Title: Imprinting Short-Distance Physics on Long Distance Observables: Can this be consistent?

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Abstract: We use the example of inflationary physics to discuss the possibility that short distance physics might be imprinted on long-distance observables. In particular, we focus on issues involving decoupling in field theory.
Imprinting Short-Distance Physics on Long-Distance Observables: Can this be Consistent?

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Outline


- Theory of Inflationary Effective States (Hael Collins, R.H.)

- Conclusions
Are Inflationary Predictions Sensitive to Very High Energy Physics?

- Why we should worry
- Toy Model 1: Non-Adiabatic Physics
- Toy Model 2: Adiabatic Physics
- Having your cake and eating it too
Why we should worry: Trans-Planckian Problem/Opportunity
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- WMAP data is fully consistent with the inflationary paradigm:
  - Quantum fluctuations in the inflaton field source metric perturbations.
  - The scale of fluctuations is stretched outside the inflationary horizon, and then re-enters the FRW horizon.
  - These decohere and provide the source for classical density fluctuations.
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- Do we need to understand QG to make use of the CMB to test inflation?
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- Can we make use of the CMB to understand high energy physics, maybe all the way to the Planck scale?
The main issue at hand is this: If we have to worry about TP in order to understand the CMB sky, or if we want to use the CMB to understand high energy physics, then we have to worry about the loss of predictive power.

This would a violation of **DECOUPLING**: We can understand physics scale by scale because the only effect of higher energy scales on lower ones is through a small number of parameters that must be measured. After this the low energy theory has predictive power on its own.

Maybe QG violates decoupling. However, if string theory is any indication, decoupling is probably a feature of QG. In fact, any non-decoupling theory of QG needs to explain how it’s effects are prevented from infiltrating known physics and thus ruining predictability.

On the other hand, if there can be some **CONTROLLED** violation of decoupling, it may be possible to use the CMB to get SOME information about high energy physics.
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We can take two different approaches to understanding TP effects in inflation:

- **Construct models:** Modified dispersion relations, stringy uncertainty relations, shortest distance prescription, de Sitter invariant alpha vacua...

- **Try to use symmetries and general principles to make generic statements**

- We can mimic TP physics by looking at field theories with a high energy sector. Then ask whether decoupling can be violated, but in a controlled fashion.
del 1: Non-Adiabatic
Toy Model 1: Non-Adiabatic Physics
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- In order think about low energy physics, we have to be able to integrate out the high energy sector.

- Implicit in this is the assumption that there are no transitions between high and low energy states.

- But in inflation, there is time evolution all over the place. Suppose the heavy field is oscillating quickly enough and couples to the light fields in such a way as to induce light-to-heavy transitions.

- What is the “low-energy EFT” now?
Consider a hybrid inflation model.

\[ \phi \rightarrow \text{inflaton} \]
\[ \chi \rightarrow \text{heavy field} \]

Inflaton rolls along trough at \( \chi = 0 \) until \( m_{\chi, \text{eff}} \equiv g\phi^2 - 2\lambda v^2 = 0 \)

Now suppose we displace the heavy field initially. It will oscillate with some frequency, which gives a time dependent mass to the inflaton fluctuations.

\[ -\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \partial_\mu \chi \partial^\mu \chi + V(\phi, \chi) \right] \]
\[ V(\phi, \chi) = \frac{1}{2} m^2 \phi^2 + \lambda (\chi^2 - v^2)^2 + \frac{1}{2} g\phi^2 \chi^2 + \tilde{\lambda} \phi^4 \]

\[ \ddot{\chi} + 3H \dot{\chi} + M^2(\phi) \chi \approx 0 \]
\[ \chi(t) \approx \chi_i \left( \frac{a(t_i)}{a(t)} \right)^{3/2} \cos [M(\phi)(t - t_i)] , \quad M(\phi) \approx g\phi^2 \]
Need to:

Choose initial inflaton zero mode conditions so that effective heavy mass is less than the Planck scale, but larger than Hubble parameter during inflation

Wait till energy density in heavy field oscillations gets damped enough so that inflaton energy density can dominate expansion

Now look at inflaton fluctuations and see how the heavy field oscillations impact the CMB fluctuations.

