Title: Interpretations of Probability in Quantum Mechanics

Date: Jul 19, 2006 09:15 AM

URL: http://pirsa.org/06070043

Abstract: Taking for granted that the mathematical apparatus for describing probabilities in quantum mechanics is well-understood via work of von Neumann, LÃ¼ders, Mackey, and Gleason, we present an overview of different interpretations of Â“probabilityÂ” in quantum mechanics bearing on physics and experiment, with the aim of clarifying the meaning and place of so-called Â“objectiveÂ” interpretations of quantum probability.

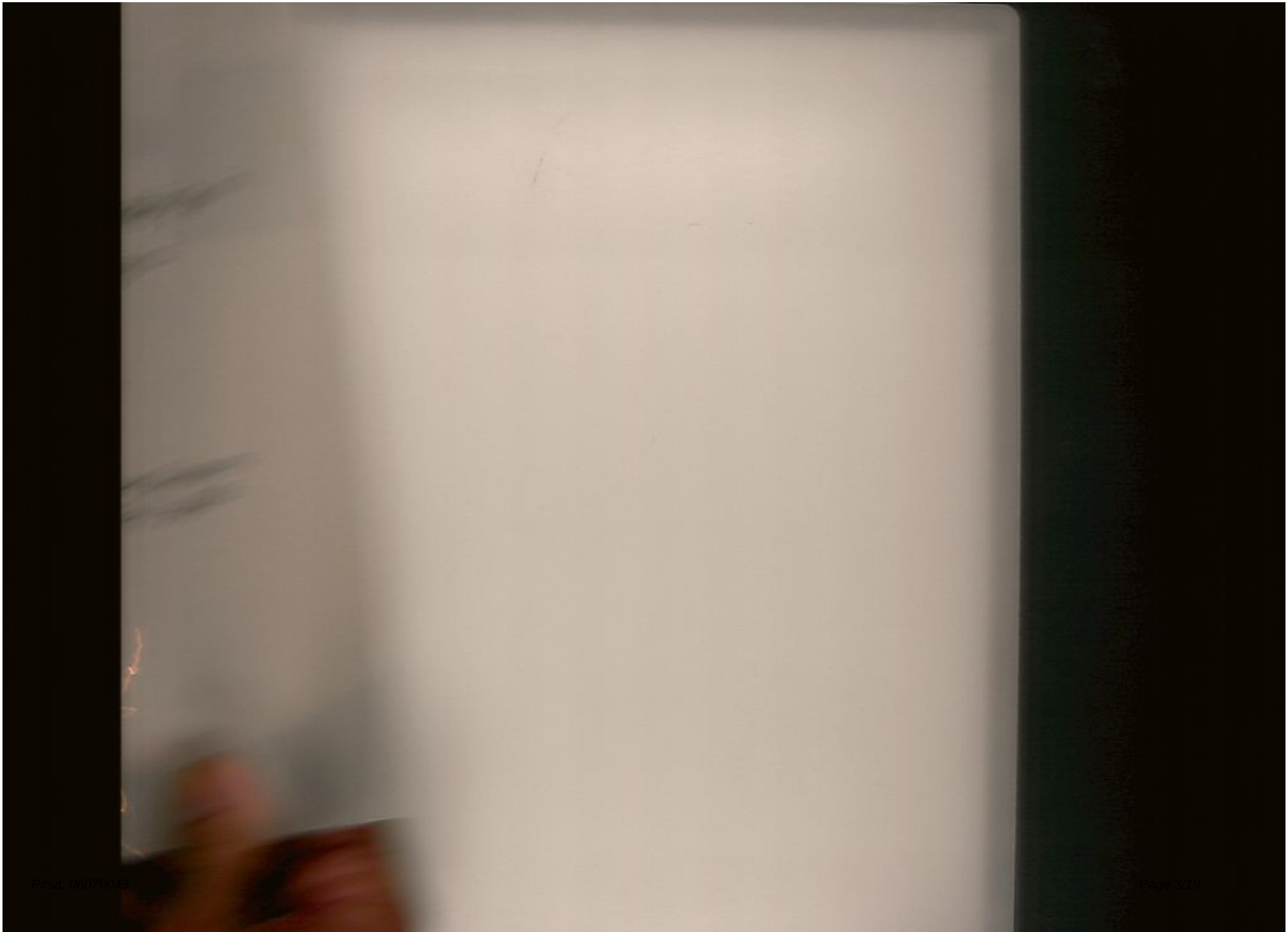
The dichotomy Â“objective/subjectiveÂ” is unfortunate, we argue, as we should distinguish two different dimensions integral to the concept of probability. The first concerns the values of probability functions, viz. what the real numbers measure, e.g. relative frequencies of experimental outcomes, or strengths of physical dispositions (objective-1), vs. degrees of belief of idealized agents (subjective-1), etc. But a second dimension is also important, concerning the domain of definition, the Â“eventsÂ” or Â“bearersÂ” of probability, what the probabilities are probabilities of. Relative frequencies of what, described how, or strengths of dispositions to do what, described how, degrees of belief in what, etc. Reminding ourselves of the quantum mechanical phenomenon of incompatible observables, we recall that contradictions are standardly avoided by describing probabilities as pertaining to Â“measurement outcomesÂ” rather than Â“possessed propertiesÂ”: thereby, subjective elements are introduced into the very description of the Â“eventsÂ”. (Interpretations qualify as objective-2 if they avoid Â“bad wordsÂ” like Â“measurementÂ” as primitive, in favor of possessed properties or physical interactions; as subjective-2 if such terms are employed in an essential way.)

