Title: Unfinished Work: A Bequest

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Abstract: Here are some topics in physics and philosophy on which my work is incomplete. I invite my friends in this assembly, and their colleagues and students, to continue the work and inform me about their progress.

1. There is a well known theorem of Wigner that a necessary condition for a quantity $M$ of a physical system $O$ to be measured without distortion (i.e., if $O$ is in an eigenstate $u$ of $M$ just prior to the measurement then it remains in $u$ immediately afterwards) is the commutation of $M$ with any additive conserved quantity. This theorem has been generalized by Stein and Shimony by relaxing the condition “without distortion”, but the natural full generalization has neither been proven nor refuted by a counter-example.

2. In two-particle interferometry, using an ensemble of pairs of particles in a pure entangled state $\gamma$, the fringe visibility $V_{12}$ of pairs counted in coincidence may be defined analogously to the fringe visibility $V_1$ of single particles, and a complementarity relation has been derived: $V_{12} + V_{12} = 1$. Generalizations of this complementarity relation to n-tuples ($n \geq 2$) of entangled particle are desired.

3. Relations have been explored among various reasonable definitions of “degree of entanglement”, but further systematization of these relations is desirable.

4. Bell’s Theorem shows that certain quantum mechanical predictions cannot be derived in any “local realistic” theory, and experiments have overwhelmingly favored quantum mechanics in situations of theoretical conflict. Is it plausible to maintain “peaceful coexistence” between the nonlocality of quantum mechanics and the locality of relativity theory by citing the impossibility of using the former to send superluminal messages? But if this strategy fails, what is the proper adjudication of the conflict between these fundamental physical theories?

5. Stochastic modification of quantum dynamics (proposed by Ghirardi-Rimini-Weber, Gisin, Pearle, Wigner, Penrose, Károlyházy, and others) has been proposed as a promising program for solving the quantum mechanical measurement problem. But theoretical refinements of the proposed modifications are desirable as well as definitive experimental tests. Two promising areas of relevant experimental research are quantum gravity and variants of the “quantum telegraph” (an ensemble of atoms undergoing transitions between the ground state and a metastable state when exposed to appropriate laser beams).

6. Corinaldesi conjectured that the boson statistics of integral spin particles and the fermion statistics of half-integral spin particles are consequences of their dynamics rather than of their kinematics (the latter usually accepted because of Pauli’s spin and statistics theorem), so that obedience to the Pauli Exclusion Principle in a freshly formed
ensemble of electrons would become increasingly strict as the ensemble ages. To test Corinaldesi’s conjecture it was proposed to form fresh ensembles of electrons by allowing a high velocity beam of Ne⁺ ions in a linear accelerator to be intersected by electrons from an electron gun, thereby neutralizing a subset of the ions. Transitions of electrons to the doubly occupied 1S shell in the complete Ne atoms will be monitored by suitable x-ray detectors at varying distances from the region of intersection, in order to scrutinize the conjectured diminution with time of violations of the Exclusion Principle. Refinements of this design and actual performance of the experiment are requested.

7. A test of the speculative conjecture that wave packet reduction is a psycho-physical phenomenon, occurring only when a conscious observer reads a measuring device, was performed in 1977 by three of my undergraduate students. They used a slow gamma emitter (about one emission per minute) monitored by a detector connected to registration devices in two separated rooms. There was a short time delay between the two registrations. The observer in the first room read his registration device in randomly chosen intervals of time and the observer in...
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E. P. Wigner, *Zeits. Phys.* 133, 104 (1952)


Discovering that the existence of an additive quantity that is conserved during the measurement of a quantity M requires the commutation of M with the component of the conserved quantity belonging to the measured system.

(Informal but suggestive paper.)


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Discovery that the existence of an additive quantity that is conserved during the measurement of a quantity M requires the commutation of M with the component of the conserved quantity belonging to the measured system.
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Consider an observable \( M \) of system 1, having a discrete spectrum whose eigenspaces constitute the family \( \{ E_n \} \). Take measurability of \( M \) by system 2 to mean the following: there is a unit vector \( \eta \in E_n \) in the Hilbert space \( \mathcal{H} \), a family \( \{ F_n \} \) of mutually orthogonal subspaces of \( \mathcal{H} \), and a time translation operator \( U \) on \( \mathcal{H} \), such that \( U(E_n \otimes \phi_h) \subseteq E_m \otimes F_k \) for each \( \eta \). In this scheme, if the object is initially in an eigenstate of \( M \), it is left in the eigenspace corresponding to the initial eigenvalue, but not necessarily in the same state: the measurement is a value-preserving, but is not required to be non-distorting. The Araki-Yamasu theorem states roughly—a qualification see below—that \( M \) is not measurable in this sense unless for every observable \( L \) of the system 1 + 2 which (1) is conserved in the measuring process, and which (2) \( L = L_0 \otimes 1 + 1 \otimes L_0 \), of observables on systems 1 and 2 respectively, we have \( 3) L_0 \otimes 1 \subseteq M \). The assumptions on \( L \) come formally to the conditions: 1) \( L U = U L \) and 2) \( L = L_0 \otimes 1 + 1 \otimes L_0 \), with 3) \( L_0 \) and \( L_0 \) self-adjoint (as they can always be chosen to be, if \( L \) is self-adjoint and 3) holds). In addition, however, the proof given by Yamasu tacitly assumed that \( L \) is bounded, since he applied \( L \) freely to vectors, without troubling over the fact that every unbounded self-adjoint operator is undefined on some vectors of the Hilbert space. The paper of Araki and Yamasu contains a sketch of a way to weaken this assumption, allowing \( L_0 \) to be unbounded; but their argument still requires the boundedness of \( L_0 \); and, indeed, there is an important conceptual distinction in the unbounded case that makes the Araki-Yamasu formal statement of the conclusion itself—in condition 3) above—inappropriate when \( L_0 \) is unbounded. (For a discussion of the Araki-Yamasu proof, with an explanation of this last remark, see Appendix A below.) The problem we are concerned with is whether the assumption of boundedness for \( L \) can be dropped entirely. To our knowledge, the problem has not been completely solved. However, some partial results are contained in the following theorem and comments.

**Theorem 1.** Let \( \{ E_n \} \) be a family of mutually orthogonal subspaces spanning \( \mathcal{H} \), \( \{ F_n \} \) a family of mutually orthogonal subspaces of \( \mathcal{H} \), \( \eta \) a unit vector in \( \mathcal{H} \), and \( U \) a unitary operator on \( \mathcal{H} \), such that:

1) for each \( \eta \) and each \( \phi \) in \( E_n \), there is an \( \eta \) in \( F_k \) with

\[
U(\phi \otimes \eta) = \phi \otimes \eta.
\]

Further, let \( L_0 \) and \( L_0 \) be self-adjoint operators on \( \mathcal{H} \) and \( \mathcal{H} \), respectively, such that, setting \( L = L_0 \otimes 1 + 1 \otimes L_0 \), we have:

2) for every real number \( r \), \( U \exp \{ i r L \} = \exp \{ i r L \} U \).

Then, for each \( \phi \) and \( r \), \( \exp \{ i r L \} E_0 \subseteq E_0 \).

**Theorem 1** establishes the Wigner limitation, in full generality, for non-distorting measurement. A stronger theorem, applying to what we call “finely distorted” measurement, is proved in Appendix B; in this theorem, condition 1) of Theorem 1 is replaced by the following:

1) for each \( \phi \) and each \( \phi \) in \( E_n \), there is a finite-dimensional subspace \( E_{n,\phi} \subseteq E_n \) such that \( U(E_{n,\phi} \otimes \phi_h) \subseteq E_{n,\phi} \otimes F_k \).

3) Neither of the theorems which attempt to express the essential discovery of Wigner implies the other: the theorem of Araki and Yamasu is more general in allowing arbitrary change of state within an eigenspace; our theorem is more general in allowing the conserved quantity \( L \) to have an unbounded spectrum for the object-system. To formulate in what seems its natural generality the proposition suggested by Wigner's original discovery, one would take (as it were) the join of these two theorems: the proposition that results from replacing 1) in our Theorem 1, by \( U(E_{n,\phi} \otimes \phi_h) \subseteq E_{n,\phi} \otimes F_k \). We have not attempted proving this proposition;
Consider an observable $M$ of system 1, having a discrete spectrum whose eigenvalues constitute the family $(E_i)$. Take measurability of $M$ by system 2 to mean the following: there is a unit vector $\eta_\alpha$ in the Hilbert space $\mathcal{H}_\alpha$ of system 2, a family $(F_\alpha)$ of mutually orthogonal subspaces of $\mathcal{H}_\alpha$, and a time translation operator $U$ on $\mathcal{H}_\alpha \otimes \mathcal{H}_\beta$, such that $U(\mathcal{E}_\alpha \otimes \{\eta_\alpha\}) = \mathcal{E}_\alpha \otimes \{U\eta_\alpha\}$ for each $\alpha$. In this scheme, if the object is initially in an eigenstate of $M$, it is left in the eigenstate corresponding to the initial eigenvalue, but not necessarily in the same state: the measurement is value-preserving but is not required to be non-distorting. The Araki-Yanase theorem states roughly—for a qualification see below—that $M$ is not measurable in this sense unless for every observable $L$ of the system 1+2 which 1) is conserved in the measuring process, and which 2) is a sum, $L = L_1 + L_2$, of observables of systems 1 and 2 respectively, we have 3) $\text{ML}_1 = L_1 M$. The assumptions on $L$ come formally to the conditions: 1) $LU = UL$ and 2) $L = L_1 \otimes 1 + 1 \otimes L_2$, with 3) $L_1$ and $L_2$ self-adjoint (as they can always be chosen to be, if $L$ is self-adjoint and 2) hold). In addition, however, the proof given by YAMASE tacitly assumed that $L$ is bounded, since he applied $L$ freely to vectors, without troubling over the fact that every unbounded self-adjoint operator is undefined on some vectors of the Hilbert space. The paper of ARAKI and YAMASE contains a sketch of a way to weaken this assumption, allowing $L_2$ to be unbounded; but their argument still requires the boundedness of $L_1$, and, indeed, there is an important conceptual distinction in the unbounded case that makes the Araki-Yanase formal statement of the conclusion itself—i.e. condition 3) above—inappropriate when $L_2$ is unbounded. (For a discussion of the Araki-Yanase proof, with an explication of this last remark, see Appendix A below.) The problem we are concerned with is whether the assumption of boundedness for $L_2$ can be dropped entirely. To our knowledge, the problem has not been completely solved. However, some partial results are contained in the following theorem and comments.