Linearized fluctuations satisfy:

\[ \ddot{\delta \phi_k} + 3H \dot{\delta \phi_k} + \left( \frac{k^2}{a^2(t)} + V''(\langle \phi \rangle) - g\chi^2(t) \right) \delta \phi_k = 0 \]

What happens to modes and acoustic oscillations?
Some notable features

Oscillations in the fluctuations are rapid, driven by high frequency of heavy field oscillations.
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Low momentum modes are suppressed since initially,

\[
\langle g\chi^2(t) \rangle \propto \langle \cos^2 Mt \rangle = \frac{1}{2}
\]

Fluctuations exhibit a peak due to excitation due to oscillations. Modes that match frequency at earliest times are the ones that are enhanced.
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Toy Model 2: Adiabatic Physics

- Suppose now that we look at adiabatically evolving situation, where the heavy fields are pinned down or evolving very slowly compared to the inflaton. Are there situations where we can beat the standard suppression of heavy field effects?

- If there are symmetry reasons why the light theory has no potential initially, and this potential only gets generated when we integrate out the heavy fields, then inflation comes about ONLY due to heavy fields, so we expect effects on the CMB to be less suppressed than expected.

- We might be able to make the suppression of heavy field effects to be of order $M_0^2 / M^2$ rather than $H^2 / M^2$ where $M_0$ can be much larger than $H$
The Model:

- Consider a global SUSY theory with chiral multiplets \( \{ \Phi_0, H_+, H_- \} \) charged under a \( U(1) \), coupled to a vector multiplet.

- Global minimum is SUSY, but it's broken if the inflaton is displaced from its minimum.

- There is a trough at \( h_\pm = 0 \) for large \( |\phi| \) along which the potential is constant.

- Need to integrate the heavy charged modes to generate a potential for \( \phi \)

\[
\begin{align*}
K &= H^*_+ H_+ + H^*_H H_- + \Phi^* \Phi \\
W &= g\Phi (H_+ H_- - v^2) \\
V &= V_F + V_D \\
V_F &= g^2 \left( |h_+ h_- - v^2|^2 + |\phi h_-|^2 + |\phi h_+|^2 \right) \\
V_D &= \frac{e^2}{2} \left( |h_+|^2 - |h_-|^2 + \xi \right)^2
\end{align*}
\]
Inflaton effective potential after integrating out the heavy sector:

\[
\begin{align*}
V_{\text{eff}}(\phi) &= V_0 + \Delta V(\phi) \\
\Delta V(\phi) &= \delta V_0 + \frac{2N}{64\pi^2} \sum_{i=\pm} M_i^4(\phi) \ln \left( \frac{M_i^2(\phi)}{\mu^2} \right) - m_i^4(\phi) \ln \left( \frac{m_i^2(\phi)}{\mu^2} \right) \\
M_{\pm}^2 &= g^2 |\phi|^2 \pm \Delta, \quad m_{\pm}^2 = g^2 |\phi|^2
\end{align*}
\]

For large \( \phi \), we have:

\[
V_{\text{eff}} \sim \text{const} + \frac{N \Delta^2}{16\pi^2} \left[ \ln \left( \frac{m^2(\phi)}{m_*^2} \right) + O\left( \frac{\Delta^2}{m^4} \right) \right]
\]

For sufficiently small parameters, we can get the requisite amount of inflation. This is the usual fine-tuning problem in inflation.

The real point is that we only get inflation at all because of heavy particle effects!

\[
(2\epsilon)^{1/2} \sim \frac{g^2 N \Delta^2}{8\pi^2 \rho} \left( \frac{\phi_* M_P}{m_*^2} \right), \quad \eta \sim \frac{g^2 N \Delta^2}{8\pi^2 \rho} \left( \frac{M_P^2}{m_*^2} \right)
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Comparison is NOT to \( H^2 \)
Conclusions Part 1: Having Your Cake and Eating it too!

- This all started with an EFT analysis by Shenker et al. that argued that just from the effective action, TP effects would be at most of order $\frac{H^2}{M^2}$ and so essentially be unobservable.