This leads to a two-by-two matrix of interpretative possibilities. The remainder of our talk consists in filling in the blanks (which the reader is invited to try for him/herself) and providing commentary on the relative advantages and disadvantages, which go to the heart of the problem of interpreting quantum theory. Given our scheme, it turns out that Â“objective version of CopenhagenÂ” makes good sense; this is one locus of Â“propensitiesÂ”, which can be made sense of, we claim, along the lines of pre-hidden-variables Bohm (his 1951 text), not to be confused with Popper. We close by noting a serious deficiency in recent Bayesian approaches to quantum probability (lying in the subj-1, subj.-2 quadrant), viz. its explanatory impoverishment. But IÂ’ve already given too many hints.

Interpretations of Probability in Quantum Mechanics

Geoffrey Hellman

Conference in honour of Abner Shimony,
Perimeter Institute, July 18-21, 2006



1 Generalized probability measures, Gleason's Theorem

States can be represented as *density matrices, W*: positive, self-adjoint operators such that, for any orthonormal basis $\{\psi_k\}$,

$$\sum_k (\psi_k, W\psi_k) = 1$$

For B a bounded linear operator and $\{\psi_j\}$ an orthonormal basis, define

$$TrWB = \sum_j (\psi_j, WB\psi_j),$$

the *trace* of WB. (The traces of WB and BW are independent of choice of o.n. basis.)

Theorem (von Neumann) If a finite expectation value $\langle B \rangle$ is defined for all bounded operators B, then there is a unique density matrix W such that

$$\langle B \rangle = TrWB.$$

1 Generalized probability measures, Gleason's Theorem

States can be represented as *density matrices*, W: positive, self-adjoint operators such that, for any orthonormal basis $\{\psi_k\}$,

$$\sum_k (\psi_k, W\psi_k) = 1$$

For B a bounded linear operator and $\{\psi_j\}$ an orthonormal basis, define

$$TrWB = \sum_j (\psi_j, WB\psi_j),$$

the *trace* of WB. (The traces of WB and BW are independent of choice of o.n. basis.)

