Abstract: The Problem of Time in Quantum Gravity and Cosmology
Review Ashtekar for
Review Ashtekar formulation
Review

Ashtekar formulation of
Review Ashtekar formulation of GR.
Review Ashtekar formulation of GR/BF+
Review Ashtekar formulation of GR / BF theory
Review, Ashtekar formulation of GR/BF theory
Review, Ashtekar formulation of GR/BF theory
Review, Ashtekar formulation of GR/BF theory

Direct Program for
Review. Ashtekar formulation of GR/BF theory

Direct program for Quant.
Review, Ashtekar formulation of GR/BF theory

Direct program for quantization...
Review. Ashtekar formulation of GR/BF theory

Direct program for quantization of Con.
Review. Ashtekar formulation of GR/BF theory

Direct Program for Quantization of
Constrained
Review. Ashtekar formulation of GR/BF theory

- Direct program for quantization of
  - Stacked sys
Review. Ashtekar formulation of GR/BF theory

Direct program for quantization of constrained systems
Review: Ashtekar formulation of GR/BF theory

Direct program for quantization of constrained systems
Review. Ashtekar formulation of GR/BF theory

Direct program for quantization of constrained systems
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Direct program for quantization of constrained systems
Review. Ashtekar formulation of GR/BF theory

Direct program for quantization of constrained systems
Review. Ashtekar formulation of GR/BF theory

Direct Program for Quantization of Constrained Systems

G\& General Rel.
Review: Ashtekar formulation of GR/BF theory

- Diffeomorphism Program for Quantization of Constrained Systems
- Quantum Gravity Relativity
Review. Ashtekar formulation of GR/BF theory

- Direct program for quantization of constrained systems
- Perturbative Relativity
Review, Ashtekar formulation of GR/BF theory
- Direct program for quantization of constrained systems
- Quantum and Relativity
Review, Ashtekar formulation of GR/BF theory

- Dirac Program for Quantization of Constrained Systems
- Quantum General Relativity
Review, Ashtekar formulation of GR/ BF theory

- Dirac Program for Quantization of Constrained Systems
- Quantum General Relativity
- Quantum Gravity
Review, Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems

- Quantum General Relativity
Review. Ashtekar formulation of GR/BF theory
- Dirac program for quantization of constrained systems
- Quantum cosmology
- General Relativity
- Quantum gravity
Review, Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems

- Quantum General Relativity

"Laplace, gravity"
Review, Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
- "Laplace's Galaxy!"
Review, Ashtekar formulation of GR/BF theory

- Direct program for quantization of constrained systems

- Quantum General Relativity
Review, Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
  - Lapidus gravity
Review, Ashtekar formulation of GR/ BF theory
- Dirac Program for Quantization of Constrained Systems
- Quantum General Relativity
  "Loop quantum gravity"
Review of Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
- "Laplace in gravity"
Review, Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
- "Lap gravity"
Review, Ashtekar formulation of GR/BF theory
  - Dirac program for quantization of constrained systems
  - Quantum General Relativity
    - Lapse function
Review. Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
  - "laplacian of flux"
Review, Ashtekar formulation of GR/ BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity (QGR)
Review, Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
- Loop quantum gravity
Review, Ashtekar formulation of GR/BF theory

- Dicr program for quantization of constrained systems
- Quantum General Relativity
- Loop quantum gravity
Review, Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
- Loop quantum gravity
Review, Ashtekar formulation of GR/BF theory

- Dirac Program for Quantization of Constrained Systems
- Quantum General Relativity
  "Lap Quantum Gravity"
Review, Ashtekar formulation of GR/BF theory
- Direct program for quantization of constrained systems
- Quantum General Relativity
  "Lagrangian gravity"
Review, Ashtekar formulation of GR/ BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
  "Lap quantum gravity"
Review, Ashtekar formulation of GR/ BF theory

- Dirac Program for Quantization of Constrained Systems
- Quantum General Relativity
- Loop Quantum Gravity
Review, Ashtekar formulation of GR/BF theory

- Dirac program for quantization of constrained systems
- Quantum General Relativity
Review, Ashtekar formulation of \( GR/BF \) theory

- Dirac program for quantization of constrained systems

- Quantum General Relativity
Review. Ashtekar formulation of GR/BF theory

- Dirac Program for Quantization of Constrained Systems
- Quantum Geometric Theory
- Quantum Gravity
Review, Ashtekar's formulation of GR/ BF theory

- Direct program for quantization of constrained systems

- Quantum General Relativity

  "Loop quantum gravity"
Review, Aspects in the foundation of LGR/BF theory

- Direct program for quantization of constrained systems

- Quantum General Relativity

"Loop quantum gravity"
Review. Aspects of quantization of GR/BF theory

- Direct program for quantization of constrained systems
- Quantum General Relativity
  "Loop Quantum Gravity"
Review. Aspects in the formulation of GR/BF theory