- What we argue is that this analysis is generically true i.e. decoupling mostly works the way you think it does, BUT...

- There are interesting loopholes in the argument;
  - If we really can’t integrate out the heavy physics, because of non-adiabatic behavior, decoupling does not apply and heavy physics can show up in the CMB.
  - If there is no tree level potential, then all of inflation comes from the heavy physics being integrated out, and although it does suppress effects, not by the usual amount.
  - Invoking decoupling is a much trickier than it would otherwise appear!
Inflaton effective potential after integrating out the heavy sector:

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V_{\text{eff}}(\phi) = V_0 + \Delta V(\phi)
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\Delta V(\phi) = \delta V_0 + \frac{2N}{64\pi^2} \sum_{i=\pm} \left[ M_i^4(\phi) \ln \left( \frac{M_i^2(\phi)}{\mu^2} \right) - m_i^4(\phi) \ln \left( \frac{m_i^2(\phi)}{\mu^2} \right) \right]
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Theory of Inflationary Effective States

- The philosophy of Effective Field Theory
- Modifications due to a cosmological background
- How to specify an effective initial state
- Boundary Renormalization and the Backreaction problem
The Philosophy of Effective Field Theory

- In EFT, we divide phenomena according to whether or not they occur at energies larger than some fixed scale $M$.
  - The fields and symmetries of the low-energy theory fix the renormalizable operators.
  - High energy physics appears as higher dimension operators, suppressed by powers of $M$.

- The physics is consistent since for experiments at a scale $E$, all high energy physics will be suppressed by powers of $E/M$.
- In principle, renormalizability of the low-energy theory would require an infinite number of operators, but in practice, how well we can measure determines the dimension of the operators we should keep.

Integrate out the W boson in the standard model to get the Fermi theory. This will be valid for $E \ll M_W$.
Modifications of EFT due to a Cosmological

- An expanding Universe requires a number of modifications to the standard EFT program.

- Time evolving background means S-matrix type calculations are not relevant. Instead of a BVP, need to turn it into an IVP.

- The initial quantum state also has to satisfy the strictures of EFT.

  - Long-distance features are set empirically

  - but short-distance structures are left general

  - Long-distance measurements should not be greatly affected by the short-distance structures.
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There are potentially two objects to which we could apply an effective description—the state and its evolution:

\[ |\phi_{\text{eff}}(\eta)\rangle = U_{I}(\eta, \eta_0) |\phi_{\text{eff}}(\eta_0)\rangle \]
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\]

There are potentially two objects to which we could apply an effective description—the state and its evolution:

Expand the field in spatial eigenmodes,

\[
\varphi = \int \frac{d^3k}{(2\pi)^3} \left[ \varphi_k e^{ik\cdot x} a_k + \varphi_k^* e^{-ik\cdot x} a_k^\dagger \right]
\]

In solving for the modes, we must make some assumption at arbitrarily large $k$,

\[
\varphi_{k< M}(\eta_0) \rightarrow \text{understood}
\]

\[
\varphi_{k> M}(\eta_0) \rightarrow ?
\]

Add some general structure that goes away for $k \ll M$

\[
\varphi_k(\eta_0) - \varphi_k^{BD}(\eta_0) \to \sum_n d_n \frac{k^n}{M^n}
\]
How to Specify an Effective Initial State

- How do we pick the usual vacuum for inflationary fluctuations?

- Solve the massless, minimally coupled KG equation in a de Sitter background

- Get two solutions for each mode. One of these will match to the flat space vacuum state in the limit \( k\eta \rightarrow -\infty \)

- This is the Bunch-Davies vacuum state; use this.

\[
\left[ \frac{d^2}{d\eta^2} + 2H \frac{d}{d\eta} + k^2 - \frac{1}{6}a^2(\eta)R \right] \delta \phi_k = 0
\]

The rationale for this is

1. At short distances we shouldn't be able to tell that we are not in flat space,
2. This vacuum is de Sitter invariant.
How to Specify an Effective Initial State (cont’d)

- But... what if the dynamics changes at some scale $M$?
- As an example, suppose the inflaton is actually a composite particle so that $M$ represents the scale of compositeness.
- Then using a prescription that requires a specification of modes down to arbitrarily short distance misses the actual physics.
- This is exactly what EFT is there for!
More reasonable: At energy scales higher than $M$, effective theory described by KG equation breaks down.

