Title: Ekpyrotic Perturbations & a Holographic Big Bang

Date: May 08, 2007  11:00 AM

URL: http://pirsa.org/07050008

Abstract: TBA
Ekpyrotic Perturbations

and

A Holographic Big Bang

- An alternative to inflation
- Scale-invariant curvature perturbations
- Non-perturbative bounce in M theory
- “Scale invariance from Scale Invariance”
Ekpyrotic Perturbations and A Holographic Big Bang

- An alternative to inflation
- Scale-invariant curvature perturbations
- Non-perturbative bounce in M theory
- "Scale invariance from Scale Invariance"
work with:

- Jean-Luc Lehners,
  Paul McFadden,
  Paul Steinhardt.

- Ben Craps,
  Thomas Hertog.
So far, observations are consistent with a spatially flat Universe, and the simplest possible perturbations:

- Gaussian
- Linear, growing mode
- Adiabatic
- Scalar
- Scale-Invariant

-as predicted by simple inflationary models,
Inflation

- Assumes start in a super-dense, $P = -\rho$ state: why?
- Cosmic singularity unresolved
- Requires fine tuned potentials $\lambda < 10^{-10}$
  
  $\rho_{DE} \sim 10^{-100} \rho_{INF}$

- Strange empty future
- Measure problem: canonical measure, with random ICs $\Rightarrow P(N) \sim e^{-3N}$

Gibbons+NT 2006
Inflation’s most specific signature - primordial tensor modes - has not yet been seen
Motivations for a radical alternative

1. The dark energy puzzle: what is its role?

2. The idea that today's universe is in a dynamical, metastable state

3. String and M theory must deal with the singularity: since all we see traces back to it, it is surely crucial to determining the physical selection of states.

4. Either time began at the singularity, or it didn't. Let's consider both options.
Inflation

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Inflation’s most specific signature - primordial tensor modes - has not yet been seen
"The Cyclic Universe"

**Inter-brane force draws branes together**, amplifying quantum wrinkles.

A trillion years after the bang: branes are empty, flat and parallel.

Two branes engage in an endless cycle of collision, rebound, stretching, and collision once again.

Wrinkled branes collide, create slightly non-uniform hot plasma, and rebound.

A microsecond after the bang: branes reach maximum separation but continue to stretch rapidly, filled with radiation.

You are here:

Dark energy takes over, driving accelerated expansion that begins to spread out.

Radiation dilutes away. Matter dominates and clusters around non-uniformities to form galaxies and stars.
Ekpyrotic perturbations

e.g. \[ V = -V_0 e^{-c\phi} \]

\( \phi \) (radion)

Scale symm: \( x^\mu \to e^{\lambda} x^\mu, \)
\( \phi \to \phi + 2\lambda/c, \bar{\eta} \to e^{2\lambda} \bar{\eta} \)

Scaling solution: \( \dot{\phi} \sim t^{-1} \)

\( |kt| \ll 1 \) Time delay mode: \( \delta \phi \sim \dot{\phi} \sim t^{-1} \)

Scaling symmetry \( \to \) \( \langle \delta \phi^2 \rangle \sim \bar{\eta} t^{-2} \int d^3k/k^3 \)

cf Massless scalar in de Sitter;
scaling background soln \( ds^2 = (-dt^2 + dx^2)/(Ht)^2 \)
scale symmetry \( x^\mu \to \lambda x^\mu \)
shift mode \( \phi \to \phi + c, \) \( c \) constant

Hence, \( \langle \delta \phi^2 \rangle \sim \bar{\eta} H^2 \int d^3k/k^3 \)
\[ \delta \phi = \frac{2}{t^2} \delta \phi - k^2 \delta \phi \]
\[ \Delta \phi - \nu \Delta \phi \]

\[ \delta \dot{\phi} = \frac{\partial}{\partial t} \delta \phi - \frac{\partial^2}{\partial t^2} \delta \phi - \kappa^2 \delta \phi \]
Ekpyrotic perturbations

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\( \phi \) (radion)

Scale symm: \( x^\mu \rightarrow e^\lambda x^\mu \),
\( \phi \rightarrow \phi + 2\lambda/c, \, \hat{h} \rightarrow e^{2\lambda} \hat{h} \)

