Title: Nuclear Theory/Heavy Ions 1

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Abstract:

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Quantum Corrections for a Bose gas at $T < T_c$

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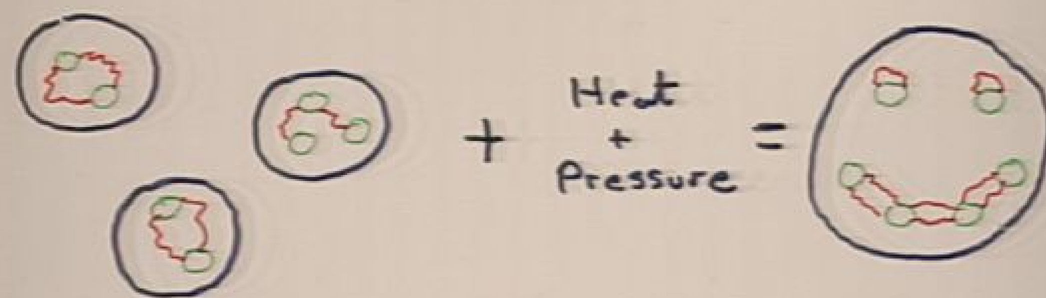
- Why finite temperature and density?
  : Quark-gluon plasma
  : Early universe

- Field theory at finite temperature
  : Formalisms
  : New problems

- Application to Bose-Einstein condensation
  : thermodynamic properties
  : behaviour near critical point

Where are finite temperature effects important?

– Quark-gluon plasma



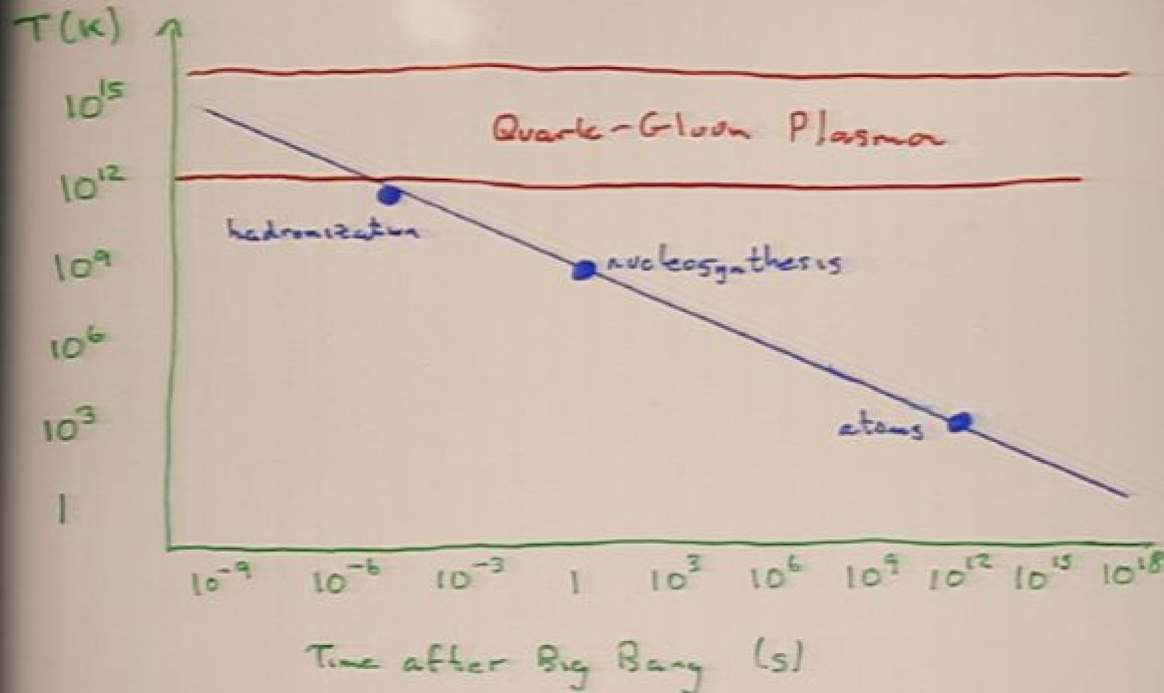
: expected to be seen in heavy ion collisions