**Theorem 1.** Let $(E_\alpha)$ be a family of mutually orthogonal subspaces spanning $\mathcal{H}_\alpha$. $(F_\alpha)$ a family of mutually orthogonal subspaces of $\mathcal{H}_\alpha$ where $\eta_\alpha$ a unit vector in $E_\alpha$, and $U$ a unitary operator on $\mathcal{H}_\alpha \otimes \mathcal{H}_\beta$, such that:

1. for each $\alpha$, and each $\alpha$ in $E_\alpha$, there is a $\gamma$ in $F_\alpha$ with

$$U(\alpha \otimes \gamma) = \alpha \otimes \gamma.$$ 

Further, let $L_1$ and $L_2$ be self-adjoint operators on $\mathcal{H}_\alpha$ and $\mathcal{H}_\beta$ respectively, such that, setting $L = L_\alpha \otimes 1 + 1 \otimes L_\beta$, we have:

2. for every real number $r$, $U \exp(\{\alpha L\}) = \exp(\{rL\})U$.

Then, for each $\alpha$ and $r$, $\exp(\{\alpha L\}) \subset F_\alpha$.

**Theorem 1** establishes the Wigner limitation, in full generality, for non-distorting measurements. A stronger theorem, applying to what we call a finitely distorting measurement, is proved in Appendix B in this theorem, condition 1) of Theorem 1 is replaced by the following:

1') for each $\alpha$, and each $\alpha$ in $E_\alpha$, there is a finite-dimensional subspace $E_\alpha \subset E_\alpha$ such that $U(\alpha \otimes \{\eta_\alpha\}) \subset E_\alpha \otimes \{\gamma\}$.

3) Neither of the theorems which attempt to express the essential discovery of Wigner implies the other: the theorem of Araki and Yanase is more general in allowing arbitrary change of state within an eigenspace; our theorem is more general in allowing the conserved quantity $L$ to have an unbounded spectrum for the object-system. To formulate in what seems its natural generality the proposition suggested by Wigner's original discovery, one would take (as it were) the join of these two theorems: the proposition that results from replacing 1)', in our Theorem 1, by $U(\alpha \otimes \{\eta_\alpha\}) \subset E_\alpha \otimes \{\gamma\}$. We have not succeeded in proving this proposition;
Consider an observable $\mathcal{M}$ of system 1, having a discrete spectrum whose eigenvalues constitute the family $(E_i)$. Take measurability of $\mathcal{M}$ by system 2 to mean the following: there is a unit vector $\psi_i$ in the Hilbert space $E_i$ of system 2, a family $(E_i)$ of mutually orthogonal subspaces of $\mathcal{M}_1$, and a unitary operator $U$ on $\mathcal{M}_1 \otimes \mathcal{M}_2$, such that $U(E_i \otimes \psi_i) \subset E_i \otimes F_i$ for each $i$. In this scheme, if the object is initially in an eigenstate of $\mathcal{M}$, it is left in the eigenstate corresponding to the initial eigenvalue, but not necessarily in the same state; the measurement is not value-preserving, but is not required to be non-distorting. The Araki-Yamada theorem states roughly—for a qualification see below—that $\mathcal{M}$ is not measurable in this sense unless for every observable $L$ of the system 1 + 2 which 1) is conserved in the measuring process, and which 2) is a sum, $L = I_0 + I_1$, of observables of systems 1 and 2, respectively, we have 3) $M_{I_1} = I_0 L$. The assumptions on $L$ come from the conditions: 1) $L = UL$ and 2) $L = I_0 \otimes 1 + 1 \otimes I_0$, with 2) $I_0$ and $I_0$ self-adjoint (as they can always be chosen to be, if $L$ is self-adjoint and 2) holds). In addition, however, the proof given by Yamada tacitly assumed that $L$ is bounded, since he applied $L$ freely to vectors, without trembling over the fact that every unbounded self-adjoint operator is unbounded on some vector of the Hilbert space. The paper of Araki and Yamada contains a sketch of a way to weaken this assumption, allowing $I_0$ to be unbounded; but their argument still requires the boundedness of $I_0$; and, indeed, there is an important conceptual distinction in the unbounded case that makes the Araki-Yamada formal statement of the conclusion itself—i.e. condition 3) above—inappropriate when $L$ is unbounded. (For a discussion of the Araki-Yamada proof, with an explication of this last remark, see Appendix A below.) The problem we are concerned with is whether the assumption of boundedness for $L$ can be dropped entirely. To our knowledge, the problem has not been completely solved. However, some partial results are contained in the following theorem and comments.

Theorem 1. Let $(E_i)$ be a family of mutually orthogonal subspaces spanning $\mathcal{M}_1$; $(F_i)$ be a family of mutually orthogonal subspaces of $\mathcal{M}_2$, $\psi_i$ a unit vector in $F_i$, and $U$ a unitary operator on $\mathcal{M}_1 \otimes \mathcal{M}_2$, such that:

1) for each $i$, and each $\sigma \in E_i$, there is an $\eta \in F_i$ with $U(\sigma \otimes \eta) = \sigma \otimes \eta$.

Further, let $I_0$ and $I_0$ be self-adjoint operators on $\mathcal{M}_1$ and $\mathcal{M}_2$, respectively, such that, setting $L = I_0 \otimes 1 + 1 \otimes I_0$, we have:

2) for every real number $r$, $\exp(\text{i} r L) = \exp(\text{i} r L) U$.

Then, for each $i$, and $r \exp(\text{i} r L) \subset E_i \otimes F_i$.

Theorem 1 establishes the Wigner limitation, in full generality, for non-distorting measurement. A stronger theorem, applying to what we call a finitely distorting measurement, is proved in Appendix B; in this theorem, condition 1) of Theorem 1 is replaced by the following:

1) for each $i$, and each $\sigma \in E_i$, there is a finite-dimensional subspace $E_{i0} \subset E_i$ such that $U(\sigma \otimes \eta) \subset E_{i0} \otimes F_i$.

3) Neither of the theorems which attempt to express the essential discovery of Wigner implies the other: the theorem of Araki and Yamada is more general in allowing arbitrary change of state within an eigenstate; our theorem is more general in allowing the conserved quantity $L$ to have an unbounded spectrum for the object-system. To formulate in what seems its natural generality the proposition suggested by Wigner's original discovery, one would take (as it were) the join of these two theorems: the proposition that results from replacing 1) in our Theorem 1, by $U(E_i \otimes \eta) \subset E_i \otimes F_i$. We have not succeeded in proving this proposition;
Consider an observable $M$ of system 1, having a discrete spectrum whose eigenspaces constitute the family $(E_n)$. Take measurability of $M$ by system 2 to mean the following: there is a unit vector $\eta$ in the Hilbert space $\mathcal{H}$ of system 2, a family $(F_n)$ of mutually orthogonal subspaces of $\mathcal{H}$, and a time translation operator $U$ on $\mathcal{H} \otimes \mathcal{H}$, such that $U(E_n \otimes \eta) \subset C \subset E_m \otimes F_n$ for each $n$. In this scheme, if the object is initially in an eigenstate of $M$, it is left in the eigenspace corresponding to the initial eigenvalue, but not necessarily in the same state: the measurement is a value-preserving, but is not required to be nondistorting. The Araki-Yamase theorem states roughly—for a qualification see below—that $M$ is not measurable in this sense unless for every observable $L$ of the system 1+2 which 1) is conserved in the measuring process, and which 2) is a sum, $L = L_0 + L_1$, of observables of systems 1 and 2, respectively, we have 3) $M_0 = L_0$. The assumptions on $L$ come formally to the conditions: 1) $L U = U L$ and 2) $L = L \otimes 1 + 1 \otimes L_0$, with 3) $L_0$ and $L_1$ self-adjoint (as they can always be chosen to be, if $L$ is self-adjoint and 2) holds). In addition, however, the proof given by Yamase tacitly assumed that $L$ is bounded, since he applied $L$ freely to vectors, without troubling over the fact that every unbounded self-adjoint operator is undefined on some vectors of the Hilbert space. The paper of Araki and Yamase contains a sketch of a way to weaken this assumption, allowing $L_0$ to be unbounded; but their argument still requires the boundedness of $L_0$; and, indeed, there is an important conceptual distinction in the unbounded case that makes the Araki-Yamase formal statement of the conclusion itself—i.e., condition 3) above—inequitable when $L_0$ is unbounded. (For a discussion of the Araki-Yamase proof, with an explication of this last remark, see Appendix A below.)

The problem we are concerned with is whether the assumption of boundedness for $L$ can be dropped entirely. To our knowledge, the problem has not been completely solved. However, some partial results are contained in the following theorem and comments.