Scaling solution: \( \dot{\phi} \sim t^{-1} \)

\( |kt| << 1 \) Time delay mode: \( \delta\phi \sim \phi \sim t^{-1} \)

Scaling symmetry \( \rightarrow \) \( \langle \delta\phi^2 \rangle \sim \hat{h} t^{-2} \int d^3k/k^3 \)

\text{cf} \quad \text{Massless scalar in de Sitter;}
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Hence, \( \langle \delta\phi^2 \rangle \sim \hat{h} H^2 \int d^3k/k^3 \)
Now include gravity

\[ ds^2 = -dt^2 (1 + 2\Phi) + a^2(t) dx^2 (1 - 2\Phi) \]

\[ \delta t = \frac{\alpha_1(x)}{a} - \frac{\alpha_2(x)}{a} \int_t^{t'} dt' a(t') , \quad \delta x^i = (1 + \alpha_2(x)) x^i \]

\[ \Phi = \alpha_1(x) \frac{\dot{a}}{a^2} + \alpha_2(x) \left( 1 - \frac{\dot{a}}{a^2} \int_t^{t'} dt' a(t') \right) \]

Long \( \lambda \), Quasi-gauge modes

Local time delay
Decaying
Growing
Decaying

Expanding U
Contracting U
How can a local time delay match on to a local spatial dilation?

Creminelli et al, Lyth, Huang...

A. 5d effects near bounce (warping of 5th dimension): 
Tolley et al., Battefeld et al., McFadden et al.

B. Additional light dofs in 4dET driven unstable:
Lehners, McFadden, Steinhardt, NT
Creminelli, Senatore
Buchbinder, Khoury, Ovrut
Koyama, Wands
Tolley, Wesley
Koyama, Mizuno, Wands

Flurry of papers 2007
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Local time delay

Local dilatation: “Curvature pertn. \( R \)”

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Flurry of papers 2007
Assume two scalar fields, $\phi_1$ and $\phi_2$, with independent, negative, steeply flattening potentials.

Relative perturbation:

$$\frac{\delta \phi_1}{\phi_1} - \frac{\delta \phi_2}{\phi_2}$$

Scale-invariant on long wavelengths.

But this converts easily to $R$.
General result:

\[ \dot{\mathcal{R}} = -\frac{H}{\dot{H}} g_{IJ}(\phi) \frac{D^2 \phi^I}{Dt^2} s^J + \frac{H}{\dot{H}} k^2 \Psi \]

where the entropy perturbation is

\[ s^I = \delta \phi^I - \phi^I g_{JK}(\phi) \frac{\phi^J \delta \phi^K}{g_{LM}(\phi) \phi^L \phi^M} \]
Heterotic M Theory

\[ \int_5 \left( \frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - Ce^{-2\phi} \right) - \sum_i \mu_i \int_4 e^{-\phi} \]

Two moduli:
radion and \( V_{cy} = e^\phi \)

Both can pick up scale-invariant perturbations before pre-bang \( \rightarrow \) entropy perturbations

Before and after boundary brane collision, minus brane hits zero of \( H \) and bounces back. This bounce converts entropy to curvature!
General result:

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perturbation pre-bang \( \rightarrow \) entropy

perturbation

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5d solution

Trajectory tangential to singularity

-described by a hard boundary ($\phi_2=0$) in the 4d effective theory

embedding in 5d static
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M-theory model for the bang

Winding M2 branes = Strings:

No blue-shift for winding membranes:
describe perturbative string states
including gravity

Weak coupling at singularity

Classical evolution of string is
regular across t=0

Calculable $T_{HBB}$ due to string creation

BUT: what about KK modes
i.e. nonperturbative string states?
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Perry, Steinhardt & NT, 2004
Berman & Perry, 2006
Niz+NT 2007
A Holographic Big Bang

IIB SUGRA on $S^5 \times \text{AdS}^5$
includes $m^2 = -4$ BF scalar

$\phi \sim \alpha r^{-2} \ln r + \beta r^{-2}$

SUSY$\rightarrow \alpha = 0$ no dynamics
If $\alpha = \alpha(\beta)$ $\rightarrow$ dynamics

Bulk collapses to a finite-time singularity

Deformed CFT on $\mathbb{R} \times S^3$
Also unstable:

