Title: Constraining Inverse Curvature Gravity with Supernovae

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Abstract: We show that the current accelerated expansion of the Universe can be explained without resorting to dark energy. Models of generalized modified gravity, with inverse powers of the curvature can have late time accelerating attractors without conflicting with solar system experiments. We have solved the Friedman equations for the full dynamical range of the evolution of the Universe. This allows us to perform a detailed analysis of Supernovae data in the context of such models that results in an excellent fit. Hence, inverse curvature gravity models represent an example of phenomenologically viable models in which the current acceleration of the Universe is driven by curvature instead of dark energy. If we further include constraints on the current expansion rate of the Universe from the Hubble Space Telescope and on the age of the Universe from globular clusters, we obtain that the matter content of the Universe is $0.07 \leq \omega_m \leq 0.21$ (95% Confidence). Hence the inverse curvature gravity models considered can not explain the dynamics of the Universe just with a baryonic matter component.
Constraining Inverse Curvature Gravity with Supernovae

O. Mena, J. Santiago and JW
PRL, 96, 041103, 2006
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The Cosmic Pie
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- Combining cosmic microwave background, Supernovae and large scale structure observations: WMAP, SCP, High-z, SNLS,..., SDSS, 2dF,...
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- Baryonic Matter 4%
- Dark Matter 21%
- Dark Energy 75%
Supernovae Measurements

SNe allow measurement of distance - redshift relation at large redshifts: The expansion of the Universe is accelerating!

- Perlmutter et al.; Riess et al.; Knop et al.; Astier et al.
Cosmological Constant
Cosmological Constant

✦ Simplest explanation; consistent with SNe, CMB, LSS, clusters of galaxies, …
Cosmological Constant

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- Cosmological Constant problem: Why $\Lambda \ll (\text{TeV})^4$?
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  Why \( \Lambda \ll (\text{TeV})^4 \) ?
- Why now: \( \Omega_m \approx \Omega_{\Lambda} \) ?
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★ Maybe something else …
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Dark Energy

simple fluid: $p = w \rho$
Dark Energy

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Deceleration parameter (flat Universe, only DE):

$$q_0 = -\frac{\ddot{a}_0}{H_0^2} = \frac{1 + 3w}{2}$$
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Quintessence

\[ \Omega_\Lambda = 0.7 \quad \rightarrow \quad \rho_\Lambda \approx 10^{-48} \text{ eV} = 10^{-121} M_{\text{pl}}^4 \]
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Dynamical dark energy

Equation of state of scalar field:

\[ w = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)} \]
Quintessence

1st try (Wetterich, Ratra and Peebles 1988, Ferreira and Joyce 1998):

\[ V(\phi) = e^{-\lambda \phi / M_{pl}} \]
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2nd try (Steinhardt, Caldwell et al. 1998):

\[ V(\phi) = M^4 e^{-M_{pl} / \phi} \quad ; \quad V(\phi) = M^{4+\alpha} / \phi^\alpha \]
Different Quintessence Models

Scalar field dark energy models (quintessence)

\[ w = \frac{p}{\rho} \]
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All models ad hoc
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but maybe something completely...
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All models ad hoc

but maybe something completely different ...
Maybe gravity is standard at short distances...
but gets modified on large distances ...
New Gravitational Action
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\[ S_{E-H} = \frac{1}{16\pi G} \int \sqrt{-g} R d^4 x \]
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But in general:

\[ S = \frac{1}{16\pi G} \int \sqrt{-g} F(R, R_{\mu\nu} R^{\mu\nu}, R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}, \ldots) d^4 x \]
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Simple approach: \( F(R) = R + m R^n \)
Well known for $n>1 \rightarrow$ early de Sitter
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Modification becomes important at low curvature and can lead to accelerated expansion
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$$3H^2 - \frac{\mu^4}{12(\dot{H} + 2H^2)^3} \left(2H\ddot{H} + 15H^2\dot{H} + 2\dot{H}^2 + 6H^4\right) = \frac{\rho_M}{M_{Pl}^2}$$
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Purely gravitational alternative to dark energy
1/R model