Temp ~ 170 MeV ; Density ~ 1 $\frac{\text{GeV}}{\text{fm}^3}$

: signatures expected to be, eg, enhanced production rates of various particles

: tentatively claimed to be found at RHIC in April, 2005

Early Universe



hadronization : protons and neutrons form

nucleosynthesis : nucleii form

Finite temperature formalisms (equilibrium)

$$\langle 0 | \mathcal{O} | 0 \rangle \rightarrow \mathrm{Tr} \langle \mathcal{O} e^{-\beta H} \rangle$$

$$\beta = \frac{1}{k_B T}$$

$$G(t_1 - t_2) = \mathrm{Tr} \langle \phi(t_1) \phi(t_2) e^{-\beta H} \rangle$$

$$= G(t_1 - t_2 - i\beta)$$

Two-point function is periodic in imaginary time, with period $i\beta$.

- Imaginary time formalism

$$t \rightarrow -i\tau$$

$$\int_{-\infty}^{\infty} dt \rightarrow -i \int_0^{\beta} d\tau$$

$$k_0 \rightarrow i 2\pi n \beta$$

$$\int_{-\infty}^{\infty} dk_0 \rightarrow \sum_{n=0}^{\infty}$$

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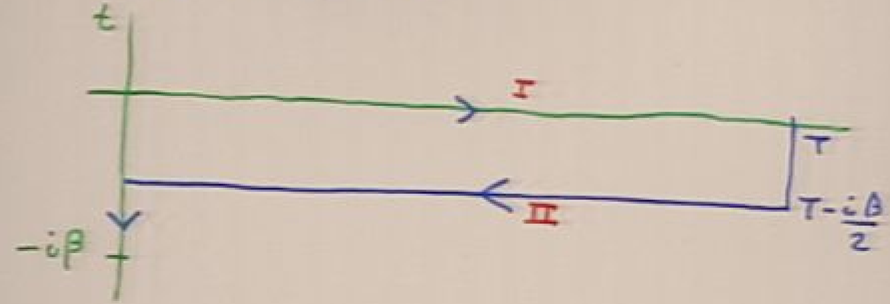
Real-time formalisms



$$G(k_0) = \begin{pmatrix} G_{\mathrm{I}\mathrm{I}}(k_0) & G_{\mathrm{I}\mathrm{II}}(k_0) \\ G_{\mathrm{II}\mathrm{I}}(k_0) & G_{\mathrm{II}\mathrm{II}}(k_0) \end{pmatrix}$$

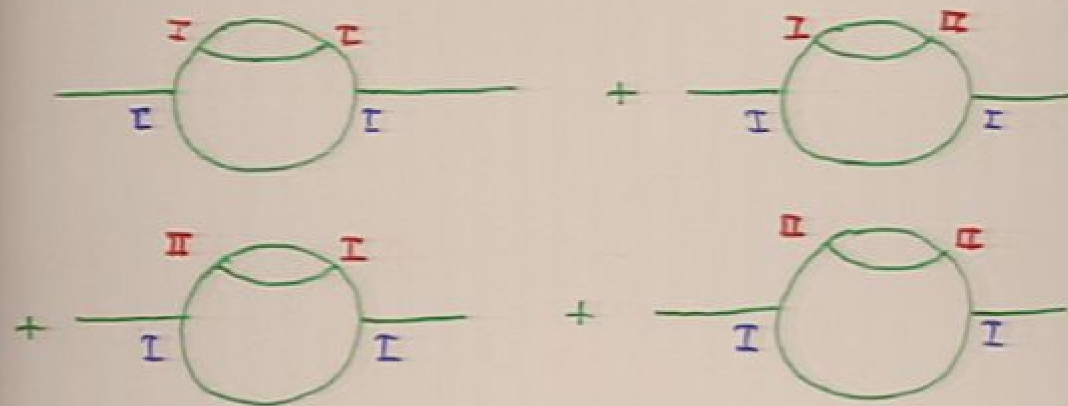
$$G_{\mathrm{I}\mathrm{I}}(k_0) = \frac{i}{k^2-m^2+i\varepsilon} + 2\pi\, n(k_0)\, \delta(k^2-m^2)$$

