschild black hole



Black branes

$$ds^2 = H^{-1/2}\left(-f(r)dt^2 + dx^2 + dy^2 + dz^2\right) + H^{1/2}\left(dr^2 + r^2 d\Omega^2\right)$$

$$H(r) = 1 + \frac{R^4}{r^4}$$

$$f(r) = 1 - \frac{r_0^4}{r^4}$$

$$h_{\mu\nu}(r,t,x,y,z,\theta) \sim e^{-i\omega t + i\vec{q}\,\vec{x}}\, H_{\mu\nu}(r,\theta)$$

$$A_\mu(r,t,x,y,z,\theta) \sim e^{-i\omega t + i\vec{q}\,\vec{x}}\, V_\mu(r,\theta)$$

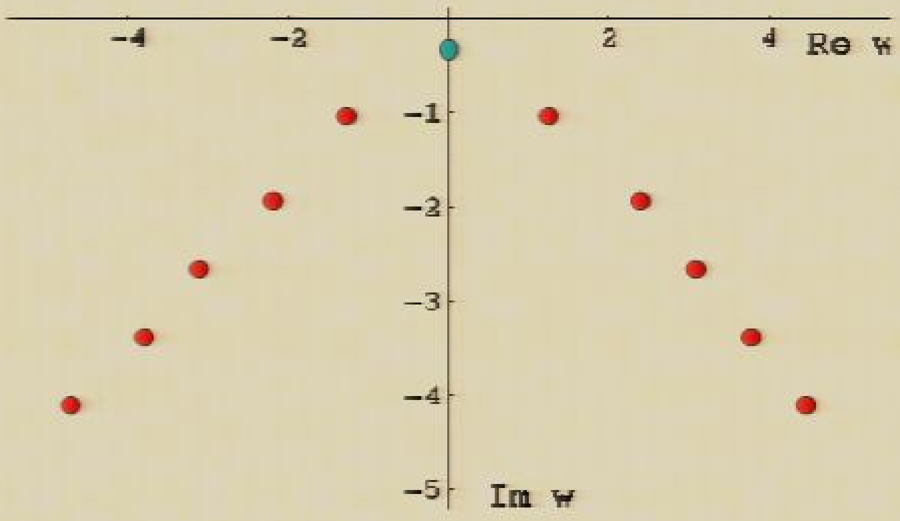
$$\phi(r,t,x,y,z,\theta) \sim e^{-i\omega t + i\vec{q}\,\vec{x}}\, S(r,\theta)$$

Quasinormal spectrum of black branes is of the form $\omega_n = \omega_n(q)$

Can have finite gap: $\omega = \omega_0 + aq + \ldots \qquad q \to 0$

Or be gapless, e.g. $\omega = v_s q - i\Gamma q^2 + \ldots \qquad q \to 0$

Quasinormal spectrum of asymptotically AdS 5d black three-brane



Vector fluctuations: $A_\mu(r,t,x,y,z,\theta) \sim e^{-i\omega t + i\vec{q}\,\vec{x}}\, V_\mu(r,\theta)$

The spectrum $\omega_n = \omega_n(q)$ has a purely imaginary frequency

The lowest quasinormal frequency of vector fluctuations in the background

$$ds^2 = G_{00}(r)dt^2 + G_{rr}(r)dr^2 + G_{xx}(r)\sum_{i=1}^{p}(dx^i)^2 + Z(r)K_{mn}(y)dy^m dy^n$$

where $\quad G_{00} = -(r-r_0)\gamma_0 + O((r-r_0)^2)$

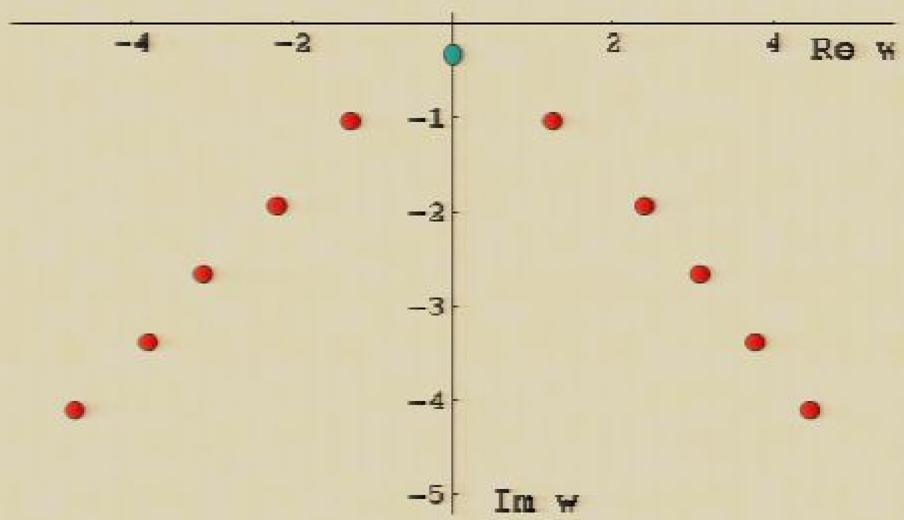
can be computed analytically for small $\quad \omega/T\,,\ \ q/T$

$$\omega = -i\,D\,q^2 + O(q^4)\,,$$

where

$$D = \frac{Z(r_0)\sqrt{-G(r_0)}}{G_{xx}(r_0)\sqrt{-G_{00}(r_0)G_{rr}(r_0)}}\int_{r_0}^{\infty} dr\,\frac{-G_{00}(r)G_{rr}(r)}{\sqrt{-G(r)}Z(r)}$$