Unstable
5d bulk

Craps, Hertog, NT

Hertog+Horowitz
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A Holographic Big Bang

AdS/CFT

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A Holographic Big Bang

\[ \alpha = \lambda \beta \] corresponds
deformation \(-\lambda \phi^4\) of
CFT \(\rightarrow\) instability
\( (\phi^2 = \text{Tr}(\phi_1^2 - \phi_2^2) ) \)
\( \lambda \) is asymptotically free

\[ V(\phi) = -\frac{16 \pi^2 \phi^4}{3 \ln(\phi/M)} \]

large \( N \rightarrow \beta \) fn is 1-loop
exact, \( V \) under good
perturbative control
Unstable CFT

\[ V(\phi) \sim + R_{AdS}^{-2} \phi^2 - \lambda \phi^4 \]

Finite \( V_3 \): need wavefn for backgrd

Self-adjoint extension

Take \( M \ll R_{AdS}^{-1} \) -> weak coupling

Conformal coupling
Key Points

* No gravity in CFT

* Finite time singularity $\rightarrow$ Ultralocality

  Quantum mechanics $\rightarrow$ natural resolution of singularity via "self-adjoint extension"

* Asymptotic freedom

* Finite $V_3 \sim$ entire background becomes quantum around singularity

* CFT is (nearly) scale invariant $\rightarrow$

  Automatically get scale-invariant pertur...
A Holographic Big Bang

\[ \alpha = \lambda \beta \text{ corresponds to deformation} - \lambda \phi^4 \text{ of CFT} \rightarrow \text{instability} \]

\[ ( \phi^2 = \text{Tr}(\phi_1^2 - \phi_2^2) ) \]

\[ \lambda \text{ is asymptotically free} \]

\[ V(\phi) = -\frac{16 \pi^2 \phi^4}{3 \ln(\phi/M)} \]

large \( N \rightarrow \beta \text{ fn is 1-loop exact, } V \text{ under good perturbative control} \]
Unstable CFT

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singularity via “self-adjoint extension”

* Asymptotic freedom
* Finite \( V_3 \) \( \sim \) entire background becomes quantum around singularity
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Key Points

* No gravity in CFT

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* Finite $V_3 \sim$ entire background becomes quantum around singularity

* CFT is (nearly) scale invariant $\rightarrow$
  Automatically get scale-invariant pertns
1. Ultralocality

$$\partial^2 \phi = -\lambda \phi^3 + \frac{1}{6} R \phi$$

**zero E bg soln:**

$$\phi = \sqrt{\frac{2}{\lambda t - t_s}}$$

**Gen soln:**

$$-d\tau^2 + h_{ij} dx^i_s dx^j_s,$$

$$h_{ij} = h_{ij}^{(0)} + 2K_{ij}\tau + K_{ik}h_{jl}^{(0)}K_{lj}\tau^2$$

$$h_{ij}^{(0)} \equiv \delta_{ij} - \delta_i t_s \delta_j t_s, \quad K_{ij} \equiv \gamma \partial_i \partial_j t_s.$$
Interpretation in linearized theory

\[ \delta \chi(t, x) = \sqrt{\frac{\lambda}{2}} (-t_s(x)) + \frac{1}{6} t^2 \nabla^2 t_s - \frac{1}{24} t^4 (\nabla^4 t_s) + \ldots + \rho(x_s) t^5 + \ldots \]

Time delay: \( \delta \phi/\phi \)

Hamiltonian density

\[ \delta \mathcal{H} = \dot{\phi} \delta \phi + V, \phi \delta \phi = \partial_t \left( \frac{\delta \phi}{\phi} \right) \phi^2 \]

As gradients become unimportant, different spatial points decouple \( \rightarrow \) QM

Self-adjoint extension matches local time delay and energy density across singularity
1. Ultralocality

zero E bg soln:

$$\phi = \sqrt{\frac{2}{\lambda t - t_s}}$$

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$$-d\tau^2 + h_{ij} dx^i_s dx^j_s,$$

$$h_{ij} = h_{ij}^{(0)} + 2K_{ij}\tau + K_{ik}h_{kl}^{(0)}K_{lj}\tau^2$$

$$h_{ij}^{(0)} \equiv \delta_{ij} - \partial_i t_s \partial_j t_s, \quad K_{ij} \equiv \gamma \partial_i \partial_j t_s.$$