$$= G^{*}_{\mathrm{II}\mathrm{II}}(k_0)$$

$$G_{\mathrm{I}\mathrm{II}}(k_0) = G_{\mathrm{II}\mathrm{I}}(k_0) = 2\pi\, \theta(k_0)\, n(k_0)\, \delta(k^2-m^2)$$

$$n(k_0) = \frac{1}{e^{\beta|k_0|}-1}$$

Avoid Matsubara sums by doubling degrees of freedom.

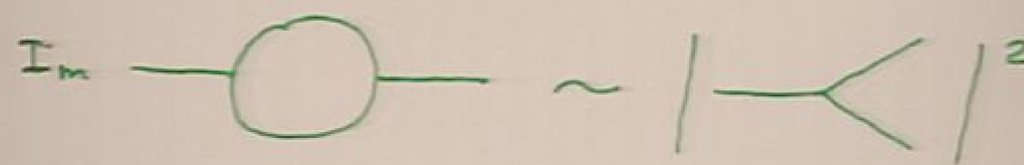


Notes :

- no additional ultraviolet finite temp divergences due to $\frac{1}{e^{\beta|k_0|}-1}$
- potentially additional infrared finite temp divergences due to $\frac{1}{e^{\beta|k_0|}-1}$ (bosons)
- ill-defined products like $\frac{i}{k^2-m^2+i\varepsilon}\,\delta(k^2-m^2)$ must cancel when all terms added
-

Finite temperature can change dramatically behaviour of theories.

For example, consider decay rates



At finite temperature, get contributions from particles coming from and going into the heat bath, and so processes forbidden at zero temp can arise at finite temp.

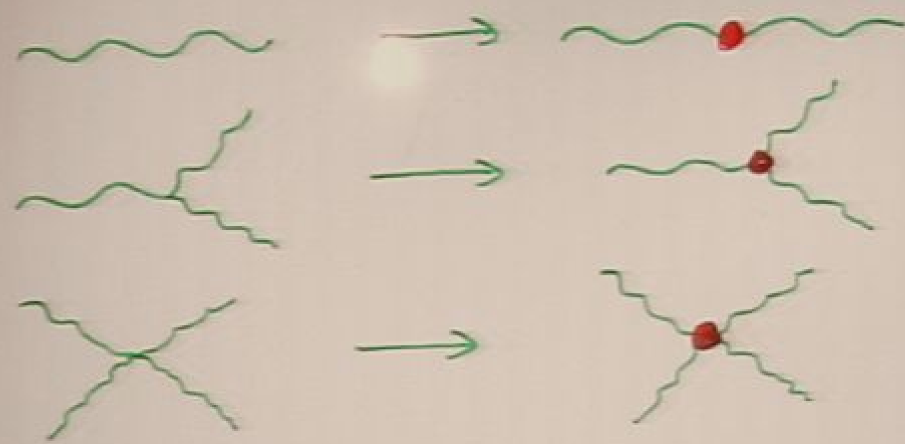
However, IR problems arise, esp. in gauge theories. Consider pure QCD at high temperature

$\mathrm{Re}\,\Sigma \sim T^2$: finite temp mass (gauge independent)

$\mathrm{Im}\,\Sigma \sim gT$: finite temp decay rate (gauge dependent)

But poles of propagator must be gauge independent.

Solution is to use Braaten-Pisarski resummation.



