Decoherence-free subspaces and spontaneous emission cancellation: necessity of Dicke limit

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Outline

- Why decoherence-free subspaces (DFS)?
- Dark states and spontaneous emission cancellation
- Decoherence-free subspaces
- A theorem about limitations of DFS
- Conclusions
Why DFS?

In quantum information processing qubits should be well isolated from their environment.

Decoherence-free subspaces consist of states that are not coupled to the environment.

DFS were first suggested by Duan & Guo '97, Zanardi & Rasetti '97, Lidar, Chuang & Whaley '98.

They could be used as an alternative/addition to quantum error correction.
Decoherence-Free Subspaces appear if a particular superposition of states is not coupled to the environment.

Consider a V-system with two excited states:

\[ |D\rangle = \Omega_2 |\rightarrow\rangle - \Omega_1 |\rightarrow\rangle \]

The state is not coupled to the ground state by this particular superposition of laser light.
Hamiltonian: \[ H_{\text{int}} = |e\rangle \left( \Omega_1 \langle - | + \Omega_2 \langle + | \right) + \text{H.c.} \]

\( \Omega_i = \) light field amplitude

Obviously \[ H_{\text{int}} |D\rangle = H_{\text{int}} \left( \Omega_2 |\rightarrow\rangle - \Omega_1 |+\rangle \right) = 0 \]

(very much like dark states)

\[ |\rightarrow\rangle \]
\[ |+\rangle \]

The state \[ |D\rangle = \Omega_2 |\rightarrow\rangle - \Omega_1 |+\rangle \]

is not coupled to the ground state by this particular superposition of laser light
To be immune against decoherence $|D\rangle$ must not couple to any mode configuration

This is possible if the transition matrix elements are the same: $d_{-g} = d_{+g} \iff$ spontaneous emission cancellation \cite{ZhuScully96}

\[
\begin{array}{c}
|g\rangle \\
\downarrow d_{-g} \\
|\rightarrow\rangle \\
\downarrow d_{+g} \\
\downarrow \downarrow |+\rangle
\end{array}
\]

However, selection rules forbid this for atoms

Molecules seem to be necessary, but ....
... another trick to achieve such a cancellation is to use more than one atom. Consider two 2-level atoms

The energy eigenstates form a V-system with equal dipole matrix elements. $|eg\rangle - |ge\rangle$ forms a DFS

\[
\begin{align*}
|e\rangle & \quad |e\rangle \\
\uparrow d_{eg} & \quad \uparrow d_{eg} \\
|g\rangle & \quad |g\rangle \\
\end{align*}
\Rightarrow
\begin{align*}
|eg\rangle & \quad |ge\rangle \\
\downarrow d_{eg} & \quad \downarrow d_{eg} \\
|gg\rangle & \quad \\
\end{align*}
\]
This procedure can be generalized to many N-level atoms [Duan & Guo '97, Zanardi & Rasetti '97, Lidar, Chuang & Whaley '98]

Theories usually employ master equations and Lie groups or Dicke states.

However, there's a catch:
How far can the atoms be apart? Surely there's a limit.
We have proven that, under very general conditions, a real DFS can only be obtained in the Dicke limit.

Dicke limit: particles are co-located

Main assumptions of the theorem:

- System is composed of particles with a finite-dimensional internal Hilbert space located at a fixed position
- Markovian reservoir (no memory)
- Reservoir invariant under translations
- Energy is exchanged in system-reservoir interaction
Some details: \[ H_{\text{int}} = \sum_n \sum_i E^i(x_n) \, d_{i,n} \]

with reservoir operators \( E^i \) and system operators \( d_{i,n} \)

Translational invariance is used to employ the Lehmann representation for reservoir operators

\[
\hat{E}^i(x_0 + x) = e^{i \hat{P} \cdot x / \hbar} \hat{E}^i(x_0) e^{-i \hat{P} \cdot x / \hbar}
\]

The Markovian master equation can be written as

\[
\dot{\rho} = -\hat{\Gamma} \rho - \rho \hat{\Gamma} - i[H, \rho] + \text{jump terms}
\]

where the decoherence matrix \( \hat{\Gamma} \) describes the (de-) excitation of any particle in the system
Energy conservation (or time averaging) is needed to keep the master equation consistent [Dumcke & Spohn 1979]

Without it the system would gain or loose energy in processes like photon-exchange between two atoms.
After lengthy calculations $\hat{\Gamma}$ can be written as

$$\hat{\Gamma} = \sum_{\Delta E} \int d^2 \hat{k} \sum_{n,m} e^{ik_0 \hat{k} \cdot (x_n - x_m)} \hat{R}_n(\Delta E) \hat{R}_m^\dagger(\Delta E)$$

• $\hat{k}$ is a unit vector (direction of the reservoir momentum)
• $\hat{R}_n(\Delta E)$ changes the energy of particle $n$ by $\Delta E$
\[ \hat{\Gamma} = \sum_{\Delta E} \int d^2 k \sum_{n,m} e^{ik_0 \hat{k} \cdot (x_n - x_m)} \hat{R}_n(\Delta E) \hat{R}_m^\dagger(\Delta E) \]

Our proof exploits that \( \hat{\Gamma} \) can only have zero eigenvalues for all reservoir modes \( \hat{k} \) if \( x_n = x_m \), i.e., in the Dicke limit. Otherwise the integral will always contain nonzero parts.
This result points out where DFS may be realized:

- Single-particle DFS (spontaneous emission cancellation)
- The integral disappears for a 1D reservoir. Waveguides may therefore allow for ordinary DFS outside the Dicke limit.
\[ \hat{\Gamma} = \sum_{\Delta E} \int d^2 \hat{k} \sum_{n,m} e^{i k_0 \hat{k} \cdot (x_n - x_m)} \hat{R}_n(\Delta E) \hat{R}_m(\Delta E) \]

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- Single-particle DFS (spontaneous emission cancellation)
- The integral disappears for a 1D reservoir. Waveguides may therefore allow for ordinary DFS outside the Dicke limit.
Conclusions

- DFS are a tool to suppress decoherence

- DFS can only exist in Dicke limit,

- single-particle DFS, or 1D reservoirs may be a way around this problem