Expand in

$$\tau \nabla_s$$

Define

$$\chi = \phi^{-1}$$

$$\chi = \left(\frac{\lambda}{2}\right)^{\frac{1}{2}} (\tau + \frac{1}{6}K_1\tau^2 + \frac{1}{18}(K_1^2 - 3K_2)\tau^3$$

$$+ \frac{1}{4}(K_3 - \frac{13}{18}K_2K_1 + \frac{7}{54}K_1^3 - \frac{1}{6}\nabla K_1)\tau^4$$

$$+ \ln \tau \left(\frac{1}{5}(-7K_4 + \frac{29}{3}K_1K_3 + \frac{11}{3}K_2^2 - \frac{67}{9}K_1^2K_2 + \frac{34}{27}K_1^4 - \frac{1}{18}(\partial K_1)^2$$

$$+ \frac{2}{9}K_1 \nabla K_1 - \frac{1}{6}\nabla K_2)\tau^5 + C_6\tau^6 + \ldots \right) + \rho(x_s)\tau^5 + D_6\tau^6 + \ldots,$$

2 arbitrary functions: $t = t_s(x_s), \phi(x_s)$.
Interpretation in linearized theory

\[ \delta \chi(t, x) = \sqrt{\frac{\lambda}{2}} \left( -t_s(x) + \frac{1}{6} t^2 \nabla^2 t_s - \frac{1}{24} t^4 (\nabla^4 t_s) + \ldots + \rho(x_s) t^5 + \ldots \right) \]

Time delay \( \frac{\delta \phi}{\phi} \) Hamiltonian density

\[ \delta \mathcal{H} = \dot{\phi} \delta \phi + V, \phi \delta \phi = \partial_t \left( \frac{\delta \phi}{\phi} \right) \phi^2 \]

As gradients become unimportant, different spatial points decouple \( \to \) QM

Self-adjoint extension matches local time delay and energy density across singularity
\[ \delta \phi = -V_{\phi} \delta \phi \]

\[ \dot{\delta \phi} = \frac{2}{t^2} \delta \phi - \frac{t^2}{\phi_0^2} \delta \phi \]

\[ \int -\frac{\phi_0^2}{2} \delta x^2 \]

\[ \delta x = \frac{4}{t^2} \delta x \cdot x^2 - k^2 \delta x \]
\[ \delta x = \frac{4}{t^2} \delta x - k^2 \delta x \]
\[ \delta x \sim \cos \omega t + \delta \]
\[ \int_{\Phi_2} X \]
\[ X \]
1. Linear terms in $t_s$ and $\rho$ completely regular (even/odd in $t$): match unambiguously across $t=0$

2. Nonlinear parts are then completely determined
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2. Nonlinear parts are then completely determined
2. WKB, SA extension

\[ p_\phi \sim \sqrt{2(E-V)} \sim \lambda^{1/2} \phi^2 V_3 \]

WKB condn \( p_\phi^{-2} \frac{dp_\phi}{d\phi} \sim_1 \lambda \phi^{-3} V_3 \sim 0 \), large \( \phi \)

Self-Adjoint extension: Reed+Simon 70's

\[ \Psi \sim e^{-iET} p_\phi^{1/2} \left( e^{i\int p_\phi d\phi} + e^{i\theta} e^{-i\int p_\phi d\phi} \right) \]

\[ p_\phi \sim \phi^2 \rightarrow |\Psi|^2 \sim \phi^{-2} \] normalisable

Halve Hilbert space \( \rightarrow \) unitary evolution, no probability lost at infinity
\[ V = -\lambda \phi^4 \]
2. WKB, SA extension

\[ p_\phi \sim \sqrt{2(E-V)} \sim \lambda^{1/2} \phi^2 V_3 \]

WKB condn: \( p_\phi^{-2} dV_3/d\phi \sim 1 \lambda \phi^{-3} V_3 \rightarrow 0 \), large \( \phi \)

Self-Adjoint extension: 