More general IC:

$$\partial_{\eta} U_k(\eta_0) = -i\omega_k(\eta_0) U_k(\eta_0)$$

$$\frac{B^D_k(\eta) + e^{\alpha_k} U_{k}^{BD^*}(\eta)}{\omega_k} \quad \frac{1}{e^{\alpha_k + \alpha_k^*}}$$

(initial state)
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\[
U_k(\eta) = N_k \left( U_k^{BD}(\eta) + e^{\alpha_k} U_k^{BD*}(\eta) \right)
\]

\[
e^{\alpha_k} = \frac{\omega_k - \omega_k}{\omega_k + \omega_k}
\]

\[
N_k = \frac{1}{\sqrt{1 - e^{\alpha_k + \alpha_k^*}}}
\]

(initial state structure function)
More reasonable: At energy scales higher than $M$, effective theory described by KG equation breaks down.

More general IC: $\partial_\eta U_k(\eta_0) = -i\omega_k(\eta_0)U_k(\eta_0)$

$U_k(\eta) = N_k(\Omega^{BD}_k(\eta) + e^{\alpha_k}U^{BD*}_k(\eta))$

$e^{\alpha_k} = \frac{\omega_k - \omega_k}{\omega_k + \omega_k}$ (initial state structure function)

$N_k = \frac{1}{\sqrt{1 - e^{\alpha_k}\dot{\alpha}_k}}$

Redshifting of scales means that effective theory can be valid only for times later than $\eta_0^{\text{earliest}}$.
More reasonable: At energy scales higher than $M$, effective theory described by KG equation breaks down.

More general IC: \[ \partial_\eta \mathcal{U}_k(\eta_0) = -i\omega_k(\eta_0) \mathcal{U}_k(\eta_0) \]

\[
\mathcal{U}_k(\eta) = N_k \left( \mathcal{U}_k^{BD}(\eta) + e^{\alpha_k} \mathcal{U}_k^{BD*}(\eta) \right) \\
e^{\alpha_k} = \frac{\omega_k - \omega_k}{\omega_k + \omega_k} \\
N_k = \frac{1}{\sqrt{1 - e^{\alpha_k + \alpha_k}}} \\
(\text{initial state structure function})
\]

Redshifting of scales means that effective theory can be valid only for times later than $\eta_0^{\text{earliest}}$

\[ \frac{a(\eta_0^{\text{earliest}})}{a(\eta_0^{\text{now}})} \sim \frac{\kappa_{\text{expt}}}{M} \]
What about propagators?
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\[-iG^\alpha_k (\eta, \eta') = \Theta (\eta - \eta') U_k^{BD} (\eta) U^*_{k}^{BD} (\eta') + \Theta (\eta' - \eta) U^*_{k}^{BD} (\eta) U_k^{BD} (\eta') + e^{\alpha_k} U_k^{BD} (\eta) U_k^{BD} (\eta')\]
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Forward propagation only for initial state information
What about propagators?

\[-iG_k^{\alpha,\kappa}(\eta, \eta') = \Theta(\eta - \eta')U_k^{BD}(\eta)U_k^{*BD}(\eta') + \Theta(\eta' - \eta)U_k^{*BD}(\eta)U_k^{BD}(\eta') + e^{\alpha_k}U_k^{BD}(\eta)U_k^{BD}(\eta')\]

Forward propagation only for initial state information

Structure function contains:

-- IR aspects, which are real observable excitations
-- UV virtual effects encoding the mistake made by extrapolating free theory states to arbitrarily high energy.
Boundary Renormalization and the Backreaction Problem
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\textit{Initial time hype\textit{s}surface splits spacetime into \textit{bulk}+\textit{boundary}.}
Boundary Renormalization and the Backreaction Problem

Initial time hypersurface splits spacetime into bulk + boundary.