**Theorem 1.** Let $(E_n)$ be a family of mutually orthogonal subspaces spanning $\mathcal{H}$, $(F_n)$ a family of mutually orthogonal subspaces of $\mathcal{H}$, $\sigma$, a unit vector in $\mathcal{H}$, and $U$ a unitary operator on $\mathcal{H} \otimes \mathcal{H}$, such that:

1) for each $m$ and each $\sigma$ in $E_m$, there is an $\eta$ in $F_\sigma$ with

$$U(\sigma \otimes \eta) = \sigma \otimes \eta.$$ 

Further, let $L_0$ and $L_1$ be self-adjoint operators on $\mathcal{H}$ and $\mathcal{H}$, respectively, such that, setting $L = L_0 \otimes 1 + 1 \otimes L_1$, we have:

2) for every real number $r$, $U \exp(rL) = \exp(rL) U$.

Then, for each $m$ and $r$, $\exp(rL)(E_m) \subset E_m$.

**Theorem 1** establishes the Wigner limitation, in full generality, for non-distorting measurement. A stronger theorem, applying to what we call a finitely distorting measurement, is proved in Appendix B; in this theorem, condition 1) of Theorem 1 is replaced by the following:

1') for each $m$ and each $\sigma$ in $E_m$, there is a finite-dimensional subspace $E_{\sigma_n} \subset E_{\sigma_m}$ such that $U(E_{\sigma_n} \otimes \eta) \subset E_{\sigma_m} \otimes F_{\eta}$.

3) Neither of the theorems which attempt to express the essential discovery of Wigner implies the other: the theorem of Araki and Yamase is more general in allowing arbitrary change of state within an eigenspace; our theorem is more general in allowing the conserved quantity $L$ to have an unbounded spectrum for the object-system. To formulate in what sense its natural generality the proposition suggested by Wigner's original discovery, one would take (as it were) the join of these two theorems: the proposition that results from replacing 1), in our Theorem 1, by $U(E_\sigma \otimes \eta) \subset E_\sigma \otimes F_\eta$. We have not succeeded in proving this proposition;
Consider an observable $M$ of system 1, having a discrete spectrum whose eigenspaces constitute the family $(E_m)$. Take measurability of $M$ by system 2 to mean the following: there is a unit vector $\eta$ in the Hilbert space $\mathcal{H}_2$ of system 2, a family $(E_m)$ of mutually orthogonal subspaces of $\mathcal{H}_2$, and a time translation operator $U$ on $\mathcal{H}_1 \otimes \mathcal{H}_2$, such that $U(E_m \otimes \{\eta\}) \subset E_n \otimes \{\eta\}$ for each $n$. In this scheme, if the object is initially in an eigenstate of $M$, it is left in the eigenspace corresponding to the initial eigenvalue, but not necessarily in the same state; the measurement is a value-preserving, but is not required to be nondistorting. The Araki-Yanase theorem states roughly—for a qualification see below—that $M$ is not measurable in this sense unless for every observable $L$ of the system 1+2 which 1) is conserved in the measuring process, and 2) is a sum, $L = L_1 + L_2$, of observables of systems 1 and 2 respectively, we have 3) $ML = L_1 M$. The assumptions on $L$ come formally to the equations: 1) $LU = UL$ and 2) $L = L_1 \otimes 1 + 1 \otimes L_2$, with 3) $I_1$ and $I_2$ self-adjoint (they can always be chosen to be, if $L$ is self-adjoint and 3) holds). In addition, however, the proof given by Yanase tacitly assumed that $L$ is bounded. He applied $L$ freely to vectors, without troubling over the fact that any unbounded self-adjoint operator is undefined on some vectors of the Hilbert space. The paper of Araki and Yanase contains a sketch of a way to weaken this assumption, allowing $I_1$ to be unbounded; but their argument still requires the boundedness of $I_2$; and, indeed, there is an important conceptual distinction in the unbounded case that makes the Araki-Yanase formal statement of the conclusion itself—i.e., condition 3) above—inappropriate when $I_1$ is unbounded. (For a discussion of the Araki-Yanase proof, with application of this last remark, see Appendix A below.) The problem we are concerned with is whether the assumption of boundedness for $L$ can be dropped entirely. To our knowledge, the problem has not been completely solved. However, some partial results are contained in the following theorem and its corollary.

Theorem 1. Let $(E_m)$ be a family of mutually orthogonal subspaces spanning $\mathcal{H}_1$, $(F_n)$ a family of mutually orthogonal subspaces of $\mathcal{H}_2$, $\sigma$ a unit vector in $\mathcal{H}_1$, and $U$ a unitary operator on $\mathcal{H}_1 \otimes \mathcal{H}_2$, such that:

\begin{enumerate}
  \item for each $m$, and $n$, in $E_m$, there is an $\eta$ in $F_n$ with 
    \[ \sigma \eta = \sigma \otimes \eta. \]
  \item Further, let $L_1$ and $L_2$ be joint operators on $\mathcal{H}_1$ and $\mathcal{H}_2$ respectively, such that, setting $L = L_1 \otimes 1 + 1 \otimes L_2$, 
    \[ L \sigma \eta = \sigma (L_2 \otimes 1 + 1 \otimes L_1) \eta. \]

Then, for each $m$, $n$, $\sigma \otimes \eta$ is a joint eigenstate of $L_1$ and $L_2$.

Theorem 2. (Corollary) Nondistorting measurement is equivalent to what we call (finite) determinism. In this theorem, condition 1) of the Araki-Yanase theorem is replaced by a sequence of internal dispositions, more conventional, and 2) is left unbounded; the corollary to this theorem is still valid.
Consider an observable \( M \) of system 1, having a discrete spectrum whose eigenspaces constitute the family \( (E_m) \). Take measurability of \( M \) by system 2 to mean the following: there is a unit vector \( r \) in the Hilbert space \( \mathcal{H}_2 \) of mutually orthogonal subspaces of \( \mathcal{H}_1 \) and a time translation operator \( U \) on \( \mathcal{H}_1 \otimes \mathcal{H}_2 \), such that \( U(E_m \otimes \langle \phi \rangle) \subseteq \mathcal{E}_m \otimes \mathcal{F}_r \) for each \( m \). In this scheme, if the object is initially in an eigenvector of \( M \), it is left in the eigenspace corresponding to the initial eigenvalue, but not necessarily in the same state: the measurement is value-preserving, but is not required to be non-distorting. The Araki-Yanase theorem states roughly—for a qualification see below—that \( M \) is not measurable in this sense unless for every observable \( L \) of the system 1 + 2 which 1) is conserved in the measuring process, and which 2) is a sum, \( L = I_0 + I_b \), of observables of systems 1 and 2 respectively, we have 3) \( \mathcal{M}_b = I_0 \cdot I_b \). The assumptions on \( L \) come formally to the conditions: 1) \( LU = UL \) and 2) \( L = I_0 \otimes I_1 + I_1 \otimes I_0 \), with 3) \( I_0 \) and \( I_1 \) self-adjoint (as they can always be chosen to be, if \( L \) is self-adjoint and 2) holds). In addition, however, the proof given by Yamabe tacitly assumed that \( L \) is bounded, since he applied it freely to vectors, without troubling over the fact that every unbounded self-adjoint operator is undefined on some vectors of the Hilbert space. The paper of Araki and Yanase contains a sketch of a way to weaken this assumption, allowing \( I_0 \) to be unbounded; but their argument still requires the boundedness of \( I_0 \); and, indeed, there is an important conceptual distinction in the unbounded case that makes the Araki-Yanase formal statement of the conclusion itself—i.e. condition 3) above—inappropriate when \( I_0 \) is unbounded. (For a discussion of the Araki-Yanase proof, with an explication of this last remark, see Appendix A below.) The problem we are concerned with is whether the assumption of boundedness for \( L \) can be dropped entirely. To our knowledge, the problem has not been completely solved. However, some partial results are contained in the following theorem and its comments.

**Theorem 1.** Let \((E_m)\) be a family of mutually orthogonal subspaces spanning \( \mathcal{H}_1 \), \((E_n)\) a family of mutually orthogonal subspaces of \( \mathcal{H}_1 \), \( r \) a unit vector in \( \mathcal{H}_2 \), and \( U \) a unitary operator on \( \mathcal{H}_1 \otimes \mathcal{H}_2 \), such that:

1) for each \( m \), and each \( \sigma \) in \( E_m \), there is an \( \eta \) in \( E_n \) with:

\[ U(\sigma \otimes \eta) = \sigma \otimes \eta. \]

Further, let \( I_0 \) and \( I_b \) be self-adjoint operators on \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \) respectively, such that, setting \( L = I_0 \otimes I_1 + I_1 \otimes I_0 \), we have:

2) for every real number \( r \), \[ \exp(rL) = \exp(rI_0). \]

Then, for each \( m \) and \( r \), \[ \exp(rI_0) \subseteq \mathcal{E}_m \otimes \mathcal{F}_r. \]

**Theorem 1** establishes the following limitation, in full generality, for non-distorting measurement. A theorem, applying to what we call a finitely distorting measurement (proved in Appendix B; in this theorem, condition 1) of Theorem 1 is replaced by the following:

1) for each \( m \), and each \( \sigma \) in \( E_m \), there is a finite-dimensional subspace \( F \subseteq \mathcal{E}_m \) such that \[ U(\sigma \otimes \langle \phi \rangle) \subseteq \mathcal{E}_m \otimes F. \]

3) Neither of the theorems of Wigner implies the other since, in allowing arbitrary chains, it is more general in allowing the spectrum for the object-system. Reconsider the proposition suggested in the text (as it were) the join of the theorems from replacing \( I_0 \); in our Theorem 1, the proposition that results from not succeeding in proving the

