Thanks to the organizers for a great, diverse and provocative conference:

Sabine Hossenfelder, Bianca Dittrich, Tomasz Konopka, and Achim Kempf
1) How far have we come?

2) Challenges

3) DSR from quantum gravity

4) Is the dark energy a quantum gravity phenomena?

5) Non-locality and quantum gravity

6) Other windows into quantum gravity phenomenology
In 1998-2000 there arose the then novel idea that Planck scale phenomena could be observed in high energy astrophysics. These measure the fate of Poincare invariance to order \( l_{pl} E \) or \( (l_{pl} E)^2 \):

How far has this idea come?
A key observational question for quantum gravity is:

*What is the symmetry of the ground state?*
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Global Lorentz and Poincare invariance are not symmetries of classical GR, they are only symmetries of the ground state with $\Lambda=0$.

Hence, the symmetry of the quantum ground state is a dynamical question.
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*Hence, the symmetry of the quantum ground state is a dynamical question.*

Three possibilities

1. Poincare invariant
2. Broken Lorentz invariance
3. Deformed Poincare invariance (DSR)
Deformed or doubly special relativity (DSR)

Principles of deformed special relativity (DSR):

1) Relativity of inertial frames
2) The constancy of $c$, a velocity
3) The constancy of an energy $E_{\text{planck}}$
4) $c$ is the universal speed of photons for $E \ll E_{\text{planck}}$
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Consequences:

- Modified energy-momentum relations
- Momentum space has constant curvature given by $E_{\text{Planck}}$
- Energy-momentum conservation becomes non-linear (Coproduct)
\[ \frac{\hbar}{m} \rightarrow 0 \]
\[ G \rightarrow 0 \]
\[ m_p \sim \sqrt{\frac{\hbar}{G}} \rightarrow \text{constant} \]
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Mathematical realizations:

1) Deformed poincare algebra is a hopf algebra
   Acts on a spacetime geometry which is non-commutative.

2) metric becomes scale dependent: $g_{ab}(E)$

Are they different?
What are the differences?
Is there a way to map them to each other?
\[ \tanh \left( \frac{m^2 E}{E_r^3} \right) \]

\[ E \sim \sqrt{\frac{m^2 E_p}{G}} \]

\[ \hbar \to 0 \]

\[ G \to 0 \]

\[ m_p \sim \sqrt{\frac{\hbar^2}{G}} \]

\[ c \hbar G \to c(E) \]
\[ h \to 0 \]
\[ G \to 0 \]
\[ c h G \to c(E_0) \]
\[ m_p \sim \sqrt{\frac{t}{G}} \]
\[ h \rightarrow 0 \]
\[ G \rightarrow 0 \]
\[ c \rightarrow h \]
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Models of DSR:

DSR is realized precisely in 2+1 gravity with matter \(\text{hep-th/030708:}\)

QFT on kappa-minkowski

Rainbow metric

Energy dependent \(h\) and \(c\)
How can we experimentally distinguish the three possibilities of exact, broken or deformed Poincare invariance?

There are two basic low energy QG effects:

1) Corrections to energy momentum relations:

\[ E^2 = p^2 + m^2 + \alpha \, l \, p \, E^3 + \beta \, l \, p^2 \, E^4 + \ldots \]
\[ v = c (1 + \alpha \, l \, p \, E^+ \ldots) \]

2) Modifications in the conservation laws.

Some basic consequences:

- Preferred frame allowed processes (photon decay)
- Modifications of thresholds (GZK, TeV photons…)
- Energy dependence of the speed of light, neutrinos …

Can they be measured to \( O(l_{\text{Planck}}) \)?
\[ E \sim \sqrt{m^2E_p} \]

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\[ t \sim h \left( \frac{m^2E}{E^3} \right) \]

\[ h \to 0 \quad G \to 0 \]

\[ M_p \sim \sqrt{\frac{E}{c^2}} \quad \text{constant} \]

\[ c, h, G \to \ell(E) \quad \ell(E) \to \text{constant} \]
Energy dependent speed of light

\[ v(E) = c (1 + a \, l_p \, E + b \, l_p^2 \, E^2 + \ldots) \]

- Accumulates for long distances
- Observable in Gamma Ray bursts.
- Present limits have \( a < 1000 \)
- GLAST will put limits \( a < 1 \)

- Could be parity even or odd

- A parity odd \( v(E) \) has been ruled out at \( O(l_p) \)
  by observations of distant polarized radio galaxies
  Also, by polarization observed in Gamma Ray Bursts

- GLAST could see \( O(l_{pl}) \) parity-even \( v(E) \)
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Broken lorentz invariance gives modified dispersion relations but unmodified conservation laws

- GZK threshold moves appreciably
- helicity odd energy dependent speed of light

Deformed lorentz invariance gives both.

