Title: Quantum Information Theory 4

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Abstract:

Entanglement in the classical limit

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Introduction

Quantum and classical mechanics differ in their descriptions of **states** and **dynamics** of a system.

States:
Quantum wave functions vs classical probability distributions/points in phase space
Quantum entanglement (non-locality) vs. classical correlations

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|10\rangle + |01\rangle\right) \neq |a\rangle|b\rangle$$

Dynamics:
Quantum and classical theory predict different evolutions.
Quantum and classical dynamics of the mean values of observables can diverge after a finite time.

Classical mechanics can lead to **nonlinear** dynamics and **chaos**.
Divergence between quantum and classical evolutions can be very fast in chaotic systems even in the macroscopic regime.

Introduction

How does classical behaviour emerge from quantum mechanics?

Open quantum systems

Decoherence: Entanglement of a system with an environment can remove coherences and lead to classical states.

Continuous measurement: 'Environment' can be a detector that continuously monitors the system. Entanglement of a system with the detector can yield a measured trajectory which agrees with classical predictions.

Decoherence is equivalent to a measurement process in which the measurement results are not monitored.

Continuous Position Measurement

Bipartite system A+B: Position of A is continuously measured:

- **Stochastic Schrödinger equation**

$$d\rho = -\frac{i}{\hbar}[H,\rho]dt - k[z,[z,\rho]]dt + \sqrt{2k}\left(z\rho + \rho z - 2\langle z\rangle\rho\right)dW$$

Measurement record $dy = \langle z\rangle dt + (8k)^{1/2}\, dW$

k : measurement strength (resolution)

dW : Wiener (Gaussian) noise process

- **Quantum Dynamics**

$$d\langle z\rangle = \langle p\rangle/m\, dt + \sqrt{8k}C_{zz}dW$$
$$d\langle p\rangle = \langle -\partial U/\partial z\rangle dt + \sqrt{8k}C_{zp}dW$$

- **Classical Dynamics**

$$dz/dt = p/m$$
$$dp/dt = -\partial U/\partial z$$

- **Conditions for recovering classical trajectories**

The **covariances**

$$C_{ab} = \frac{\langle\hat{a}\hat{b}\rangle + \langle\hat{b}\hat{a}\rangle}{2} - \langle\hat{a}\rangle\langle\hat{b}\rangle$$

must remain **small** relative to the phase space of the dynamics.

Entanglement

- **Stochastic Schrödinger equation**

$$d\rho = -\frac{i}{\hbar}[H,\rho]dt - k\left[z,[z,\rho]\right]dt + \sqrt{2k}\left(z\rho + \rho z - 2\langle z\rangle \rho\right)dW$$

- **Evolution of measured subsystem A**

$$d\tilde{\rho}_1 = -\frac{i}{\hbar}\mathrm{Tr}_2\left([H,\rho]\right)dt + k(2z\tilde{\rho}_1 z - z^2\tilde{\rho}_1 - \tilde{\rho}_1 z^2)dt + \sqrt{2k}\left(z\tilde{\rho}_1 + \tilde{\rho}_1 z - 2\langle z\rangle\tilde{\rho}_1\right)dW$$

- **Linear entropy of measured subsystem A**

$$S = 1 - \mathrm{Tr}(\tilde{\rho}_1^{\,2})$$

$$dS = dS_0 - 8k\left\langle \tilde{\rho}_1\left(z - \langle z\rangle\right)^2\right\rangle dt - 4\sqrt{2k}\left\langle \tilde{\rho}_1^2\left(z - \langle z\rangle\right)\right\rangle dW$$

- **For Gaussian states**

$$dS_G = dS_0 - 16k\frac{\beta(1-\beta)}{(1+\beta)^3}C_{zz}dt \qquad \beta = S_G/(2 - S_G)$$

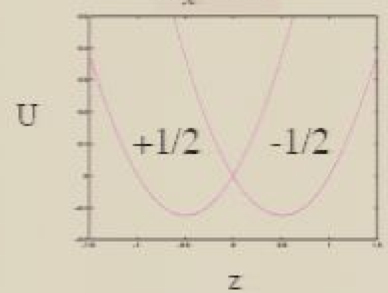
Large entanglement can persist in the classical limit, specially in chaotic systems

Spin-boson system

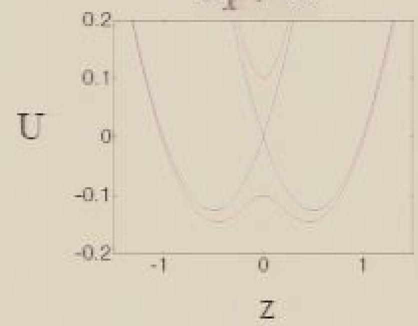
$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 z^2 + B_z z J_z + B_x J_x$$