Theorem (von Neumann) If a finite expectation value $\langle B \rangle$ is defined for all bounded operators B, then there is a unique density matrix W such that

$$\langle B \rangle = TrWB.$$

(Here $\langle\rangle$ satisfies the conditions that if B is self-adjoint (and positive), $\langle B\rangle$ is real (and non-negative), $\langle cB\rangle = c\langle B\rangle$, $\langle A+B\rangle = \langle A\rangle + \langle B\rangle$, $\langle 1\rangle = 1$, and for mutually orthogonal projectors E_k, $\left\langle \sum_k E_k \right\rangle = \sum_k \langle E_k\rangle$.)

Then a quantum state can be taken as represented by a unique density matrix W. Taking B as ranging over projection operators, E, the finite expectation $\langle E\rangle$ is a normalized, generalized probability, where the last condition above generalizes "disjoint additivity". Recall that projection operators are in 1-1 correspondence with the closed linear subspaces of the Hilbert space representing the system. The subspaces in turn correspond to quantum "propositions" of the form "observable A has a value lying in Borel set b". A density matrix W in effect assigns a probability $p(S)$ to each such subspace S via

$$p(S) = \left\langle E^S \right\rangle = TrWE^S,$$

for all S, where E^S is the projector onto S. Where the state is pure (usually represented as a vector φ), $W =$

$|\varphi\rangle\langle\varphi|$.If the state is mixed, it can be taken as a density matrix $W = \sum_k w_k |\varphi_k\rangle\langle\varphi_k|$, where the φ_k are orthonormal and the weights w_k are positive summing to 1. Thus the quantum pure and mixed states furnish generalized probability measures on the closed linear subspaces of Hilbert space. Are there in principle any other gpm's? If the (separable) Hilbert space is of dimension 3 or greater, the answer is **no.** That is

Gleason's Theorem Let a Hilbert space be of dimension > 2 and let $\langle E\rangle$ map projectors E to non-negative real numbers, with $\langle 1\rangle = 1$ and obeying additivity for any set of mutually orthogonal projectors; then there is a unique density matrix W such that

$$\langle E\rangle = TrWE,$$

for every projector E.

Probability Concepts in Some Interpretations of QM

	Obj-2	Subj-2
Obj-1	Modal Interps.	Textbook (e.g. Bohm '51)
Subj-1	Bohmian Mechanics	Instrumentalist CI, Bayesian

Obj-1, Obj-2: Modal Interpretations:

- Give up eigenvalue-eigenstate link.
- Assign more precise values than that rule allows, but not "too many".
- Probabilities are of *possession of properties*, but latter are quantum-mechanical.
- Good measurements reveal these but "measurement" is not primitive. Rules are framed in terms of QM-interactions or the mathematical form of states.
- Ultimate physical randomness is recognized: pure states give measures of strength of dispositions to actualize various values in interactions.
- Challenges: Lorentz-invariance; extension to quantum field theory.

Also: GRWP:

Propensities for collapse events (random) (Obj 1)

These allow assigning values of q.m.-magnitudes.

Obj-1, Obj-2: Modal Interpretations:

- Give up eigenvalue-eigenstate link.
- Assign more precise values than that rule allows, but not "too many".
- Probabilities are of *possession of properties*, but latter are quantum-mechanical.
- Good measurements reveal these but "measurement" is not primitive. Rules are framed in terms of QM-interactions or the mathematical form of states.
- Ultimate physical randomness is recognized: pure states give measures of strength of dispositions to actualize various values in interactions.
- Challenges: Lorentz-invariance; extension to quantum field theory.

Also: GRWP:

Propensities for collapse events (random) (Obj 1)

These allow assigning values of q.m. magnitudes. i.e. probs of possession of props (Obj. 2)

Probability Concepts in Some Interpretations of QM

	Obj-2	Subj-2
Obj-1	Modal Interps.	Textbook (e.g. Bohm '51)
Subj-1	Bohmian Mechanics	Instrumentalist CI, Bayesian

Obj-1, Subj-2: Textbook (e.g. Bohm, 1951):

- Eigenvalue-eigenstate link respected.
- Probabilities are *of* measurement outcomes, classically described; "recording apparatus" taken as given (Subj-2).
- However, probabilities measure *strengths of complex physical "tendencies" or dispositions* (Obj-1) to reveal values of measured observables.
- Ultimate physical randomness in the realization process is allowed for and recognized. (Quantum pure states are "complete".)
- Drawbacks: Need for "cut" between quantum and classical domains. How make sense of wave-function for whole universe? Randomized phase factors reflecting measurement interactions yield only a FAPP solution of the measurement problem.