Quasinormal spectrum of asymptotically AdS 5d black three-brane



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The same dispersion relation was previously obtained

in a very different setting

D.T.Son, P.K.Kovtun, A.S., hep-th/0309213

$$ds^2 = G_{00}(r)dt^2 + G_{rr}(r)dr^2 + G_{xx}(r)\sum_{i=1}^{p}(dx^i)^2 + Z(r)K_{mn}(y)dy^m dy^n$$

The membrane paradigm: introduce a stretched horizon at $r = r_* \simeq r_0$

Define conserved currents on the stretched horizon by $j^\mu = n_\nu F^{\mu\nu}|_{r_*}$

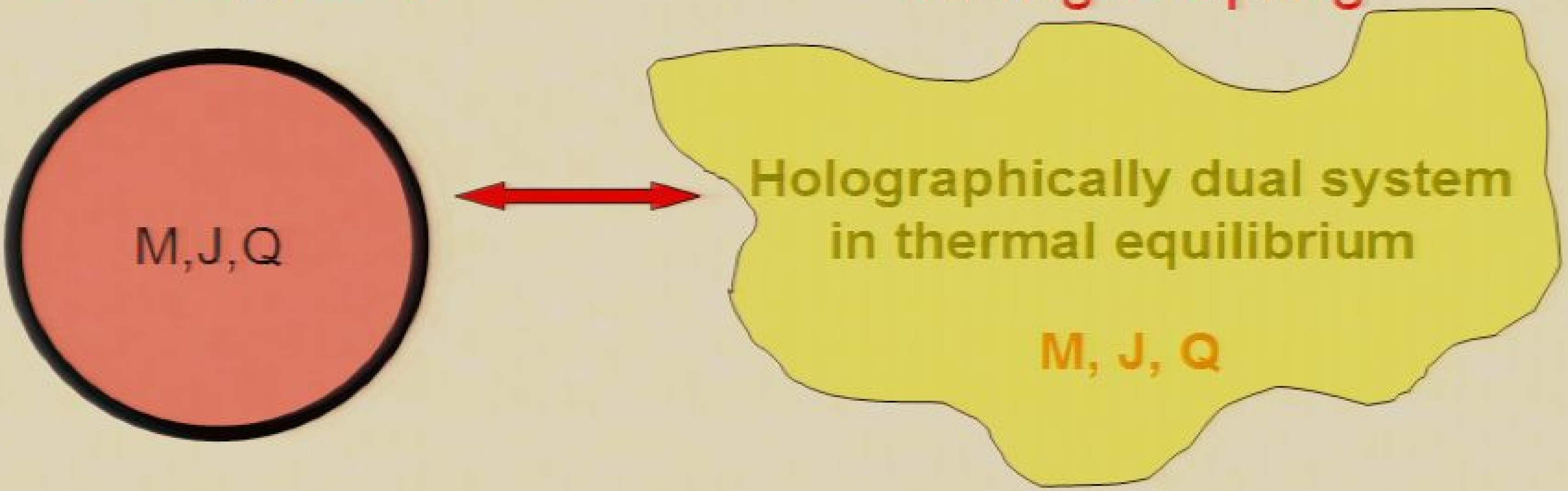
Then the charge density j^0 satisfies the diffusion equation

$$\partial_t j^0 = D \nabla j^0 ,$$

where $$D = \frac{Z(r_0)\sqrt{-G(r_0)}}{G_{xx}(r_0)\sqrt{-G_{00}(r_0)G_{rr}(r_0)}} \int_{r_0}^{\infty} dr \frac{-G_{00}(r)G_{rr}(r)}{\sqrt{-G(r)Z(r)}}$$

10-dim gravity

4-dim gauge theory – large N, strong coupling



T_{Hawking} $S_{\text{Bekenstein-Hawking}}$ $\Longleftrightarrow$ T S

Gravitational fluctuations $\Longleftrightarrow$ Deviations from equilibrium

$g^{(0)}_{\mu\nu} + h_{\mu\nu}$ $\Longleftrightarrow$????

$$j_i = -D\partial_i j^0 + \cdots$$

$"\Box" \ h_{\mu\nu} = 0$ and B.C. $\Longleftrightarrow$ $\partial_t j^0 + \partial_i j^i = 0$