- GZK threshold as in ordinary special relativity
- possible helicity even energy dependent speed of light

To distinguish the three possibilities we need three experiments:

- AUGER tests GZK
- MAGIC, GLAST tests energy dependence of photons
- Detection of polarized photons from distant sources tests helicity dependence
\[ E^2 = p^2 + m^2 + E^3 + E^4 \]

\[ \frac{E_{cr}}{E_{cl}} \sim \frac{E_x}{m} \tan h \left( \frac{m^2 E}{E^3} \right) \]

\[ \text{as } h \to 0 \]
\[ E^2 = P^2 + m^2 + E_3^2 + E_4^2 \]

\[ \frac{E_{cr}}{E_{PL}} \sim \frac{E}{m_{Pl}} \tanh \left( \frac{m^2 E}{E_3^3} \right) \]

\[ \frac{E_{cm}}{E_{PL}} \]

\[ h \rightarrow 0 \]

\[ G \rightarrow 0 \]

\[ m_p \sim \sqrt{\frac{h}{G}} \]
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The GZK threshold:

Cosmic ray protons scattering off the microwave background.

Special relativity predicts a threshold at $3 \times 10^{19}$ ev.

An effect of a $O(l_{\text{Planck}})$ modified $E-p$ relation is to move it $O(1)$

$$E^2 = p^2 + m^2 + \alpha l p E^3 + \beta l p^2 E^4 + \ldots$$

Prediction from Lorentz Inv
+ uniform sources ................

AGASA reported events over the GZK threshold!
GZK: AGASA, Sugar saw anomalous events

HIRES didn’t
Fig. 6. The energy spectrum of EHECRs measured by the Pierre Auger experiment compared with the AGASA\textsuperscript{38} and HiRes-I\textsuperscript{39} results.
Correlation of the Highest-Energy Cosmic Rays with Nearby Extragalactic Objects

The Pierre Auger Collaboration*
Fig. 2. Aitoff projection of the celestial sphere in galactic coordinates with circles of radius 3.1° centered at the arrival directions of the 27 cosmic rays with highest energy detected by the Pierre Auger Observatory. The positions of the 472 AGN (318 in the field of view of the Observatory) with redshift $z \leq 0.018 (D < 75$ Mpc) from the 12th edition of the catalog of quasars and active nuclei (12) are indicated by red asterisks. The solid line represents the border of the field of view (zenith angles smaller than 60°). Darker color indicates larger relative exposure. Each colored band has equal integrated exposure. The dashed line is the supergalactic plane. Centaurus A, one of our closest AGN, is marked in white.
Tentative but pretty compelling conclusions:

- There is a GZK cutoff
- Lorentz symmetry breaking is dead, at least at first order
- DSR and good old fashioned SR are fine.
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• The relativity of inertial frames is good up to $\gamma \sim 10^{11}$ !!!

  1 sec dilates to 10,000 years!!
  1 cm contracts to 100 fermi!!
  Hubble scale contracts to light days!!
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Probing Quantum Gravity using Photons from a Mkn 501 Flare Observed by MAGIC

We use the timing of photons observed by the MAGIC gamma-ray telescope during a flare of the active galaxy Markarian 501 to probe a vacuum refractive index \( z = 1 - (E/M_{QC})^n \), \( n = 1, 2 \), that might be induced by quantum gravity. The peaking of the flare is found to maximize for quantum-gravity mass scales \( M_{QC1} \sim 0.4 \times 10^{18} \) GeV or \( M_{QC2} \sim 0.6 \times 10^{11} \) GeV, and we establish lower limits \( M_{QC1} > 0.26 \times 10^{18} \) GeV or \( M_{QC2} > 0.39 \times 10^{11} \) GeV at the 95\% C.L. Monte Carlo studies confirm the MAGIC sensitivity to propagation effects at these levels. Thermal plasma effects in the source are negligible, but we cannot exclude the importance of some other source effect.
FIG. 2: The $\tau_1$ distribution from fits to the ECFs of 1000 realizations of the July 9 flare with photon energies smeared by Monte Carlo.
More flares??