+ accelerated attractor: [CDDETT]
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  - de Sitter (unstable)
  - Future Singularity
  - power law acceleration $a(t) \sim t^2$
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- Observational consequences similar to dark energy with $w = -2/3$
General $f(R)$ actions
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\[ \text{e.g. } \mu^{2(n+1)}/R^n, \text{ with } n>1 \text{ have late-time acceleration} \]
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Non-Cosmological Constraints on $f(R)$ Theories

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- f(R) models in Einstein frame (\( \omega = 0 \)):
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✧ Simplest model ($\propto 1/R^n$) ruled out by observations of distant Quasars and the deflection of their light by the sun with VLBI: $\omega > 35000$ [Chiva (‘03), Soussa, Woodard (‘03), ...]
The New Model
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\[ S = \frac{1}{16\pi G} \int dx \sqrt{-g} \left[ R - \frac{\mu^6}{(aR^2 + bR_{\mu\nu}R^{\mu\nu} + cR_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})} \right] \]

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- Late time accelerated attractor [CDDTT’04]
Example $1/R_{\mu\nu}R^{\mu\nu}$ Model
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- If one chooses: $c = -4b$ in action, there are **NO GHOSTS**: I. Navarro and K. van Acoleyen 2005
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\[ P = R_{\mu} R^{\mu} \]
\[ \alpha = R_{\mu \nu \sigma \delta} R^{\mu \nu \sigma \delta} \]
\[ F(R, Q-4\pi) \]
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- In general $F(R, Q-4P)$ with $Q = R_{\mu\nu} R^{\mu\nu}$ and $P = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$ has no ghosts, however…
We are still afraid of Tachyons
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- For higher inverse powers $1/(aR^2+bP+cQ)^n$ there is hope!
Solar Systems Tests
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- Linear expansion around Schwarzschild metric
Solar Systems Tests

Linear expansion around Schwarzschild metric

\[ \phi(r) \simeq - \left[ 1 - \frac{\alpha}{2} \left( \frac{r}{r_c} \right)^{6n+4} \right] \frac{GM}{r} \]

Navarro et al. 2005
Non-Cosmological Tests
Non-Cosmological Tests

with critical radius
Non-Cosmological Tests

with critical radius

\[ r_c^{6n+4} = \frac{(GM)^{2n+2}}{\mu^{4n+2}} \]
Non-Cosmological Tests

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✦ for solar system: 10pc !
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Modified Friedman Equation
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\[ \frac{H'' \mathcal{F}_1(H, H') + \mathcal{F}_2(H, H')}{{\mathcal{F}_3}(H, H')} + \frac{\mu^6}{H^4} + H^2 = \frac{8\pi G}{3} \rho \]
Modified Friedman Equation

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**Modified Friedman Equation**

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- **Effectively dependent on 3 extra parameters**:

  \[
  \alpha = \frac{12a + 4b + 4c}{12a + 3b + 2c} \quad \hat{\mu} = \frac{\mu}{[12a + 3b + 2c]^{1/6}} \quad \sigma = \text{sign} (12a + 3b + 2c)
  \]
Dynamical Analysis

- $\sigma$ is fixed by the dynamical behavior of the system

$\bar{\omega}_m$
Dynamical Analysis

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$$\alpha_1 = \frac{8}{9}, \quad \alpha_2 = \frac{4(11 - \sqrt{13})}{27} \approx 1.01, \quad \alpha_3 = \frac{20(2 - \sqrt{3})}{3} \approx 1.79, \quad \alpha_4 = \frac{4(11 + \sqrt{13})}{27} \approx 2.16$$

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\[\bar{\omega}_m\]
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  \]