4.31 of Stein/Strum
Consider an observable $M$ of system $1$, having a discrete spectrum whose eigenspaces constitute the family $(E_m)$. Take measurability of $M$ by system $2$ to mean the following: there is a unit vector $\eta_n$ in the Hilbert space $H_2$ of system $2$, a family $(F_n)$ of mutually orthogonal subspaces of $H_2$, and a time translation operator $U$ on $H_2 \otimes H_2$ such that $U(|\eta_n\rangle \otimes \phi) \in E_n \otimes F_m$ for each $m$. In this scheme, if the object is initially in an eigenstate of $M$, it is left in the eigenspace corresponding to the initial eigenvalue, but not necessarily in the same state: the measurement is a value-preserving one, but is not required to be non-distorting. The Araki-Yanase theorem states roughly—for a qualification see below—that $M$ is not measurable in this sense unless for every observable $L$ of the system $1 + 2$ which $1)$ is conserved in the measuring process, and which $2)$ is a sum, $L = L_1 + L_2$, of observables of systems $1$ and $2$ respectively, we have $3)$ $M L_m = L_m M$. The assumptions on $L$ come formally to the conditions: $1)$ $L U = U L$ and $2)$ $L = L_1 \otimes 1 + 1 \otimes L_2$, with $3)$ $L_1$ and $L_2$ self-adjoint (as they can always be chosen to be, if $L$ is self-adjoint and $3)$ holds). In addition, however, the proof given by Yanase tacitly assumed that $L$ is bounded, since he applied $L$ freely to vectors, without troubling over the fact that every unbounded self-adjoint operator is defined on some vectors of the Hilbert space. The paper of Araki and Yanase contains a sketch of a way to weaken this assumption, allowing $L_1$ to be unbounded; but their argument still requires the boundedness of $L_1$; and, indeed, there is an important conceptual distinction in the unbounded case that makes the Araki-Yanase formal statement of the conclusion itself—i.e. condition $3)$ above—inappropriate when $L_1$ is unbounded. (For a discussion of the Araki-Yanase proof, with an explanation of this last remark, see Appendix A below.) The problem we are concerned with is whether the assumption of boundedness for $L$ can be dropped entirely. To our knowledge, the problem has not been completely solved. However, some partial results are contained in the following theorem and comments.

**Theorem 1.** Let $(E_m)$ be a family of mutually orthogonal subspaces spanning $H_2$, $(F_n)$ a family of mutually orthogonal subspaces of $H_2$, $\eta_n$ a unit vector in $F_n$, and $U$ a unitary operator on $H_1 \otimes H_2$, such that:

1) for each $m$, and each $\sigma$ in $E_m$, there is an $\eta$ in $F_n$ with

$$U(\sigma \otimes \eta) = \sigma \otimes \eta.$$  

Further, let $L_1$ and $L_2$ be self-adjoint operators on $H_1$ and $H_2$, respectively, such that, setting $L = L_1 \otimes 1 + 1 \otimes L_2$, we have:

2) for every real number $r$, $U \exp(irL) = \exp(irL)U$.

Then, for each $m$ and $r$, $\exp\{irL\}|E_m\rangle \in E_m$.

---

**Theorem 1** establishes the Wigner limitation, in full generality, for non-distorting measurement. A stronger theorem, applying to what we call a finitely distorting measurement, is proved in Appendix A; in this theorem, condition 1) of Theorem 1 is replaced by the following:

1') for each $m$, and each $\sigma$ in $E_m$, there is a finite-dimensional subspace $E_\sigma \subseteq E_m$ such that $U(\sigma \otimes \phi) \in E_\sigma \otimes F_m$.

3) Neither of the theorems which attempt to express the essential discovery of Wigner implies the other. The theorem of Araki and Yanase is more general in allowing arbitrary changes of state within an eigenspace; our theorem is more general in allowing the observed quantity $L$ to have an unbounded spectrum for the object-system. However, in what seems its natural generality the proposition suggested by Wigner's original discovery, one would take (as it were) the joint of these two theorems: the proposition that results from replacing 1) in our Theorem 1 by $U(\sigma \otimes \phi) \in E_\sigma \otimes F_m$. We have not succeeded in proving this proposition.
Consider an observable $\mathcal{M}$ of system 1, having a discrete spectrum whose eigenvalues constitute the family $(E_n)$. Take measurability of $\mathcal{M}$ by system 2 to mean the following: there is a unit vector $\eta_n$ in the Hilbert space $\mathcal{H}_n$ of system 2, a family $(F_n)$ of mutually orthogonal subspaces of $\mathcal{H}_n$, and a time translation operator $U$ on $\mathcal{H}_1 \otimes \mathcal{H}_2$ such that $U(E_n \otimes \eta_n) \subseteq E_n \otimes F_n$, for each $m$. In this scheme, if the object is initially in an eigenstate of $\mathcal{M}$, it is left in the eigenspace corresponding to the initial eigenvalue, but not necessarily in the same state: the measurement is a value-preserving process, but is not required to be $\varepsilon$-nondistorting $s$. The Araki-Yamasaki theorem states roughly—for a qualification see below—that $\mathcal{M}$ is not measurable in this sense unless for every observable $L$ of the system 1+2 which (1) is conserved in the measuring process, and which (2) is a sum, $L = L_1 \otimes 1 + 1 \otimes L_2$, of observables of systems 1 and 2 respectively, we have $\beta \langle \eta \mid L \otimes 1 \rangle = L_1 \otimes L_2$. The assumptions on $L$ come formally to the conditions: 1) $LU = UL$, and 2) $L = L_1 \otimes 1 + 1 \otimes L_2$, with $L_1$ and $L_2$ self-adjoint (as they can always be chosen to be, if $L$ is self-adjoint and 2 holds). In addition, however, the proof given by Yamasaki tacitly assumed that $L_2$ is bounded. Since he applied $L$ freely to vectors, without troubling over the fact that every unbounded self-adjoint operator is undefined on some vectors of the Hilbert space. The paper of Araki and Yamasaki contains a sketch of a method to weaken this assumption, allowing $L_2$ to be unbounded; but their argument still requires the boundedness of $L_2$; and, indeed, there is an important conceptual distinction in the unbounded case that makes the Araki-Yamasaki formal statement of the conclusion itself—i.e., condition 3 above—appropriately inapplicable when $L_2$ is unbounded. (For a discussion of the Araki-Yamasaki proof, with an explanation of this last remark, see Appendix A below.) The problem we are concerned with is whether the assumption of boundedness for $L$ can be dropped entirely. To our knowledge, the problem has not been completely solved. However, some partial results are contained in the following theorem and comments.

Theorem 1. Let $(E_n)$ be a family of mutually orthogonal subspaces spanning $\mathcal{H}_1$, $(F_n)$ a family of mutually orthogonal subspaces of $\mathcal{H}_2$, $\eta_n$ a unit vector in $E_n$, and $U$ a unitary operator on $\mathcal{H}_1 \otimes \mathcal{H}_2$, such that:

(i) for each $w$ and each $v$ in $E_n$, there is an $q$ in $F_n$ with $U(v \otimes \eta_n) = v \otimes q$.

Further, let $L_1$ and $L_2$ be self-adjoint operators on $\mathcal{H}_1$ and $\mathcal{H}_2$ respectively, such that, setting $L = L_1 \otimes 1 + 1 \otimes L_2$, we have:

(ii) for every real number $r$, $U \exp[irL] = \exp[irL]U$.

Then, for each $w$ and $r$, $\exp[irL](E_n) \subseteq E_n$.

Theorem 1 establishes the Wigner limitation, in full generality, for nondistorting measurement. A stronger theorem, applying to what we call $\varepsilon$-finite $\varepsilon$-distorting $s$ measurement, is proved in Appendix B; in this theorem, condition (1) of Theorem 1 is replaced by the following:

1') for each $w$ and each $v$ in $E_n$, there is a finite-dimensional subspace $E_n' \subseteq E_n$ such that $U(E_n' \otimes \eta_n) \subseteq E_n' \otimes F_n$.

3) Neither of the theorems which attempt to express the essential discovery of Wigner implies the other: the theorem of Araki and Yamasaki is more general in allowing arbitrary change of state within an eigenspace; our theorem is more general in allowing the conserved quantity $L$ to have an unbounded spectrum for the object-system. To formulate in what seems its natural generality the proposition suggested by Wigner's original discovery, one would have to modify the join of those two theorems: the proposition that results from replacing it, in our Theorem 1, by $U(E_n' \otimes \eta_n) \subseteq E_n' \otimes F_n$. We have not succeeded in proving this proposition;

FIG. 2. Schematic two-particle four-beam interferometer, using beam splitters $H_1$ and $H_2$. 
The single-particle fringe visibilities $V_i$, and $V_j$, can be determined by inspection from Eqs. (61), (77), and (78), together with $a \neq b$. Clearly,

$$[P(U_i)]_{\text{max}}-[P(U_j)]_{\text{max}}=a^2,$$

$$[P(U_i)]_{\text{min}}-[P(U_j)]_{\text{min}}=b^2. \tag{79a}$$

Hence,

$$V_i=a^2+b^2-2a^2b^2. \tag{80a}$$

We note that $P(U_i)$ achieves its maximum and minimum when $a$ has the respective values 1 and 0. When $a$ is unity (hence $b$ is zero), a photon in $G$ will go with certainty into $U_1$ and a photon in $C$ will go with certainty into $L_i$. When $a$ is zero (hence $b$ is unity), the exit states are reversed. A similar statement can be made concerning photon 2, relating the vectors $(D)$ and $(D')$ to exit in vectors $U_1$ and $U_2$.