GLAST launches in “May”
Too tempting not to say:

If both experiments are correct we have

- GZK cutoff
- Parity even energy dependent speed of light.
- No parity odd variation in $c$

These are the signatures of DSR!!
Some questions for Lorentz violators:

EFT+ birefringence results imply no first order in $l_{pl}$ violation.

• If the MAGIC results are a real variation in the speed of light, how can this then be lorentz violation?

With respect to EFT, saying it isn’t so doesn’t make it not so.

• Are there really broken lorentz violating theories that EFT does not apply to? Why doesn’t EFT apply also to open quantum systems?

• Are there interesting theories with Lorentz violation at second order and not at first order?

• If there really is a preferred frame, what about dimension 1,2,4 operators? Why should Einstein be so wrong but SR work so well?
Some questions for DSRers:

• Shouldn’t there be a version of EFT appropriate to DSR or deformed Poincare symmetry?

• Is DSR a more general category of theories than deformed Poincare symmetry?

• Is it so hard to write a full interacting QFT with DSR?

• Are there really different versions of DSR? Are the deformed Poincare, energy dependent metric, energy dependent h and c different theories or different representations of one class of theories?

• Is there a universal version of DSR, with parameters to represent different versions?
A question for everyone:

Effective field theory versus non-locality?
Questions for quantum gravityists:

Can DSR be derived from some version of quantum gravity?

Would the result be generic, or theory dependent?
Energy dependence of the metric is a consequence of quantum gravity.
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Assumptions:

- Ashtekar variables
  \[ \{ A^i_a(x), \tilde{E}^b_j(y) \} = \rho \delta^b_a \delta^j_i \delta^d(x, y) \]

  \[ [\rho] = \text{length}^2 \]
Energy dependence of the metric is a consequence of quantum gravity.

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• Connection rep:

\[ \hat{E}_i^a(x) = -i\hbar \rho \frac{\delta}{\delta \mathcal{A}_a^i(x)} \]

\[ <A|\Psi> = \Psi(A) \]
Energy dependence of the metric is a consequence of quantum gravity. hep-th/05010901

Assumptions:

• Ashtekar variables

\[ \{ A^a_i(x), \dot{E}_j^b(y) \} = \rho \delta^b_a \delta^j_i \delta^d(x, y) \]

[\rho] = length^2

• Connection rep:

\[ \dot{E}^a_i(x) = -i\hbar \rho \frac{\delta}{\delta A^a_i(x)} \]

\[ \langle A|\Psi\rangle = \Psi(A) \]

• Semiclassical states:

\[ \Psi_0(A) = e^{i\frac{S(A)}{\hbar}} \]

S(A) = Hamilton-Jacobi function
$$\Phi(A, \frac{SS}{8A} = \varepsilon) = 0$$

$$A(\varepsilon, E\Phi)$$

$$S(A)$$
Energy dependence of the metric is a consequence of quantum gravity.

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  \[
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  \[S(A) = \text{Hamilton-Jacobi function}\]

- Matter fields:
  \[
  \Psi [A, \phi] = \Psi_0 [A] \backslash [A, \phi]
  \]
Consider a solution to the Hamilton-Jacobi equations for GR: 

\[ S[A] = \int_{\Sigma} S[A] \]
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Gradient flow along \( S(A) \) gives a classical solution:

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Consider a solution to the Hamilton-Jacobi equations for GR:

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On the classical solution there is a time coordinate \( T \) proportional to \( S(A) \):

\[ \frac{d}{dT} = \mu \frac{\delta}{\delta S[A]} \quad \text{[\( \mu \)=length}^{-1}] \]
\[ \Phi(A, \frac{SS}{SSA} = \varepsilon) = 0 \]

\[ T = S(h) \]
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On the classical solution there is a time coordinate \( T \) proportional to \( S(A) \):

\[
\frac{d}{dT} = \mu \frac{\delta}{\delta S[A]} \quad [\mu]=\text{length}^{-1}
\]

\( E^a_i \) is a densitized frame field, related to frame 1-form \( e^a_i \).