Bulk divergences should be able to be absorbed by bulk counterterms only
Boundary Renormalization and the Backreaction Problem

Initial time hypersurface splits spacetime into bulk+boundary.

Bulk divergences should be able to be absorbed by bulk counterterms only

Need to show that new divergences due to short-distance structure of initial state are indeed localized to boundary.
Boundary Renormalization and the Backreaction Problem

Initial time hypersurface splits spacetime into bulk + boundary.

Bulk divergences should be able to be absorbed by bulk counterterms only.

Need to show that new divergences due to short-distance structure of initial state are indeed localized to boundary.

Renormalization condition:
Set time-dependent tadpole of inflaton fluctuations to zero: \( \langle \alpha_k(\eta) | \psi(x) | \alpha_k(\eta) \rangle = 0 \)
Boundary Renormalization
Boundary Renormalization
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\[ e^{\alpha_k} = \sum_{n=0}^{\infty} d_n \left( \frac{H_{\text{inf}}}{\Omega_k(\eta_0)} \right)^n + \sum_{n=0}^{\infty} c_n \left( \frac{\Omega_k(\eta_0)}{M} \right)^n \]

\[ \Omega_k(\eta) \approx \frac{\sqrt{k^2 + m_{\text{eff}}^2}}{a(\eta)} \]
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IR piece: Divergences can be cancelled by renormalizable boundary counterterms
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UV piece: Need non-renormalizable boundary counterterms
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\[ \Omega_k(\eta) \approx \sqrt{\frac{k^2 + m_{\text{eff}}^2}{a(\eta)}} \]

**IR piece:** Divergences can be cancelled by renormalizable boundary counterterms

**Example:** $\lambda \phi^4$ theory

\[ \frac{\partial}{\partial \eta} U_k(\eta_0) = -i k U_k(\eta_0) \]

\[ + i \left[ \frac{2d_0}{1 + d_0} k + \frac{2H(\eta_0)d_1}{1 + d_0} \right] U_k(\eta_0) \]

\[ + \mathcal{O}\left( \frac{H^2(\eta_0)}{k} \right) U_k(\eta_0) \]
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*Example:* \( \lambda \phi^4 \) theory

*IR:* Marginal or relevant operators

\[ d_0 \rightarrow \phi \partial_\eta \phi \]

\[ d_2 \rightarrow K \phi^2 = 3H \phi^2 \]

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**IR piece:** Divergences can be cancelled by renormalizable boundary counterterms

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Example: \( \lambda \phi^4 \) theory

**IR:** Marginal or relevant operators

\[ d_0 \rightarrow \phi \partial_\eta \phi \]
\[ d_2 \rightarrow K \phi^2 = 3H \phi^2 \]

**UV:** Irrelevant operators

\[ c_1 \rightarrow \phi \ddot{\phi}, \dot{\phi}^2, K \phi \dot{\phi}, \dot{K} \phi^2, K^2 \phi^2 \]
Stress Energy Tensor Renormalization
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- Can corrections to initial state back-react to even prevent inflation from occurring?

- Effective field theory approach should eat up such divergences to leave a small backreaction
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- Effective field theory approach should eat up such divergences to leave a small backreaction

\[
\rho = \frac{1}{2} \frac{1}{a^2} \int \frac{d^3 \vec{k}}{(2\pi)^3} \left\{ U'_k U'^*_k + (k^2 + a^2 m^2) U_k U^*_k \right. \\
+ f^*_k [U'_k U'_k + (k^2 + a^2 m^2) U_k U_k] \right\},
\]

\[
p = -\rho + \frac{1}{a^2} \int \frac{d^3 \vec{k}}{(2\pi)^3} \left\{ U'_k U'^*_k + \frac{1}{3} k^2 U_k U^*_k \right. \\
+ f^*_k [U'_k U'_k + \frac{1}{3} k^2 U_k U_k] \right\},
\]
Stress Energy Tensor Renormalization (Cont’d)
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The Procedure:

1. Expand metric about FRW,

\[ g_{\mu\nu} = a^2(\eta) \left[ \eta_{\mu\nu} + h_{\mu\nu}(\eta, \vec{x}) \right] \]
Stress Energy Tensor Renormalization (Cont’d)