If we turn now to the two-particle fringe visibility $V_{ij}$, as pointed out in Ref. [10], one cannot capture the intuitive meaning of two-particle fringe visibility by using the attractive definition

$$V_{ij}=[P(U_i,U_j)]_{\text{max}}-[P(U_i,U_j)]_{\text{min}} \quad \text{(No)} \tag{81a}$$

this expression would yield a nonzero value even if $(i,j)$ is a product state, for in that case $P(U_i)$ is the product of $P(U_i)$ and $P(U_j)$, and these vary respectively with $T_i$ and $T_j$. As in Ref. [10] we define a "corrected" joint probability $P(U_i,U_j)$ by subtracting the product $P(U_i)P(U_j)$ from $P(U_i,U_j)$ and adding a constant as a compensation against excessive subtraction:

$$P(U_i,U_j)=P(U_i,U_j)-P(U_i)P(U_j)+\frac{1}{2}. \tag{82a}$$

By Eqs. (77), (77), and (78),

$$P(U_i,U_j)=a^2b^2\cos\theta+|\cos|\sin\theta\sin\sigma\sin\varphi+\frac{1}{2}. \tag{82b}$$

where

$$a=\cos\frac{\varphi}{2}, \quad b=\sin\frac{\varphi}{2}, \quad c=\cos\frac{\sigma}{2}, \quad d=\sin\frac{\sigma}{2}. \tag{82c}$$

A rationale for the term $\frac{1}{2}$ in Eq. (82a) is the fact that $\frac{1}{2}$ is the least real number such that $P(U_i,U_j)-P(U_i)P(U_j)+x$ is non-negative for all two-particle and all unitary mappings $T_i$ and $T_j$ of the classes under consideration, as can be checked from Eqs. (77), (77), and (78). We can parallel the derivation of Ref. [10] and define the two-particle fringe visibility $V_{ij}$ as

$$V_{ij}=[P(U_i,U_j)]_{\text{max}}-[P(U_i,U_j)]_{\text{min}} \quad \text{(No)} \tag{83a}$$

To find the extreme of $P(U_i,U_j)$ we use Eq. (82b) and set partial derivatives to zero: first,

$$a^2+b^2=1. \tag{84}$$

If $a^2+b^2=0$, then Eq. (84) can be satisfied only if one of the two following conditions is satisfied: (i) $\sin\theta=0$, in which case

$$0=\frac{dP_{U_i}}{d\theta}=-a^2b^2\sin\theta+|\cos|\sin\theta\sin\varphi. \tag{85a}$$

$$0=\frac{dP_{U_j}}{d\varphi}=-a^2b^2\cos\theta-|\cos|\sin\theta\sin\varphi. \tag{85b}$$

and

$$V_{ij}=\frac{1}{2}a^2b^2. \tag{86a}$$

If $a^2b^2=0$, then Eqs. (85a) and (85b) imply $\sin\theta=|\cos|\sin\theta=0$, which is possible only if one of the two conditions is satisfied: $\sin\theta=|\cos|\sin\theta=0$, with $m$ an integer, in which case Eq. (85) is again satisfied, or $\sin\theta=|\cos|\sin\theta=0$, which is again all mod 2, in which case

$$V_{ij}=\frac{1}{2}a^2b^2. \tag{87}$$

But

$$V_{ij}=0. \tag{88}$$

and therefore a review of all the cases (i), (ii), and (iib) yields

$$P(U_i,U_j)=\frac{1}{2}a^2b^2. \tag{89a}$$

$$P(U_i,U_j)=\frac{1}{2}a^2b^2. \tag{89b}$$

Note that in the neglected case of $\beta=0$ those equations continue to hold, as done Eq. (82a), because $\beta$ was assumed $\leq\beta$. It follows that without exception

$$V_{ij}=\frac{1}{2}a^2b^2. \tag{90}$$

By Eqs. (80) and (90),

$$V_i^2+V_j^2=4a^2b^2\left(1-a^2b^2\right) \quad \text{(i=1,2)}$$

$$=a^2b^2(1-a^2b^2). \tag{91}$$

which is the expression for the complementarity of one-particle and two-particle visibilities promised in the Introduction (slightly generalized, since $\beta=1$ or $\beta$).

In Ref. [10] a more restricted set of transducers was considered than the class permitted here. Each $T_i$ was taken to consist of a symmetric beam splitter with reflectivity $r$ and transmissivity $t$ both equal to $1/\sqrt{2}$, together with a phase shifter in one beam incident upon the beam splitter. The small letters $v_i$, ($i=1,2$) and $w_i$ denote the one-particle and two-particle visibilities under this restriction. It was shown that for a large class [11] of two-particle vectors $\langle 0 \rangle$, the inequality

$$a_i^2+b_i^2=1. \tag{92}$$

This is the complementarity in two-particle interferometry.
It is understood that (60) should be symmetrized since photons are bosons, but the results that we obtain without explicit symmetrization would not be changed by writing a symmetrized version of Eq. (60a), provided that the subspace spanned by \( |A\rangle, |A'\rangle \) is orthogonal to that spanned by \( |B\rangle, |B'\rangle \), and likewise for \( |U_1\rangle, |U_2\rangle \) and \( |L_1\rangle, |L_2\rangle \).

By the well-known theorem of Schmidt (12), |\( \Theta \rangle \) can be expressed as

\[
|\Theta\rangle = a|C\rangle|D\rangle + b|C'\rangle|D'\rangle,
\]

where \( |C\rangle \) and \( |C'\rangle \) constitute an orthonormal basis in the subspace spanned by \( |A\rangle \) and \( |A'\rangle \), while \( |D\rangle \) and \( |D'\rangle \) constitute an orthonormal basis in the subspace spanned by \( |B\rangle \) and \( |B'\rangle \). The coefficients \( a \) and \( b \) can be chosen to be real by using phase options for the vectors \( |C\rangle, |C'\rangle, |D\rangle, \) and \( |D'\rangle \), and

\[
a^2 + b^2 = 1.
\]

The most general unitary unimodular mapping \( T_1 \) relating the specified domain and co-domain for photon 1 can be expressed in terms of the \( |C\rangle, |C'\rangle \) basis as

\[
T_1|C\rangle = a|U_1\rangle + b|L_1\rangle,
\]

\[
T_1|C'\rangle = b^*|U_1\rangle + a^*|L_1\rangle,
\]

where \( a \) and \( b \) are real numbers whose squares sum to unity. Likewise,

\[
T_1|D\rangle = c|U_1\rangle + d|L_1\rangle
\]

\[
T_1|D'\rangle = -d^*|U_1\rangle + c^*|L_1\rangle
\]

c and \( d \) are real numbers whose squares sum to unity. The paired transducers is represented by

\[
T = T_1 \otimes T_1^*.
\]

which is unitary unimodular mapping from the space initially associated with the photon pair 1+1 into the space of output states. From Eqs. (67a)–(70) we obtain

\[
\begin{align*}
|\Theta\rangle &= \gamma |A\rangle|B\rangle + \gamma |A'\rangle|B'\rangle, \\
&= \gamma |U_1\rangle|U_2\rangle + \gamma |L_1\rangle|L_2\rangle,
\end{align*}
\]

and

\[

\Phi = \phi_1 + \phi_2 + \phi_3 + \phi_4.
\]

where
The single-particle fringe visibilities $V_i$ and $V_j$ can be determined by inspection from Eqs. (9), (77), and (78), together with $u = 0$. Clearly,

$$\{V_i\} = \{P(U_i)\} = n^2.$$  
(79a)

$$\{P(U_i)\} = \{P(U_i)\} = n^2.$$  
(79b)

Hence,

$$V_i = \frac{1}{2} \left( 2 - n^2 \right) (i = 1, 2).$$  
(80)

We note that $V_i$ achieves its maximum and minimum when $a$ has respective values $1$ and $0$. When $a$ is unity (hence $b$ is unity too), a photon in $C$ will go with certainty into $U_1$ and vice versa, in which case $V_1$ will go with certainty into $U_1$. When $a$ is unity (hence $b$ is unity), the exit states are reversed. A similar statement can be made concerning photons 2, relative to the superposition vectors $(D')$ and $(D'')$ to enter in beams $U_2$ and $L_2$.

We turn now to the two-particle fringe visibility $V_{12}$. As pointed out in Section 3, one cannot capture the intuitive meaning of two-particle fringe visibility by using the attractive definition:

$$V_{12} = \frac{P(U_1, U_2)}{2P(U_1)P(U_2)} = \frac{1}{2} \
$$

this expression would give a product state, for in the product of $P(U_1)$ and $P(U_2)$, $a$ is multiplied with $b$ and $T_i$. As is shown in Ref. 77, the probability $P(U_1, U_2)$ and $P(U_1)P(U_2)$ from $P(U_i)$ compensation against $T_i$ as

$$P(U_i) = \frac{1}{2} \left( 2 - n^2 \right) (i = 1, 2).$$  
(81a)

$$P(U_i) = \frac{1}{2} \left( 2 - n^2 \right) (i = 1, 2).$$  
(81b)

By Eqs. (77), (78), and (79),

$$P(U_1, U_2) = \frac{1}{2} \left( 2 - n^2 \right)^2 (i = 1, 2).$$  
(82)

But $P(U_1, U_2)$ is therefore a review of all the cases $0$, $\lambda$, and $\lambda$ and $\lambda$.

$$P(U_1, U_2) = \frac{1}{2} \left( 2 - n^2 \right)^2 (i = 1, 2).$$  
(83a)

$$P(U_1, U_2) = \frac{1}{2} \left( 2 - n^2 \right)^2 (i = 1, 2).$$  
(83b)

Hence, for the complementarity of one-particle visibilities promised in the

$$P(U_1, U_2) = \frac{1}{2} \left( 2 - n^2 \right)^2 (i = 1, 2).$$  
(84)

$$P(U_1, U_2) = \frac{1}{2} \left( 2 - n^2 \right)^2 (i = 1, 2).$$  
(84a)