The metric \( g \) is

\[
g = -dT^2 + \sum_i e^0_i \otimes e^0_i
\]
Variations of $A$ are along the solution and orthogonal to it:

\[
\frac{\delta}{\delta A^i_a(x)} = \frac{1}{M} \tilde{E}_{i0}^a \frac{\delta}{\delta S} + \frac{\delta}{\delta a^i_a} \tilde{E}_{0}^{ai} \delta a_{ai} = 0
\]

$M=\text{length}^2$
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$M=$length$^2$

We act on the product state: $\Psi[A, \phi] = \Psi_0[A] \chi[A, \phi]$

Since $S(A)$ is a coordinate on $C$ $\chi[A, \phi] = \chi[S, a^{ai}_a, \phi]$
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By construction: $\tilde{E}^a_i \Psi_0[A] = \tilde{E}^{0a}_i \Psi_0[A]$
Variations of $A$ are along the solution and orthogonal to it:

$$\frac{\delta}{\delta A^i_a(x)} = \frac{1}{M} \hat{E}_{i0}^a \frac{\delta}{\delta S} + \frac{\delta}{\delta a^i_a} \quad \hat{E}_0^{ai} \delta a_{ai} = 0$$

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We act on the product state:

$$\Psi[A, \phi] = \Psi_0[A] \chi[A, \phi]$$

Since $S(A)$ is a coordinate on $C$

$$\chi[A, \phi] = \chi[S, a_{ai}, \phi]$$

By construction:

$$\hat{E}_i^a \Psi_0[A] = \hat{E}_i^{0a} \Psi_0[A]$$

$$\hat{E}_i^a(x) \chi[A, \phi] = -\imath \hbar \rho \frac{\delta \chi[A, \phi]}{\delta A^i_a(x)}$$

$$= \left( \hat{E}_i^{0a} \frac{\hbar \rho}{M} \frac{\delta}{\delta S(x)} - \imath \hbar \rho \frac{\delta}{\delta a_{ai}(x)} \right) \chi[S, a_{ai}, \phi]$$
Putting it all together, we have

\[ \frac{i\hbar \rho}{M} \frac{\delta}{\delta S(x)} = \frac{i\hbar \rho}{M \mu} d \]

But dimensionally \( \frac{h \rho}{M \mu} \) is a time. But there is only one time in the problem, so

\[ \frac{h \rho}{M \mu} = \alpha l_{Pl} \]
Putting it all together, we have \[
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But dimensionally \(\frac{\hbar \rho}{M \mu}\) is a time. But there is only one time in the problem, so \(\frac{\hbar \rho}{M \mu} = \alpha l_{Pl}\)

Hence, on all semiclassical states we find to leading order:
\[
\hat{E}_i^a(x) \Psi[A, \phi] = \Psi_0[A] \hat{E}_i^{0a} \left( 1 - i \alpha l_{Pl} \frac{d}{dT} \right) \chi[T, a_{ai}, \phi]
\]
Putting it all together, we have
\[ \frac{\nu \hbar \rho}{M} \frac{\delta}{\delta S(x)} = \frac{\nu \hbar \rho}{M \mu} \frac{d}{dT} \]

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Semiclassical states of definite frequency must exist:
\[ \chi[T, a_{ai}, \phi] = e^{-i \omega T} \chi_\omega[a_{ai}, \phi] \]
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Hence, on all semiclassical states we find to leading order:
\[ \tilde{E}_i^a(x) \Psi[A, \phi] = \Psi_0[A] \tilde{E}_i^{0a} \left( 1 - i\alpha l_{Pl} \frac{d}{dT} \right) \chi[T, a_{ai}, \phi] \]

Semiclassical states of definite frequency must exist:
\[ \chi[T, a_{ai}, \phi] = e^{-i\omega T} \chi_\omega[a_{ai}, \phi] \]
Hence: in the semiclassical approximation, in the presence of an energy eigenstate matter state, the frame field has become energy dependent:

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\[ \tilde{E}^{0a}_i (x, T) \rightarrow \tilde{E}^{0a}_i (x, T, \omega) = \tilde{E}^{0a}_i (x, T)(1 - \alpha l_{Pl} \omega) \]
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Translated to the metric we have:

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Hence: in the semiclassical approximation, in the presence of an energy eigenstate matter state, the frame field has become energy dependent:

\[ \tilde{E}_i^\alpha(x) \Psi[A, \phi] = \Psi_0[A] \tilde{E}_i^0\alpha (1 - \alpha l_P \omega) \chi_\omega[T, a_{ai}, \phi] \]

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This implies modified dispersion relations

\[ m^2 = -g(\omega)^{\mu \nu} k_\mu k_\nu u = \omega^2 - \frac{k_i^2}{(1 - \alpha l_P \omega)} \]
Queries:

• *Is this DSR or broken Lorentz invariance?*
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The dynamics is given by the Wheeler-deWitt equation

\[
\hat{H}(x)\Psi[A, \phi] = 0
\]

*This is the statement that there is no preferred slicing.* Hence there is no preferred frame. Hence this is NOT lorentz symmetry breaking

*Hence it must be DSR.*
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**Hence it must be DSR.**

• Can the value of \( \alpha \) be predicted?
Queries:

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\[ \hat{H}(x)\Psi[A, \phi] = 0 \]

This is the statement that there is no preferred slicing. Hence there is no preferred frame. Hence this is NOT lorentz symmetry breaking. **Hence it must be DSR.**

• *Can the value of \( \alpha \) be predicted?*

In principle, in a more detailed treatment.
• What is the meaning of energy dependence of the metric for a general state?
What is the meaning of energy dependence of the metric for a general state?

Go back to the definition of $E_{i}^{a}$ as an operator:

$$\tilde{E}_{i}^{a}(x)\Psi[A, \phi] = \Psi_{0}[A]\tilde{E}_{i}^{0a} \left(1 - i\alpha l_{Pl} \frac{d}{dT}\right)\chi[T, a_{ai}, \phi]$$
What is the meaning of energy dependence of the metric for a general state?

Go back to the definition of $E^a_i$ as an operator:

$$\hat{E}^a_i(x) \Psi [A, \phi] = \Psi_0 [A] \hat{E}^0_i \left( 1 - \imath \alpha l_{Pl} \frac{d}{dT} \right) \chi [T, a_{ai}, \phi]$$

Is this a form of DSR that predicts an energy dependent speed of light?
What is the meaning of energy dependence of the metric for a general state?

Go back to the definition of $E^a_i$ as an operator:

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Is this a form of DSR that predicts an energy dependent speed of light?

Yes, because only the spatial components of the metric are operators:

$$g = -dT^2 + \sum_i e^0_i \otimes e^0_i$$

So:

$$m^2 = -g(\omega)^{\mu\nu}k_\mu k_\nu u = \omega^2 - \frac{k_i^2}{(1 - \alpha l_P \omega)}$$
Hence, DSR in the form of an energy dependent metric, is a consequence of quantum gravity, in the semiclassical approximation.

Hence, a parity even energy dependent speed of light is a prediction of quantum gravity, in the semiclassical approximation.

\[
\hat{E}_i^a(x) \Psi[A, \phi] = \Psi_0[A] \hat{E}_i^{0a} \left( 1 - \alpha l_P l_\omega \right) \chi_\omega[T, a_{ai}, \phi]
\]

Or, more generally:

\[
\hat{E}_i^a(x) \Psi[A, \phi] = \Psi_0[A] \hat{E}_i^{0a} \left( 1 - i\alpha l_P \frac{d}{dT} \right) \chi[T, a_{ai}, \phi]
\]
What is the scale of quantum gravity effects?
Are they only at the Planck scale?
The vacuum energy is a quantum effect.
Is the dark energy then a quantum gravity effect?
Hence, DSR in the form of an energy dependent metric, is a consequence of quantum gravity, in the semiclassical approximation.

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\]
\[ \Psi = 0 \]

\[ = \left( H_{\text{phys}} + H_{\text{int}} (\phi) \right) \Psi \nu \chi \]

\[ \Rightarrow \frac{\sqrt{\lambda}}{1/t} - H_{\text{int}} (\phi) \chi = 0 \]
\[ H \bar{\psi} = 0 \]

\[ = \left( \hat{H} + H \right) \psi_{\nu} \chi \]

\[ \Rightarrow \frac{\hbar}{\Delta t} \left[ \mathcal{S} \right] (\mathcal{E}_0) \chi = 0 \]
Hence, DSR in the form of an energy dependent metric, is a consequence of quantum gravity, in the semiclassical approximation.

Hence, a parity even energy dependent speed of light is a prediction of quantum gravity, in the semiclassical approximation.

\[ \hat{E}^a_i(x) \Psi[A, \phi] = \Psi_0[A] \hat{E}^{0a}_i (1 - \alpha l_P \omega) \chi_\omega[T, a_{ai}, \phi] \]

Or, more generally:

\[ \hat{E}^a_i(x) \Psi[A, \phi] = \Psi_0[A] \hat{E}^{0a}_i \left( 1 - i\alpha l_P \frac{d}{dT} \right) \chi[T, a_{ai}, \phi] \]