Spin 1/2

$B_x = 0$:



$B_x \neq 0$:



- **Entanglement** between spin and motional degrees of freedom:

$$|\psi\rangle = |\uparrow\rangle|\psi_{left}\rangle + |\downarrow\rangle|\psi_{right}\rangle$$

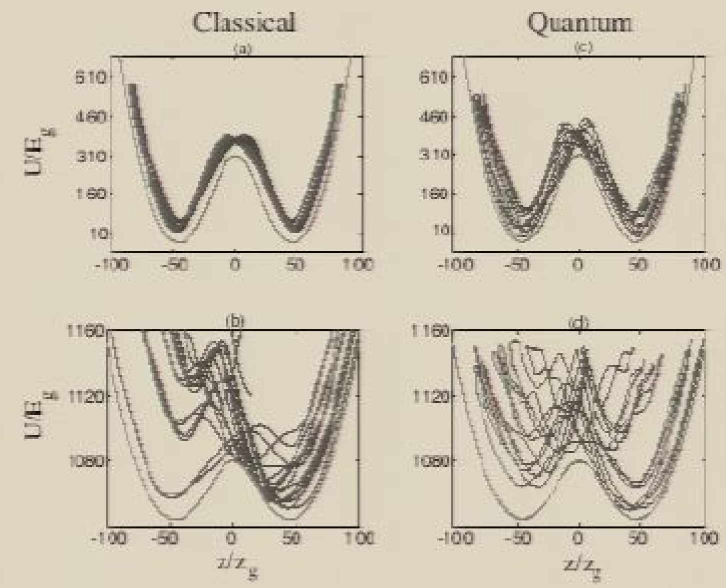
- **Classical:** Particle with a **magnetic moment** in a harmonic trap + magnetic field:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 z^2 + B_z z\mu_z + B_x\mu_x$$

$B_x = 0$: Integrable motion. $B_x \neq 0$: **Chaos**

Emergence of Chaotic Dynamics

$J = 200\hbar$



• Measured trajectories can recover mixed phase space in chaotic regime

• The largest classical **Lyapunov exponent** characterizing the classical chaos can be recovered from the quantum trajectories.

S. G et al PRA **69**, 052116 (2004).

S. G. et al PRA *A* **67**, 052102 (2003).

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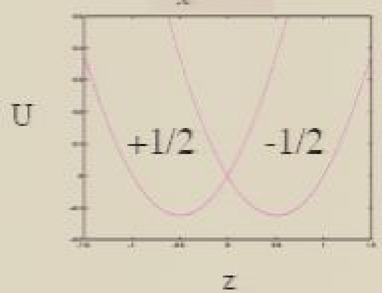

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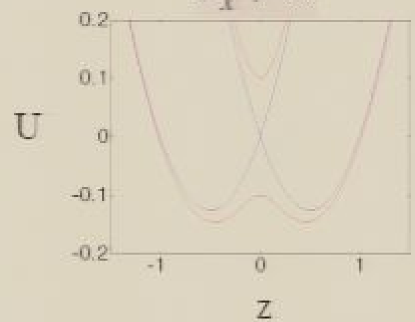

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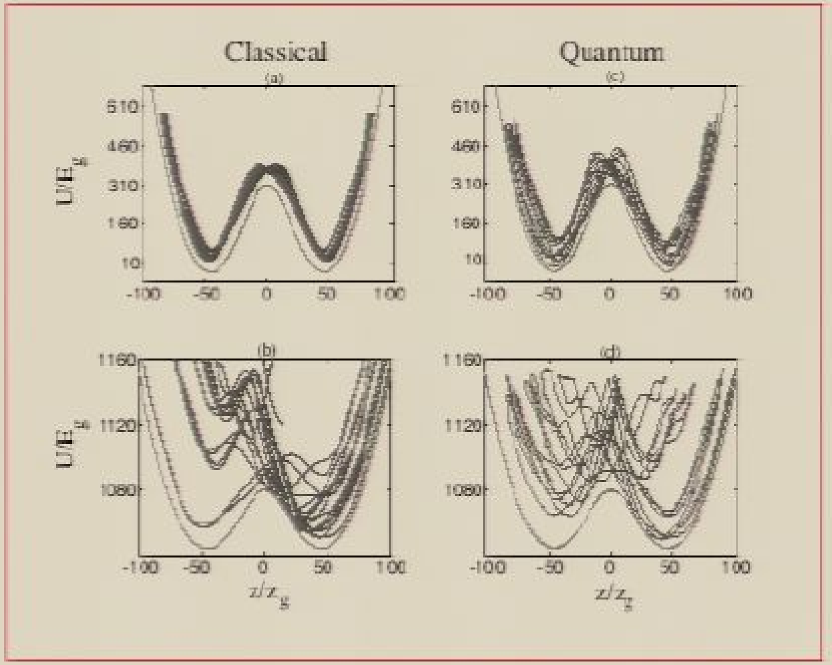