The restricted set of transducers was class permitted here. Each $T_i$ was of a symmetric beam splitter with transmittivity both equal to $\frac{1}{2}$ and hence shifter in one beam incident upon the

$$P(U_1, U_2) = \frac{1}{2} \left( 2 - n^2 \right)^2 (i = 1, 2).$$  
(84b)

The small letters $\lambda_1$ ($i = 1, 2$, and $\lambda_2$) particle and two-particle visibilities under

$$P(U_1, U_2) = \frac{1}{2} \left( 2 - n^2 \right)^2 (i = 1, 2).$$  
(84c)

It was shown that for a large class of transducers $(O)$, the inequality

$$P(U_1, U_2) = \frac{1}{2} \left( 2 - n^2 \right)^2 (i = 1, 2).$$  
(84d)
The single-particle fringe visibilities $V_i$ and $V_j$ can be determined by inspection from Eqs. (19), (177), and (178), together with $\alpha \neq \beta$. Clearly,

$$\left[ P(U_i)_{\text{max}} \right] - \left[ P(U_j)_{\text{max}} \right] = \alpha^2 \beta^2. \quad (79a)$$

$$\left[ P(U_i)_{\text{max}} \right] = \left[ P(U_j)_{\text{max}} \right] = \alpha \beta. \quad (79b)$$

From these,

$$V_i = \frac{\alpha - \beta}{\alpha + \beta} = q \quad (i = 1, 2). \quad (80)$$

We note that $P(U_i)$ achieves its maximum and minimum when $a$ has the respective values 1 and 0. When $a$ is unity (b = 0), a photon in C will go with certainty into $U_1$, and a photon in $U_1$ will go with certainty into $U_2$. When $a$ is zero (b = unity), the exit states are reversed. A similar statement can be made concerning photon 2, rotating the vectors $|D\rangle$ and $|D\rangle^*$ to exit in beams $U_1$ and $U_2$. We turn now to the two-particle fringe visibility $V_{11}$. As pointed out in Ref. [10], one cannot capture the intuitive meaning of two-particle fringe visibility by using the attractive definition

$$V_{11} = \frac{\left[ P(U_1, U_1)_{\text{max}} \right] - \left[ P(U_1, U_1)_{\text{min}} \right]}{\left[ P(U_1, U_1)_{\text{max}} \right] + \left[ P(U_1, U_1)_{\text{min}} \right]}. \quad (81)$$

This expression would yield a nonzero value even if $\{O\}$ is a product state, i.e., in that case $P(U_1, U_1)$ is the product of $P(U_1)$ and $P(U_1)$, and those vary respectively with $T_1$ and $T_2$. As in Ref. [10], we define a "corrected" joint probability $P(U_1, U_1)$ by subtracting the product of $P(U_1)$ from $P(U_1, U_1)$ and adding a constant as a compensation against excessive subtraction:

$$P(U_1, U_1) = P(U_1, U_1) - |\alpha|^2 |\beta|^2 \cos \varphi \sin \varphi \sin \varphi \cos \varphi + \frac{1}{2}. \quad (82a)$$

By Eqs. (72), (73), and (78),

$$P(U_1, U_1) = \alpha^2 \beta^2 \cos \varphi \sin \varphi \sin \varphi + \frac{1}{2}. \quad (82b)$$

where:

$$s = \cos \frac{\theta}{2}, \quad b = \sin \frac{\theta}{2}, \quad \alpha = \cos \frac{\varphi}{2}, \quad \beta = \sin \frac{\varphi}{2}. \quad (82c)$$

A rationale for the term $\frac{1}{2}$ in Eq. (82a) is the fact that $\frac{1}{2}$ is the least real number $\gamma$ such that $P(U_1, U_1) - |\alpha|^2 |\beta|^2 \cos \varphi \sin \varphi \sin \varphi + \frac{1}{2}$ is non-negative for all two-particle states of the form of Eq. (56a) and all unitary mappings $T_1$ and $T_2$ of the classes under consideration, as can be checked from Eqs. (72), (73), and (78). We now parallel Ref. [10] and define the two-particle fringe visibility $V_{11}$ as

$$V_{11} = \frac{\left[ P(U_1, U_1)_{\text{max}} \right] - \left[ P(U_1, U_1)_{\text{min}} \right]}{\left[ P(U_1, U_1)_{\text{max}} \right] + \left[ P(U_1, U_1)_{\text{min}} \right]}. \quad (83)$$

To find the extrema of $P(U_1, U_1)$ we use Eq. (82b) and set partial derivatives to zero: first,

$$a^2 + b^2 \leq 1. \quad (92)$$

This is the complementarity in two-particle interference.
The single-particle fringe visibilities $v_1$ and $v_2$ can be determined by inspection from Eqs. (9), (77), and (7B), together with $\phi \geq \beta$. Clearly,

$$\begin{align*}
[\psi(U_1)]_\text{max} &= [\psi(U_2)]_\text{max} = a^2. \\
[\psi(U_3)]_\text{max} &= b^2.
\end{align*}$$

Hence,

$$v_i = \frac{a^2 - b^2}{a^2 + b^2} = a^2 - b^2 \quad (i = 1, 2).$$

We note that $\psi(U_i)$ achieves its maximum and minimum when $a$ has the respective values 1 and 0. When $a$ is unity (hence $b$ is zero), a photon in C will go with certainty into $U_1$, and a photon in $C_2$ will go with certainty into $U_2$. When $a$ is zero (hence $b$ is unity), the exit states are reversed. A similar statement can be made concerning phonon 2, relating the vectors $(D)$ and $(D')$ to exit in beams $U_3$ and $L_3$.

We turn now to the two-particle fringe visibility $v_{13}$. As pointed out in Ref. [10], one cannot capture the intuitive meaning of two-particle fringe visibility by using the attractive definition

$$v_{13} = \frac{[\psi(U_1U_3)]_\text{max} - [\psi(U_2U_3)]_\text{max}}{[\psi(U_1U_3)]_\text{max} + [\psi(U_2U_3)]_\text{max}} \quad (81)\; .$$

This expression would yield a nonzero value even if $[0]$ is a product state, for in that case $P(U_1U_3)$ is the product of $P(U_1)$ and $P(U_3)$, and these vary respectively with $T_1$ and $T_3$. As in Ref. [10] we define a "corrected" joint probability $P(U_1U_3)$ by subtracting the product $P(U_1)P(U_3)$ from $P(U_1U_3)$ and adding a constant as a compensation against excessive subtraction,

$$P(U_1U_3) = P(U_1U_3) - P(U_1)P(U_3) + 1.$$  

By Eqs. (72), (77), and (7B),

$$P(U_1U_3) = a^2b^2 \cos \alpha \cos \beta + a|\sin \alpha| \sin \beta - 1.$$  

where

$$a = \cos \frac{\alpha}{2}, \quad b = \sin \frac{\alpha}{2}, \quad c = \sin \frac{\beta}{2}, \quad d = \sin \frac{\beta}{2}.\quad (82c)$$

$$A$$

is a rationale for the term $\frac{1}{2a}$ in Eq. (82a) is the fact that $\frac{1}{2}$ is the least real number such that $P(U_1U_3)$ is non-negative for all two-particle vectors of the form of Eq. (66) and all unitary mappings $T_1$ and $T_3$ of the classes under consideration, as can be checked from Eqs. (72), (77), and (7B). We now parallel Ref. [10] and define the two-particle fringe visibility $V_{13}$ as

$$V_{13} = \frac{[\psi(U_1U_3)]_\text{max} - [\psi(U_2U_3)]_\text{max}}{[\psi(U_1U_3)]_\text{max} + [\psi(U_2U_3)]_\text{max}} \quad (83).$$

To find the extrema of $\psi(U_1U_3)$ we use Eq. (82a) and set partial derivatives to zero. First,

$$0 = \frac{\partial [\psi(U_1U_3)]}{\partial \phi} = \frac{1}{2} a b \sin \alpha \sin \beta \sin \phi.\quad (84)$$

If $a b \neq 0$, then Eq. (84) can be satisfied only if one of the two following conditions is satisfied: (i) $\sin \alpha \sin \beta = 0$, in which case

$$0 = a b \sin \phi \leq a + b \sin \phi,$$

or (ii) $\sin \phi = 0$, in which case

$$P(U_1U_3) = a b \cos \phi \cos \alpha + |\alpha| \sin \phi \sin \phi,$$

and

$$P(U_1U_3) = a b \cos \phi \cos \alpha - |\alpha| \sin \phi.$$  

Note that in the neglected case of $b = 0$ these equations continue to hold, as do $E(83)$, because $a$ was assumed $\geq b$. It follows that without exception

$$V_{13} = 2a b.$$  

By Eqs. (80) and (90),

$$\frac{V_1^2 + v_2^2}{v_1^2 - v_2^2} = 4 a b (a^2 - b^2)^2 \quad (i = 1, 2)$$

which is the expression for the complementarity of one-particle and two-particle visibilities promised in the Introduction (slightly generalized, since $a + b$ or 0).

In Ref. [10] a more restricted set of transducers was considered than the class permitted here. Each $T_i$ was taken to consist of a symmetric beam splitter with reflectivity $r$ and transmissivity $t$ both equal to $1/\sqrt{2}$, together with a phase shifter in one beam incident upon the beam splitter. The small letters $v_i, i = 1, 2$, and $v_{12}$ denote the one-particle and two-particle visibilities under this restriction. It was shown that for a large class of two-particle vectors $(0, 0)$, the inequality

$$\alpha^2 + \beta^2 \leq 1,\quad (92)$$

is
3. The problem of obtaining definite measurement results.