- For \( \alpha < \alpha_1 \): both values of \( \sigma \) are acceptable
- For \( \alpha_1 < \alpha < \alpha_2 \): \( \sigma = +1 \) hits singularity in past

\[\tilde{\omega}_m\]
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- For $\alpha_3 < \alpha < \alpha_4$: no longer stable attractor and singularity is reached in the future through an accelerated phase. For small $\bar{\omega}_m$ this appears in the past.
Solving the Friedman Equation for n=1
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$$H_{\text{approx}} = H_E \left( 1 - \frac{1}{2} \frac{H''_E F_1(H_E, H'_E) + F_2(H_E, H'_E)}{F_3(H_E, H'_E)} \frac{\mu^6}{H_E^4} \right)$$
Perturbative Solution for $\alpha=1$

$$u = \log H \quad x = \log a$$

$$6\ddot{u} + 15\dot{u}^2 + 34\dot{u} + 8 + 18\Delta e^{4u} (2 + \ddot{u})^6 \left[ e^{2(\ddot{u} - u)} - 1 \right] = 0$$

$$\Delta \equiv 12a + 3b + 4c = 4(3a - c) \quad \text{for } \alpha = 1$$

$$\ddot{u} = \log \dot{H} = \log \left( H_0 \sqrt{\Omega_r e^{-4x} + \Omega_m e^{-3x}} \right)$$

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Solution and Conditions

\[ \epsilon = -\frac{1}{9H_0^6\Delta\Omega_m} \frac{40\Omega_r + 37\Omega_m e^x}{\Omega_r + \Omega_m e^x} e^{9x} \]

\[ \epsilon \ll 1 \]

\[ 6\frac{\ddot{\epsilon}}{1+\epsilon} + 9 \left( \frac{\dot{\epsilon}}{1+\epsilon} \right)^2 + 34\frac{\dot{\epsilon}}{1+\epsilon} \ll \ddot{u} + 15\dot{u}^2 + 34\dot{u} + 8 \]

\[ \frac{\dot{\epsilon}}{1+\epsilon} \ll 2 + \dot{u} \]
Specific Conditions

\[ a \ll \left( \frac{-9H_0^6 \Delta \Omega_m^3}{37} \right)^{1/9} \sim O(1) \]

For example with \( \Delta = -4 \) at \( a = 0.2 \):

\[ 1.2 \times 10^{-3} \ll 1 \]

\[ 0.99 \ll 9.24 \]

\[ 0.011 \ll 0.5 \]

In general all 3 conditions break down at \( a > 0.1 - 0.2 \)
Dynamics of best fit model

\[ \nu = -\frac{H^2}{\dot{H}} \]

\[ a \sim t^p \rightarrow \nu = p \]
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- Use approximate solution as initial condition at $z=\text{few (7)}$ for numerical solution (approximation very accurate and numerical codes can cope)
Fit to Supernovae Data
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Include intrinsic magnitude of Supernovae as free parameter: Degenerate with value of $H_0$ or better absolute scale of $H(z)$. Measure all dimensionful quantities in units of $\mu$. 
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  \[ \sigma = -1, \quad 0.89 \leq \alpha \leq 1.10 \]  
  low

  \[ \sigma = +1, \quad 1.10 \leq \alpha \leq 2.16 \]  
  high
Fit to Riess et al (2004) gold sample; a compilation of 157 high confidence Type Ia SNe data.

$$\alpha = 0.9 , \quad \bar{\omega}_m = 0.105 , \chi^2 = 184.9$$

$$\alpha = 2.15 , \quad \bar{\omega}_m = 0.085 , \chi^2 = 185.2$$

very good fits, similar to $\Lambda$CDM ($\chi^2 = 183.3$)
Combining Datasets
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![Graph showing marginalized range $0.07 < \omega_m < 0.21$ (95%)](image-url)
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![Diagram with confidence regions for different scenarios]
Combining with the Cosmic Microwave Background?
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marginalized $0.07 < \omega_m < 0.21$ (95% c.l.); require dark matter
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Small scale CMB anisotropies are mainly affected by the physical cold dark matter and baryon densities and the angular diameter distance to last scattering

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Angular Diameter Distance to Last Scattering

For the brave:
Angular diameter distance to last last scattering with WMAP data - might as well be bogus!
Including Perturbations in 1/R modes

$\Lambda$CDM

Bean et al.
2006

SDSS data

no simultaneous small scale agreement and CMB

1/R shifted to fit small scales

1/R same normalization as $\Lambda$CDM
But also, ...

Song, Hu, Sawicki 2006

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Perimeter Institute, Waterloo
April, 2007
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