The Procedure:

1. Expand metric about FRW,

2. Construct interaction Hamiltonian linear in fluctuations,

\[ g_{\mu\nu} = a^2(\eta) \left[ \eta_{\mu\nu} + h_{\mu\nu}(\eta, \vec{x}) \right] \]
Stress Energy Tensor Renormalization (Cont’d)

The Procedure:

1. Expand metric about FRW,

\[ g_{\mu\nu} = a^2(\eta) \left[ \eta_{\mu\nu} + h_{\mu\nu}(\eta, \vec{x}) \right] \]

2. Construct interaction Hamiltonian linear in fluctuations,

\[ H_1(\eta) = \frac{1}{2} a^2(\eta) \int d^3 \vec{x} \, h^{\mu\nu} \left\{ -2 \tilde{G}_{\mu\nu} + T^{\text{cl}}_{\mu\nu} + \tilde{T}_{\mu\nu} \right\} \]
Stress Energy Tensor Renormalization (Cont’d)

The Procedure:

1. Expand metric about FRW,
2. Construct interaction Hamiltonian linear in fluctuations,

\[ g_{\mu\nu} = a^2(\eta) \left[ \eta_{\mu\nu} + h_{\mu\nu}(\eta, \vec{x}) \right] \]

\[ H_I(\eta) = \frac{1}{2} a^2(\eta) \int d^3\vec{x} h^{\mu\nu} \{ -2 \tilde{G}_{\mu\nu} + T^{cl}_{\mu\nu} + \tilde{T}_{\mu\nu} \} \]

\[ \langle 0_{\text{eff}}(\eta) | h^+(\eta, \vec{x}) | 0_{\text{eff}}(\eta) \rangle = \langle 0_{\text{eff}} | T \left( h^+(\eta, \vec{x}) e^{-i \int_{\eta_0}^{\eta} d\eta' [H_I^+(\eta') - H_I^-(\eta')]} \right) | 0_{\text{eff}} \rangle \]

\[ = \frac{1}{2} \int_{\eta_0}^{\eta} d\eta' a^2(\eta') \left\{ [\Pi^\geq_{\mu\nu}, \lambda^\rho(\eta, \eta' ; \vec{0})] - [\Pi^\leq_{\mu\nu}, \lambda^\rho(\eta, \eta' ; \vec{0})] \right\} \times \left[ 2 \tilde{G}_{\lambda\rho}(\eta') - T^{cl}_{\lambda\rho}(\eta') - T_{\lambda\rho}(\eta') \right] + \cdots. \]
Stress-Energy and Backreaction
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For TP corrections

\[ f_k = d_n \frac{\omega _k^n}{(aM)^n} \]
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\[
\rho_{\text{surf}}(\eta_0) \propto \frac{H^4}{16\pi^2} \frac{H^n}{M^n} \frac{d_n^*}{\epsilon} \left[ 1 + \mathcal{O}\left(\frac{m^2}{H^2}\right) + \mathcal{O}\left(\frac{H'}{H^2}\right) \right]
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\]

\[
\frac{\rho_{\text{surf}}^R}{\rho_{\text{vac}}} \sim \frac{1}{16\pi^2} \frac{H^2}{M_{\text{pl}}^2} \frac{H^n}{M^n}
\]

Backreaction is under control!
Conclusions: Part 2

- To extract maximum information early Universe from the CMB we need to know how to reliably calculate all relevant effects.
- There is a real possibility of using the CMB power spectra to get information about possible trans-Planckian physics effects.
- We now have an effective initial state that allows for reliable, controllable calculations. We’ve shown that as expected, backreaction effects are small after renormalization of the effective theory.
Final Thoughts

- Can we imprint short-distance physics on long-distance observables? Can this happen in a controlled fashion? Can we actually calculate anything reliably if so?

- The answers are yes, yes and yes, though the generic situation is that of large suppression of new effects.

- Boundary EFT for the initial state gives a way to parametrize the new physics and calculate reliably.

- QG on the sky might be a reasonable thing to look for!
Stress-Energy and Backreaction
Stress-Energy and Backreaction