Here is a highly idealized formulation of the problem, which suffices for present purposes. Suppose that \( \psi \) and \( \psi_2 \) are normalized vectors representing states of a microscopic object in which a physical quantity \( A \) has distinct values \( \psi_1 \) and \( \psi_2 \); that \( \psi_3 \) is a normalized vector representing the initial state of a measuring apparatus; that \( \psi_0 \) and \( \psi_0 \) are normalized vectors representing states of the measuring apparatus in which a macroscopic quantity \( \beta \) (such as the position of a pointer needle) has distinct values \( \beta_1 \) and \( \beta_2 \) respectively; and finally that the interaction between the microscopic object and the measuring apparatus is such that

\[
\psi_3 \otimes \psi_0 \rightarrow \psi_i \otimes \psi_j \quad (i = 1 \text{ or } 2),
\]

where the arrow stands for temporal evolution during an interval of specified duration \( \tau \). Under these circumstances, if it is known that initially the microscopic object was either in the state represented by \( \psi_2 \) or the state represented by \( \psi_0 \), but it was not known which, then the missing information can be obtained simply by examining the quantity \( \beta \) of the apparatus at time \( \tau \) after the interaction commenced and ascertaining whether the value of this macroscopic quantity is \( \beta_1 \) or \( \beta_2 \). Furthermore,

the observation of \( \beta \) will permit the inference of the value of \( A \) both at the beginning and at the termination of the measurement. Thus far there is no conceptual difficulty.

Suppose, however, that initially the microscopic object is prepared in the state represented by the superposition \( c_1 \psi_1 + c_2 \psi_2 \), where the sum of the absolute squares of \( c_1 \) and \( c_2 \) is 1, and neither \( c_1 \) nor \( c_2 \) is zero. States of this kind are physically possible, according to the formalism of quantum mechanics, and indeed it is often experimentally feasible to prepare such states. The linear dynamics of quantum mechanics implies then that

\[
(c_1\psi_1 + c_2\psi_2) \otimes \psi_0 \rightarrow c_1\psi_1 \otimes \psi_1 + c_2\psi_2 \otimes \psi_2.
\]

In the state represented by the sum on the right hand side of this process, the macroscopic quantity \( \beta \) does not have a definite value. This fact in itself is peculiar, because our ordinary experience indicates that macroscopic physical quantities always have definite values. Furthermore, there is a conceptual difficulty in understanding the quantum formalism. The standard interpretation of the superposition \( c_1\psi_1 + c_2\psi_2 \) is that the quantity \( A \) has an indefinite value, but in the event that \( A \) is actualized, there is a probability \( |c_1|^2 \) that the result will be \( \psi_1 \) and a probability \( |c_2|^2 \) that it will be \( \psi_2 \). Now if the quantum dynamics precludes a definite measurement result, what sense does it make to speak of the probabilities of various outcomes? A literal and realistic interpretation of the quantum dynamics undermines the literal and realistic interpretation of the quantum state!

Possible strategies for a solution:

1. Realistically describe measurement apparatus as initially not in a definite quantum state; use statistical operators instead.
2. Refrain from supposition of a precise measurement.
3. Relax description of measurement process (esp. positive operator valued measures).
4. Modify Hilbert space structure.
one can give the present formulation of approximate measurement in the following conditions:

(a) \( \{ E_n \} \) is a finite or denumerably infinite family of mutually orthogonal subspaces spanning \( \mathcal{X}_1 \), \( \{ E_n \} \) is a family of mutually orthogonal subspaces of \( \mathcal{X}_1 \), and \( U \) is a unitary operator on \( \mathcal{X}_1 \otimes \mathcal{X}_2 \);

(b) \( T \) is a statistical operator on \( \mathcal{X}_2 \) such that for every \( m \) and every \( n \) \( m \in E_m, (P_m \otimes T)U^{-1} \) can be expressed in the form \( \sum r \alpha_m \beta_m P_m \), where \( \alpha_n \in \mathcal{X}_1 \otimes \mathcal{X}_2 \), and the \( \alpha_m \) are non-negative real numbers summing to 1 such that

\[
\sum r \alpha_m = \epsilon_m < 1.
\]

The theorem of Sec. II implies that if these two conditions are satisfied and if the number of subspaces \( E_m \) is greater than one, then there exist initial states of the object for which the final statistical state of the object and \( T \) are such that the apparatus is not expressible as a mixture of eigenstates of the apparatus observable.

II. A THEOREM ON MEASUREMENT

It will be convenient for proving the theorem of this section to use the Dirac bra and ket notation, in which \( (\phi | \psi) = 1 \) implies that \( |\psi \rangle \) is the projection operator \( P_{\psi} \).

The theorem is the following:

Hypotheses:

(i) \( \{ \xi_j, \eta_j \} \) are normalized orthogonal vectors of \( \mathcal{X}_1 \), \( \{ E_n \} \) is a family of mutually orthogonal subspaces of \( \mathcal{X}_1 \), \( U \) is a unitary operator on \( \mathcal{X}_1 \otimes \mathcal{X}_2 \), and \( T \) is a statistical operator on \( \mathcal{X}_2 \);

(ii) there exist orthonormal sets \( \{ \xi_j \}, \{ \eta_j \} \) such that

\[
E_n = \mathcal{X}_1 \otimes E_n, \quad \text{for } j = 1, 2,
\]

and

\[
(U | P_j \otimes T)U^{-1} = \sum \psi_j | \xi_j \rangle \langle \xi_j | \eta_j \rangle.
\]

and for some value of \( n \),

\[
\sum \psi_j | \xi_j \rangle = \sum \psi_j | \eta_j \rangle.
\]

Conclusion: If \( u \) is defined as \( g_i \xi_i + g_j \eta_j \), with both \( g_i \) and \( g_j \) nonzero, then there exists no orthonormal set \( \{ \psi_n \} \) with \( \psi_n \in \mathcal{X}_1 \otimes \mathcal{X}_2 \) and no coefficients \( \{ b_m \} \) such that \( \sum b_n = 1 \) and

\[
(U | P_j \otimes T)U^{-1} = \sum b_n | \psi_n \rangle \langle \psi_n | \beta_n \rangle.
\]

This theorem shows that strategies 1 and 2 for obtaining definite measurement results will not succeed.
The foregoing theorem was generalized in two steps: first, from the standard observables of QM, which are self-adjoint operators in a Hilbert space, and second by the spectral theorem, representable as unique projection-valued measures, to "sharp observables" which are associated with a more general set of projection-valued measures.

A first extension of the set of observables is obtained by admitting more general projection-valued (PV) measures, defined with respect to a measurable space \((\Omega, \Sigma)\), where \(\Omega\) is a set and \(\Sigma\) is a \(\sigma\)-algebra of subsets of \(\Omega\); i.e., a PV measure is a map \(E\) from \(\Sigma\) into the lattice of projections such that \(E(\emptyset) = 0\), \(E(\Omega) = 1\), \(E(\emptyset) = 0\) (denoting the null and unit operators in \(H\), respectively), and \(E(\cup_i X_i) = \sum_i E(X_i)\) for any countable collection of mutually disjoint sets \(X_i \in \Sigma\). These conditions ensure that for any state operator \(T\) of \(S\), the map \(X \mapsto \langle T X | X \rangle = \rho(T, X)\) is a probability measure on \((\Omega, \Sigma)\). Since \(\Omega\) concerns the set of possible values of the physical quantity represented by \(E\), \((\Omega, \Sigma)\) is called the state space of that observable. The case of a spectral measure is recovered by choosing the real line for \(\Omega\) and the Borel sets for \(\Sigma\). The introduction of more general value spaces \((\Omega, \Sigma)\) and of observables as PV measures on them proves convenient for a variety of purposes, such as, for example, the description of joint measurements of several commuting observables. Henceforth, we shall use the term "sharp observable" for an observable represented by a general PV measure (Busch et al., 1991) in anticipation of a further generalization in Section 3 to "unsharp observables."

The second step is to "unsharp observables", associated with projective-operator-valued measures (POVM measures):

The map \(E \equiv X \mapsto E(X)\) is a positive-operator-valued (POV) measure if the following conditions are satisfied: For each \(X \in \Sigma\), \(E(X)\) is an operator on the underlying Hilbert space \(H\) such that \(0 \leq E(X) \leq I\) (the ordering being in the sense of expectation values; i.e., \(A \leq B\) if and only if \(\langle A | \psi \rangle \leq \langle B | \psi \rangle\) for all \(\psi \in H\)) whenever \(E(\emptyset) = 0\) and \(E(\Omega) = I\), and \(E(\Omega) = \sum E(X)\) for any countable pairwise disjoint family \(\{X_i\} \subseteq \Sigma\). These properties of \(E\) ensure that the map \(\rho(T) : X \mapsto \langle T X | X \rangle = \rho(T, X)\) is a probability measure for each state \(T\). The special case of POVM measures is recovered if the additional property of idempotency, \(E(X)^2 = E(X)\), is violated.

Note that the idempotency condition can be written as \(E(X) = E(X)E(X) = E(X)\). The condition \(E(\Omega) = I\) is equivalent to the condition \(E(X)^2 = E(X)\). If a POVM measure is a sharp observable if and only if for any two disjoint sets \(X, Y\), the operators \(E(X), E(Y)\) satisfy \(E(X)E(Y)\) = 0. Such projections are orthogonal in each other in the sense that their ranges are mutually orthogonal subspaces. By contrast, for all other POVM measures there will be sets \(X, Y\) such that \(E(X)\) and \(E(Y)\) are non-orthogonal. Such POVM measures shall be called "unsharp observables."
A rough operational definition of "projection-valued measure": a rule for partitioning an arbitrary quantum state into projections onto eigenspaces of some physical quantity of a system and then labeling the projected pieces with the eigenvalues associated with the eigenspaces. (Must be refined in case of continuous spectra.)