Such a resummation scheme leads to gauge-independent poles in the propagator.

Would like a physical system, with known experimental data, to test finite temperature field theory on.

– Bose – Einstein condensation

– based on work by Bose, Einstein in 1925 predicted that a boson system, when cooled, would collapse to a system where all particles are in their lowest energy state.

$$E \sim k_B T \sim \tfrac{1}{2} m v^2 \Rightarrow m v \sim \sqrt{2 m k_B T}$$

$$\lambda \sim \frac{h}{p} \sim \frac{h}{\sqrt{2 m k_B T}}$$

At room temp, λ << interatomic spacing, so the particles are uncorrelated.

At low temp, $\lambda \sim$ interatomic spacing, so the particles lose their identity, and become governed by Bose-Einstein satatistics.

The Bose-Einstein condensate is described by a single macroscopic wave function, and is a very nice system to study fundamental quantum mechanics (eg, interference between two nearby condensates).

- Experimental discoveries

  - 1938 : ^4He at $T \sim 2.2K$ becomes a superfluid :
    - zero viscosity
    - quantized vortices
    - zero entropy
    - infinite thermal conductivity
  - not considered a "pure" Bose-Einstein condensate, as it's a fluid, and so relatively strong interactions are present

  - first "true" Bose-Einstein condensate found in 1995 by cooling 2000 Rubidium atoms to 170 nK using laser cooling (1997 Nobel prize). This led to further systems being discovered (eg, ^{23}Na).

  - won the 2001 Nobel prize.

Will examine ^4He superfluid / Bose Einstein condensate as a testing ground for finite temperature field theory.
- strong interactions (like QCD), which tests perturbation theory
- look at system near the critical point, which tests equilibrium assumption.

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Model

$$H = H^{(0)} + V^{(2)}$$

$$H^{(0)} = -\frac{1}{2m}\int d^3x \; \psi^+(t,\vec{r})\,\Delta\,\psi(t,\vec{r}) - \mu N$$

$$V^{(2)} = \frac{1}{2}\int \psi^+(t,\vec{r})\,\psi^+(t,\vec{r}\,')\,U^{(2)}(\vec{r}-\vec{r}\,')\,\psi(t,\vec{r}\,')\,\psi(t,\vec{r})$$

$$U^{(2)}(\vec{r}-\vec{r}\,') = \frac{4\pi\hbar^2 f}{m}\,\delta^{(3)}(\vec{r}_1 - \vec{r}_2)$$

f = s-wave scattering length

$\Delta = \vec{\nabla}^2$

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\Delta - \mu\right]\psi + \frac{4\pi\hbar^2 f}{m}\psi^+\psi\psi$$

Let

$$\psi = \psi' + \Theta$$

ψ' = normal phase operator

$\Theta \sim \sqrt{n_0}$ = condensed phase operator

The Green functions of interest are

$$G(\vec{r}_1,t_1;\vec{r}_2,t_2) = -i n_0 - i G'(\vec{r}_1,t_1;\vec{r}_2,t_2)$$

$$G'(\vec{r}_1,t_1;\vec{r}_2,t_2) = -i \langle 0,\beta | T\left[\psi'(\vec{r}_1,t_1)\,\psi'^{\dagger}(\vec{r}_2,t_2)\right] | 0,\beta\rangle$$

$$F(\vec{r}_1,t_1;\vec{r}_2,t_2) = -i \langle N-2 | T\left[\psi'(\vec{r}_1,t_1)\,\psi'(\vec{r}_2,t_2)\right] | N\rangle$$

Use perturbation theory to calculate these Green functions. Diagrams involved are

: normal phase

: condensate

: interaction

Tree level (no quantum corrections) corresponds to mean field approximation).

Must go to 2nd order in perturbation theory.