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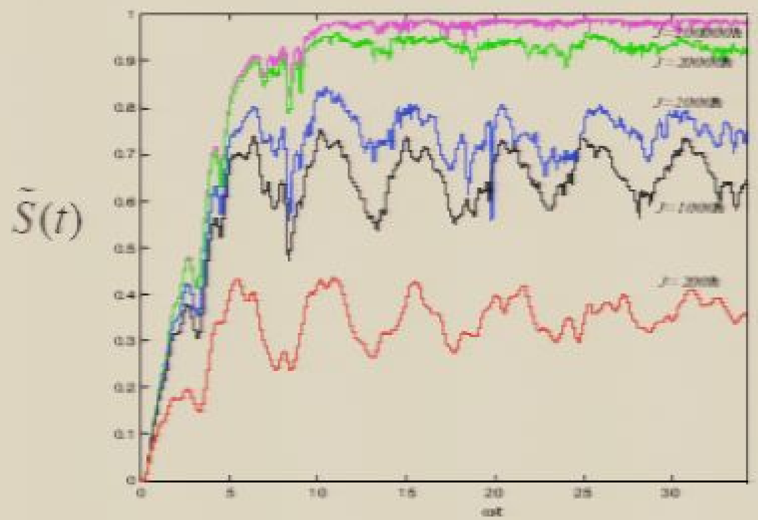
Entanglement

Linear Entropy of each subsystem:

$$\tilde{S}(t) = \frac{S(t)}{S_{max}} = \frac{1 - \mathrm{Tr}[\rho_{red}^2]}{1 - 1/(2J+1)}$$



• In the regime where classical **dynamics** is recovered, the underlying **states** can be highly entangled.

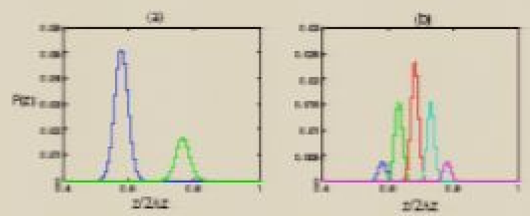
See also M. J. Everitt et al, New Journal of Physics 7, 64 (2005)

Entanglement

- Entanglement is related to the **overlap** of the spinor components of the wave function.

$$|\psi\rangle = \sum_{m=-J}^{J} \alpha_m |\phi_m\rangle |m\rangle$$

$$S = 1 - \sum_{m,n} \left| \alpha_m^* \alpha_n \langle \phi_m | \phi_n \rangle \right|^2$$



- **Weak** measurement does not resolve all the non-overlapping wave packets

$$S_{Gaussian} = 1 - \frac{\hbar/2}{\sqrt{C_{zz}C_{pp} - C_{zp}^2}}$$

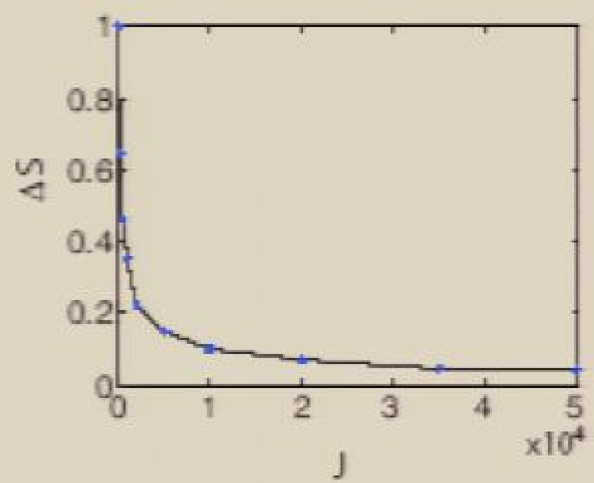
- S(t) can be large even if the covariances are small relative to the total phase space.

Entanglement

$$dS = dS_0 - 8k\left\langle \tilde{\rho}_1\left(z-\langle z\rangle\right)^2\right\rangle dt - 4\sqrt{2k}\left\langle \tilde{\rho}_1^2\left(z-\langle z\rangle\right)\right\rangle dW$$

- **Degree of change** in entanglement **decreases** for a constant measurement strength.

$$\Delta S = \sup_t\left(1-\frac{S(t)}{S_0(t)}\right)$$



Summary

- In coupled quantum systems, a weak continuous measurement can cause **strong back action** due to **entanglement**.

- In the large action limit, classical chaos can be quantitatively recovered from the quantum trajectories.

- The **strong localization** and **weak back action** conditions for the QCT set bounds on the **covariances**.

- Classical trajectories can be recovered even when there is a large amount of **entanglement.**

- The **change in entanglement** is related to the measurement back action.

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