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\[ \Psi \sim e^{-iET} p_\phi^{-1/2} (e^{i\int p_\phi d\phi} + e^{i\Theta} e^{-i\int p_\phi d\phi}) \]

\[ p_\phi \sim \phi^2 \rightarrow |\Psi|^2 \sim \phi^{-2} \text{ normalisable} \]

Halve Hilbert space \( \rightarrow \) unitary evolution, no probability lost at infinity
\[ V = -\lambda \phi^4 \]
Large $\phi$ at small time

Wavefunction may be calculated using complex classical solutions

\[ p + 2i\phi |l|^2 = p_0 + 2i\phi_0 |l|^2 \]

\[ \Psi \sim (e^{iS_1} + e^{i\theta} e^{iS_2}) \]

\[ \sim e^{-\left(\frac{\phi}{2} |l|^2\right)} \quad \phi < \frac{\lambda}{\delta} t \]

\[ \sim e^{-\left(\frac{1}{1/2} \lambda \delta^2 t^2\right)} \phi^{-1} \cos(\phi^3 + \theta) \quad \phi > \frac{1}{\delta} \]

$\phi$ is infinite $\rightarrow$ classical bg never exists!
\[ V = -\lambda \phi^4 \]
Large $\phi$ at small time

Wavefunction may be calculated using complex classical solutions

$$p+2 \ i\phi \ l^2 = p_0 + 2i \ \phi_0 \ l^2$$

$$\Psi \sim (e^{iS_1} + e^{i\theta} e^{iS_2})$$

$$\sim e^{-\left(\frac{\phi^2}{2} + \frac{\phi^4}{2}\right)}$$

$$\sim e^{-\left(\frac{1}{2} \frac{\phi^2}{\lambda \delta^2}\right)} \phi^{-1} \cos(\phi^3 + \theta)$$

$\phi < \frac{\lambda}{\delta} t$

$\phi > \frac{\lambda}{\delta} t$

$\phi$ is infinite $\rightarrow$ classical bg never exists!
\[ V = -\lambda \phi^4 \]
But for an initially localized wavepacket, large $\phi$ tail unimportant except near singularity, $|t-t_s| \sim \lambda^{1/2} R_{AdS}$

What happens at the singularity?

Example: free particle, incoming Gaussian wavepacket hits brick wall
But for an initially localized wavepacket, large $\phi$ tail unimportant except near singularity, $|t-t_s| \sim \lambda^{1/2} R_{AdS}$

What happens at the singularity?

Example: free particle, incoming Gaussian wavepacket hits brick wall
The bg/flucn split in $\phi$ fails totally near the singularity, but $\chi_c = \langle \chi \rangle$ is convergent at large $\phi$ so a bg/flucn split in $\chi$ is reasonable.

$$\chi = \phi^{-1} \quad \Rightarrow \quad \chi \partial^2 \chi - 2(\partial \chi)^2 = \lambda$$

Let

$$\chi = \langle \chi \rangle + \delta \chi \quad \Rightarrow \quad \ddot{\delta \chi} - 4 \frac{\langle \dot{\chi} \rangle}{\langle \chi \rangle} \delta \chi = -k^2 \delta \chi$$

But $\langle \chi \rangle$ finite for all $t$ (QM reflection)

$\Rightarrow$ particle creation in $\delta \chi$ is exponentially suppressed in UV, i.e. for $k > \delta t_s^{-1} \sim \lambda^{-1/2} R_{AdS}^{-1}$.
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Initial Conditions

\[ \phi \sim \lambda^{1/2} R_{AdS}^{-1} \]
\[ t_s \sim R_{AdS} \]

zero energy start

QM spreading: e.g. free particle

\[ \delta \phi^2 \sim \ell^2 + (\delta p/m)^2 \]
\[ t^2 \sim \ell^2 + (\hbar/ml)^2 t^2 \]

Minimise for given \( t \): \( \ell^2 \sim \hbar/mt \)

In our case, minimal spread achieved by

\[ \delta \phi \sim R_{AdS} : \text{time delay} \]
\[ \delta t_s \sim \lambda^{1/2} R_{AdS} \]
Away from singularity, $\phi = \phi_c + \delta\phi$ is reasonable

$$S \approx \int d^4x \left( -\frac{1}{2}(\partial\phi)^2 + \lambda\phi^4 \right)$$

$$\lambda = \frac{16\pi^2}{3\ln(\phi/M)} \equiv \frac{\lambda_0}{l}$$

Zero Energy soln (attractor)

$$\phi = \frac{l^{\frac{1}{2}}}{\sqrt{2\lambda_0}} \frac{1}{(-t)} \left( 1 + \frac{1}{2l} - \frac{1}{4l^2} \ldots \right)$$