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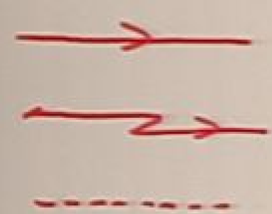

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Procedure

1. Fix T and the gas density n (equal to density of ^4He at room temp)

2. Choose $\mu = -aT$, where $a > 0$

3. Find n_0 iteratively in

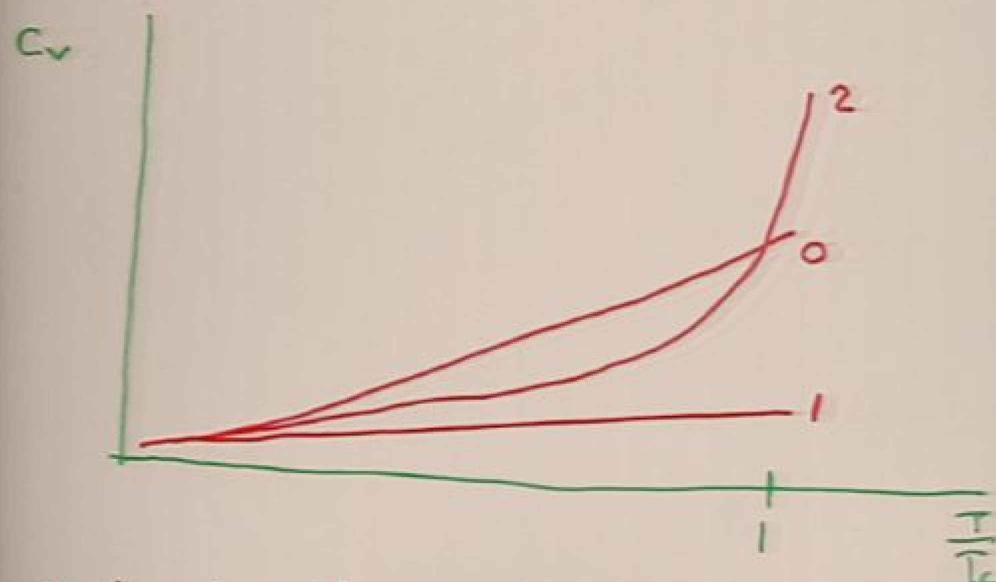
$$n = n_0 + i\,G(t=-0, \vec{r}=0)$$

4. Find the value of $F = F(n_0, \mu, T, n)$, and search for a minimum

5. Use this value of μ at the minimum in (2), and repeat steps (3) and (4) until self-consistency is reached.

Once the free energy is found, thermodynamic properties like the internal energy, specific heat, number of particles, and entropy can be computed.

The chemical potential is non-zero only in 2^{nd}



Must go to 2nd order to get correct C_v

Critical exponent

$$m_0 = \frac{N_0}{N} = \frac{\text{number of condensed phase particles}}{\text{total number of particles}}$$

As $T \to T_c$,

$$m_0 = A\,(T_c - T)^{\beta}$$

Use an ultraviolet cutoff

$$\frac{p_c^2}{2m} = \alpha n_0 U + \frac{3}{2} k_B T \quad \ldots \alpha = 1,\ U = \frac{4\pi\hbar^2 f}{m}$$

No need for an infrared cutoff except at T_c.
Can examine how things change as the cutoff changes; find, for a range of $\frac{1}{2} < \alpha < 2$,

A : $0.88 \to 0.84$
T_c : $2.12 \to 2.15$
β : $0.30 \to 0.38$ (classically, $\beta = \frac{1}{2}$)

Could also do a similar fit for a range of the scattering length f.

Experimental values agree with these ranges.

Conclusions

- used ^4He as a testing ground for thermal field theory (also, it's an interesting system in its own right).

  : relatively strongly interacting system

  : system near the phase transition is not, presumably, changing very little

- Nevertheless, thermal field theory is able to describe successfully properties of the system that can be experimentally measured

- One lesson to be drawn is that perturbation theory can be used in such systems, but in this example, must go beyond the leading order to get even the right qualitative behaviour.

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