Obj-1, Subj-2: Textbook (e.g. Bohm, 1951):

- Eigenvalue-eigenstate link respected.
- Probabilities are *of* measurement outcomes, classically described; "recording apparatus" taken as given (Subj-2).
- However, probabilities measure *strengths of complex physical "tendencies" or dispositions* (Obj-1) to reveal values of measured observables.
- Ultimate physical randomness in the realization process is allowed for and recognized. (Quantum pure states are "complete".)
- Drawbacks: Need for "cut" between quantum and classical domains. How make sense of wave-function for whole universe? Randomized phase factors reflecting measurement interactions yield only a FAPP solution of the measurement problem.

Obj-2, Subj-1: Bohmian (hidden) mechanics

(http://plato.stanford.edu/entries/qm-bohm/#eqs):

- Eigenvalue-eigenstate link not respected, definite positions assigned to systems at all times.

- Probabilities are *of* possessed position properties; all quantum "observables" reduced to these.

- Probabilities measure relative frequencies but relative to ensembles reflecting our ignorance of actual underlying positions and velocities.

- Ultimate randomness not recognized; positions evolve deterministically, but epistemically are (largely) inaccessible: Heisenberg uncertainty relations respected.

- No quantum/classical cut; no measurement problem. Main challenges: high degree of non-locality; how extend to relativisitic quantum fields?

Subj-1, Subj-2: Instrumentalist CI, Bayesian

- Eigenvalue-eigenstate link respected, if values are assigned to microsystems at all!

- Probabilities measure relative frequencies in experimentally selected ensembles, or degrees of belief of rational betting agents.

- Probabilities are *of* "measurement outcomes", "pointer readings" or "appearances" thereof.

- Appears to avoid theoretical problems (e.g. realization of definiteness, measurement problem, etc.); but provides no physical explanation of observed frequencies. Seems to renounce enterprise of physics!

Does the Subj-Subj quadrant really offer refuge from the measurement problem?

That problem arises from three assumptions:

1. "Universal applicability of quantum mechanics" (ultimately including our states of awareness)

2. Eigenvalue-eigenstate Link (no value of a quantum observable if system is not in an eigenstate of it)

3. Our subjective awareness of definite outcomes of "pointer systems".

By 1, let Ψ be a superposition of triple-product states representing a joint system consisting of quantum object (e.g. electron) + pointer system (for an observable, e.g. spin) + person (say Jeff Bub) observing pointer. Under suitable assumptions, at the conclusion of, say, a spin mmt, Jeff Bub is not in a state of experiencing any definite outcome, by assumption 2, which directly contradicts (what we may suppose to be) an instance of fact 3.

The Everettian response, that our subjective sense of definiteness may be illusory (in analogy with illusions we have about time, say, corrected by relativistic physics), is, we claim, ineffectual against "Descartes' demon": even if it only *appears* to us that we are experiencing definiteness of pointer systems, that appearance itself suffices to generate a contradiction (adjusting Ψ, and appealing to 1 and 2).

Bohrian "solutions" essentially give up 1, Universality; modal interpretations give up 2, the eigenvalue-eigenstate link. As Shimony has reminded us, taking account of work of Leggett, et al, 1 is an outstanding empirical-physical question. We are simply not now in a position to solve the problem.

Finally, consider Glashow's question:

"Ok [after running through the above argument], I guess I *do* understand what the measurement problem is, after all, and I see that it's genuine. But I don't have to work on it, do I?"

Well, at least not in Canada (still a "free country")!