Pertns

$$\ddot{\delta\phi} = \frac{6}{t^2} \left( 1 + \frac{5}{12l} - \frac{2}{3l^2} \ldots \right) \delta\phi - k^2\delta\phi.$$

Evolve incoming modes until they become ultralocal (‘frozen’), then match across singularity using QM SA extension
3. Mode Mixing, Particle Creation

At leading order in log, no mode mixing and no particle creation. But at next order,...

Mode Evolution

\[ \delta \phi^{(1)} = l^{\frac{1}{2}} f^{(1)}(kt) + l^{-\frac{1}{2}} g^{(1)}(kt) + \ldots, \]
\[ \delta \phi^{(2)} = l^{-\frac{1}{2}} f^{(2)}(kt) + l^{-\frac{3}{2}} g^{(2)}(kt) + \ldots, \]

Evolve incoming pos freq mode, match across t=0, compute Bog. coeff ft

\[ f^{(1)} = \cos kt \left( 1 - \frac{3}{(kt)^2} \right) - 3 \frac{\sin kt}{kt}, \]
\[ f^{(2)} = \sin kt \left( 1 - \frac{3}{(kt)^2} \right) + 3 \frac{\cos kt}{kt} \]

\[ \beta \approx -i \frac{\pi}{\ln(k/\sqrt{\lambda M})} \]
Particle Production

Density of created particles

\[ \rho_c = \int \frac{d^3k}{(2\pi)^3} k|\beta|^2 \sim R_{AdS}^{-4} \]

A small perturbation on \( V \) where UV cutoff kicks in

\( \phi \) returns close to its original value

After N bounces

This falls to the point where QFT fails, after

\[ V(\phi_{min}) = -NR_{AdS}^{-4} \]

\[ N \sim \lambda_m^{-3} \]
Scale-Invariant Perturbations

"improved"

\[ T_{\mu\nu} \]

\[ \langle \delta T_{00}(r, t)\delta T_{00}(0, t) \rangle \sim \frac{1}{\ln^2(1/Mr)} \frac{1}{t^2 r^6} \]

\[ \langle \delta T_{0i}(r, t)\delta T_{0i}(0, t) \rangle \sim \frac{1}{\ln^2(1/Mr)} \left( \frac{1}{t^2 r^6} + \frac{1}{t^4 r^4} \right) \]

\[ \langle \delta \bar{T}_{ij}(r, t)\delta \bar{T}_{ij}(0, t) \rangle \sim \frac{1}{\ln^2(1/Mr)} \frac{1}{t^6 r^2} \]

\[ \langle \frac{\delta \rho(r, t)}{P + \rho} \frac{\delta \rho(0, t)}{P + \rho} \rangle \sim \frac{1}{\ln^2(1/Mr) \ln(1/Mt)} f(r/t) \]

These will determine bulk correlators and hence cosmological perturbations.
Amplitude $\sim \lambda^3$ naturally small
Tilt: red, from running of $\lambda$
Gaussian (NG $\sim \lambda$)
Scalar, Adiabatic
Scale-Invariant Perturbations

"improved" $T_{\mu\nu}$

i.e.

These will determine bulk correlators and hence cosmological perturbations
Amplitude $\sim \lambda^3$ naturally small
Tilt: red, from running of $\lambda$
Gaussian (NG $\sim \lambda$)
Scalar, Adiabatic
• Finite density of radiation produced
• GLASS perturbations without tuning

In progress:
• Translation of perturbations into bulk
• Model with 4d bulk, 3d CFT
• Glue onto positive dark energy phase to get realistic cyclic model
Summary

* The cyclic model is (an attempt at) a more complete cosmological model than inflation, incorporating dark energy, dealing with singularity

* Possible to generate realistic curvature perturbations before the bang, even within 4dET

* Main phenomenological difference: inflation $\rightarrow$ scale-invariant tensors