A "position-operator valued measure" is also a rule for partitioning an arbitrary quantum state and labeling the pieces, but in general labeling with less information than eigenvalues — less than the whole truth but not a distortion of the truth.
By means of an auxiliary theorem, the superposition
of superpositions, this theorem alone from Phys.Rev.
B7, Part II, is generalized from general sharp
observable to sharp observable.
Thus, strategy 3 for achieving definite experimental
results will not succeed.
By means of an auxiliary theorem, the Independence of Superpositions, the theorem alone from Phys. Rev. 1(1), Part II, is generalized from general, sharp, observable and then to unsharp, observable. Thus, strategy 3 for obtaining definite, experimental results will not succeed.

We started this paper by outlining the experimental facts of the wave-particle dualism and then set up the measurement problem as follows: it is possible to derive the wave-function collapse by measurement, as described in eq. (3.9), by applying quantum mechanics to the total system consisting of an object quantum particle and an apparatus system? Unlike the conventional Copenhagen interpretation, we consider the wave-function collapse by measurement as a dephasing process that provokes the disappearance of the phase correlations existing among the different eigenstates of the observable to be measured. We repeatedly emphasized that the process is characterized by the lack of the off-diagonal components of the final total density matrix as a result of the dephasing process, and not by a simple orthogonal decomposition of the apparatus wave function.

Our answer to the measurement problem is affirmative. In fact, we have explicitly derived the wave-function collapse by measurement, eq. (3.9), by taking into account the statistical fluctuations in the measuring apparatus, in the limit of infinite number of degrees of freedom of the apparatus system.

It is very important to remark that the exact wave-function collapse takes place only in the infinite $N$ limit ($N$ being the number of degrees of freedom) and is to be regarded as an asymptotic process, like a phase transition. However, in practice, a finite but very large $N$ suffices to produce the wave-function collapse, as was repeatedly discussed and was shown by numerical simulations. Of course, as long as we keep $N$ finite, the present theory yields only an approximate wave-function collapse, even though the exact collapse can be approximated up to any desired accuracy by increasing $N$.

Do not forget that the present theory describes the exact wave-function collapse as an asymptotic limit.

For fixed and finite $N$, coherence among the branch waves engendered by the spectral decomposition is partially lost, and the measurement is not perfect. In this case, we are facing an imperfect measurement. Up to what extent a measurement is imperfect depends on the details of the physical process taking place in the detector.

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References: M. Namiki & S. Passazion, Physics Reports 222, no. 6, Sept. 1993, p. 405. See also pp. 337-342.

Questions: (1) Is the mathematical treatment of infinite products of Hilbert spaces correct? (2) Does the mathematical structure complete the approach of B. B. Grifiths, M. Gell-Mann & J. Hartle, R. Forman & J. V. Zuber, who emphasize loss of phase relations because of interaction with immense environment? (3) Is the approximation when $N$ is very large but finite sufficient to yield definite measurement results? (4) Is the philosophical question: can actuality be achieved? A sufficient yield probabilities $0$ and $1$?
4. Proposed experiment to test Cornaladesi’s conjecture that the Pauli Principle holds after a “relaxation time” in a freshly constituted ensemble of electrons. The onset of the principle is due to interactions in the ensemble.

Ernesto Cornaladesi has conjectured that the symmetry of integral spin particles under exchange and the antisymmetry of half-integral spin particles under exchange are not kinematic principles but are rather time-dependent consequences of interactions among the particles. Hence, a freshly constituted ensemble of electrons may exhibit violations of the Pauli Exclusion Principle (PP), but as the ensemble ages, the violations become more and more infrequent. An experiment is proposed to test Cornaladesi’s conjecture. A beam of Na⁺ ions, accelerated in a linear accelerator to 1000 eV, is scanned by a beam of electrons from an electron gun at variable positions along the direction of flow of the ions. Some of the ions capture electrons, at a rate monitored by detectors sensitive to the photons emitted in the capture process. A PP violating electron can make a transition to the doubly occupied 1s level, emitting a photon of approximately 1 keV. A rate of detection of such photons, which diminishes with the distance of the detector from the point of capture, and hence with the age of the ensemble, permits in principle the calculation of the equilibration constant of Cornaladesi’s conjecture. Reasonable assumptions about the parameters of the experimental arrangement indicate that if the conjecture is correct and the equilibration constant is not shorter than $10^{-15}$, the proposed experiment can determine the value of this constant.

PP is a corollary of Pauli’s theorem of the connection between spin and statistics, Phys. Rev. 52, 746-748. That theorem assumes local validity of Lorentz invariance. If local validity of Lorentz invariance is experimentally confirmed, a possible explanation is limited validity of Lorentz invariance in the small.

Request: can any one help to achieve a performance of the proposed experiment?
Fig. 1

Micrometer screw moves $X$ in steps of $10^{-4}$ cm.
Lens moves $X$ in steps of $10^{-6}$ cm.
Detectors $A_n$ and $A'$ at $X$, both movable.
Detectors $A_1$, $A_2$, ..., $A_{10}$ fixed, separated by 20 cm.
Diameter of each detector: 2.5 cm.
Diagram not drawn to scale.
5. How should one treat the tension between the nonlocality exhibited in careful experimental tests of Bell's Inequality and the locality implicit in the special theory of relativity?


Bell's rejection of this anthropocentric variety of peaceful coexistence:

"The obvious definition of 'local causality' does not work in quantum mechanics, and this cannot be attributed to the 'incompleteness' of that theory."¹

Exponents have looked to see if the relevant predictions of quantum mechanics are in fact true. The consensus is that quantum mechanics works excellently, with no sign of an error. It is often said then that experiment has decided against the locality inequality. Strictly speaking that is not so. The actual experiments depart too far from the ideal, and only after the various deficiencies are 'corrected' by theoretical extrapolations do the actual experiments become critical. There is a school of thought which stresses this fact, and advocates the idea that better experiments may contradict quantum mechanics and vindicate locality. I do not myself entertain that hope. I am too impressed by the quantitative success of quantum mechanics for the experiments already done, to hope that it will fail for more nearly ideal ones.

Do we then have to fall back on "no signalling faster than light" as the expression of the fundamental causal structure of contemporary theoretical physics? That is hard for me to accept. For one thing we have lost the idea that correlations can be explained, or at least this idea awaits reformulation. More importantly, the "no signalling..." notion rests on concepts which are desperately vague, or vaguely applicable. The assertion that "we cannot signal faster than light" immediately provokes the question:

Who do we think we are?

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Who do we think we are?

For who can make 'measurements', we who can manipulate 'external fields', we who can 'signal' at all, even if not faster than light? Do we include chemists, or only physicists? plants, or only animals? pocket calculators, or only supercomputers?
Abstract. One of the most important and most frequently discussed theological problems related to cosmology is the creation problem. Unfortunately, it is usually considered in a context of a rather simplistic understanding of the initial singularity (often referred to as the Big Bang). This review of the initial singularity problem considers its evolution in twenty-first-century cosmology and develops methodological rules of its theological (and philosophical) interpretations. The recent work on the "noncommutative structure of singularities" suggests that on the fundamental level (below the Planck's scale) the concepts of space, time, and localization are meaningless and that there is no distinction between singular and nonsingular states of the universe. In spite of the fact that at this level there is no time, one can meaningfully speak about dynamics, albeit in a generalized sense. Space, time, and singularities appear only in the transition process to the macroscopic physics. This idea, explored here in more detail, clearly is not an atemporal understanding of creation.


Nevertheless, dynamics in a generalized sense is definable in a physics based on noncommutative geometry. Likewise, the concept of probability based upon the statistics of individual events has to be abandoned, but a generalized concept of probability can be defined.

Ordinary physics is recovered by a restriction to the noncommutative algebra’s center, which is commutative — a restriction that gives birth to space, time, and multiplicity.

The generalized dynamics of the fundamental noncommutative regime has two limiting cases, one of which is standard quantum mechanics and one is standard general relativity.

The nonlocality of EPX correlations in the first limit and the singularities and horizon problem that appear in the second limit are the residue of the global character of the fundamental level — an appealing method of resolving well-known problems.

The generalization of physical concepts resulting from using noncommutative geometry at the fundamental level has profound implications for philosophy. At the fundamental level the world is characterized by timelessness and nonlocality. If the concept of causality is to be retained, it has to be generalized. "It seems that the essence of causality is a dynamical nexus rather than the distinction between the cause and its effect, and their temporal order"
Attractive Features of Michael
Heller's approach to physics in the
extreme microscopic domain.

1. By abandoning the concepts of point and neighborhood, it offers a fundamental role to nonlocality.

2. By recasting ordinary space-time structure on a sufficiently large scale it promises a "correspondence relation" between the extreme microscopic physics and the physics of atoms, nuclei, and elementary particles.

3. The conjunction of (1) and (2) may provide an answer to the problem of peaceful coexistence of QM and relativity theory.

4. Even though differential operations are not definable in the extreme microscopic domain, there is an algebraic surrogate: Leibniz's Rule: \( A(fg) = (Af)g + f(Ag) \). Some kind of dynamics is possible.

5. It endows the primitive world with enough structure to permit the kind of evolutionary cosmic process envisaged by Prigogine, Whithead, Wheeler, & Smolin to take place. It does not claim "law without law", but the primitive law envisaged is minimal.

6. and maybe fertile — i.e., capable of admitting transience (?) and protomateriality (?)

7. The mathematics is learnable, because there are elementary (?) expansions.
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