Quasinormal spectrum $\Longleftrightarrow$ $\omega = -iDq^2 + \cdots$

Consequences

AdS/CFT context: since quasinormal frequencies = poles of the retarded correlators in the dual quantum field theory

D. Birminghem, I. Sachs, S. Solodukhin, hep-th/0112055
D. Son, A.S., hep-th/0205051
P. Kovtun, A.S., hep-th/0506184

we obtain an explicit expression for a diffusion constant of U(1) current for a large class of theories with gravity duals, e.g. $D = \frac{1}{2\pi T}$ for $\mathcal{N} = 4$ SYM

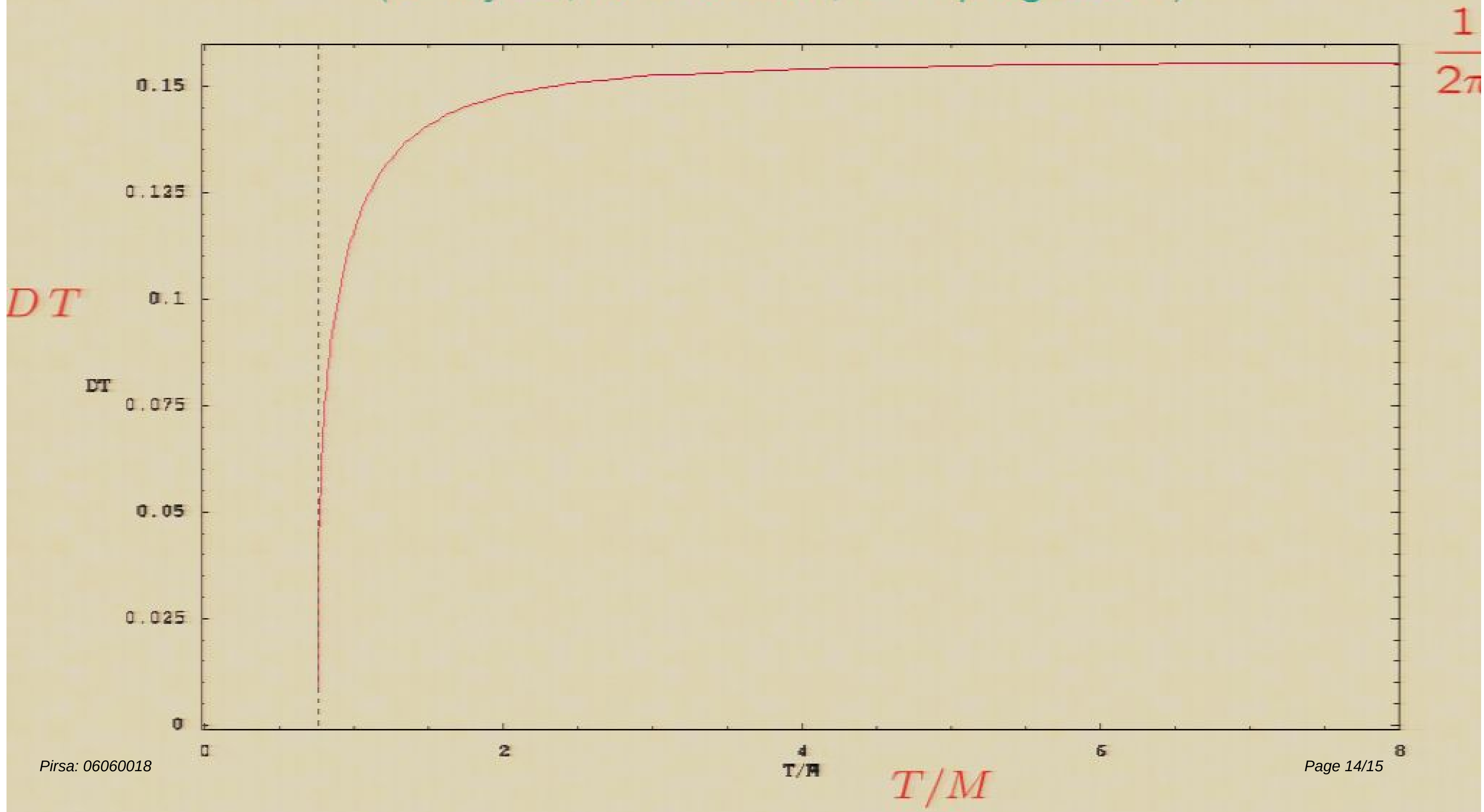
GR context: for gravitational vector-type perturbation, A.Buchel and J.Liu's universality result for D (hep-th/0311175) now mean that there exists a universal gravitational quasinormal frequency

$$\omega = -\frac{i}{4\pi T} q^2 + O(q^4)$$

Remark: this completes yet another (third) proof of the universality of the shear viscosity/entropy density ratio in QFTs with gravity duals

Flavor currents diffusion coefficient as a function of temperature/quark mass in strongly coupled thermal SYM with fundamental fermions

(R. Myers, R. Thomson, … in progress…)



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