- Direct program for quantization of constrained systems
- Quantum General Relativity
  "Loop quantum gravity"
compact, no boundary
\[ \exists \text{ compact, not bounded} \]

\[ \alpha \geq 0 \]
\[ d = \text{spins} = 3 \]

\[ G = \text{SU}(2) \]

\[ E \quad A \in G \]

\[ \exists \text{ compact, } \frac{\text{no boundary}}{\text{compact}} \]

\[ \sigma \zeta = 0 \]
\[ d = \text{spreading} = 3 \]

\[ G = SU(2) \]

\[ E^a A_i \in G \]

\[ \exists \text{compact, no boundary} \]

\[ \sigma \Sigma = 0 \]
\[ \Pi_i \rightarrow E_i \]
$$G = SU(2)$$

$$\{ a_{\hat{a}} E_i (x) \} = S^R S^L S^3 (x, \vec{y})$$
\[ d = spire = 3 \]

\[ E^r A_{\mu} \in G \]

\[ G = SU(2) \]

\[ \{ A_{\mu}(y), E^r_{\tau}(y) \} = \delta^r_{\tau} \]

\[ \phi^r = 3 \text{ constraints} \]

\[ \text{compact, no boundary, no metric} \]
φ = 3 constraints
\[ \lambda \in \mathcal{E} \]
$\mathcal{H} = \mathcal{E}_{\text{Constraints}}$

$\beta$-F Theory Constraints

$G^i = (\Omega^a E^b)^i = 0$
$\mathcal{H} = 3 \text{ constraints} \quad \forall \mathbf{k} \in \mathbb{G}$

\( C^i = \left( \mathcal{O}_a \mathcal{E}^i \right)^i = 0 = \mathcal{E}^{\mathbf{m}} + \epsilon^{\mathbf{m} \mathbf{n}} A_{\mathbf{n}} \mathcal{E}^i \)
$H = \Sigma \text{Constraints}$

$E' = E^{\Sigma + E}$

$C^i = (\Omega, E')^i = 0 = \hat{E}^{\tau i} + \hat{E}^{\alpha} A_{\bar{\alpha}}$
\[ \beta - F \text{ Theory Constraints} \]

\[ G^i = (\omega a E^a)^i = 0 = \dot{E}^i + \epsilon_{i3}^{\epsilon} \]

\[ J^i = F_{ab}^i (A) - A \epsilon_{abc} E^c = 0 \]
Ashtek formulation constraints

\[ G^c = 0 \]
Ashtekar-Fadmulation Constraints

\[ \mathcal{A} = 0 \]

\[ D_a = F_{ab} E^b \]

"diffeomorphism constraint"

\[ C = \epsilon^{ijk} E_i E_j \left[ F_{abk} + \Lambda \varepsilon_{abc} \tilde{E}^c \right] = 0 \]

Hamiltonian constraint
connection to ADM variables
connection to ADM

$g_{ab} \rightarrow g_{ab}$
$C = \sum_i E_i E_j \left[ F_{\text{asy}} + \nabla \right]$

connection to ADM variables

$g_{ab} \rightarrow g_{ab}$ spatial slices
Connection to ADM variables

$g_{ab} \rightarrow g_{a'b'}$ spatial slicers

$\Pi_{a'b'} \sim \sqrt{\lambda + \bar{g}_- E_0} \, E_{a'}$
Connection to ADM variables.

$\gamma_{ab} \rightarrow \eta_{ab}$ spatial slices.

$\rho_{45}(x), \Pi^{ij}(y)$

$\prod_{ab} \sim \sqrt{\text{det} g} \cdot \tilde{\eta} \cdot \theta_{35}$

$\delta_{ab} \delta_{cd} \delta^{3}(x, y)$
Translation? $E_a E_b$
Translation:

\[ E^a E^b = g_{ab} \]
Translation?

\[ E_i^a \tilde{E}^b_i = (\text{det } g) Z^{ab} \]
Translation?

\[ \tilde{\mathcal{E}}^a \tilde{\mathcal{E}}^b = (\det \mathcal{E}) Z_{\alpha \beta} \]

\[ A_i = \]
Translation?

\[ \tilde{E}_i^a \tilde{E}_i^b = \text{det}(\Omega) Z_{ab} \]

\[ A^i_a = \Gamma^i_a (\Omega) \]
Translation:

\[ \hat{E}^a_i \hat{E}^b_i = (\text{det} \Gamma) Z^{ab} \]

\[ A_a^i = \Gamma^i_a(\xi) + i \frac{\text{tr} \Pi_{ab} \hat{E}^{b}{}_{i}}{\text{det} \Gamma} \]
\( E^a_i E^b_i = (\det e)^2 g^{ab} \)

\( G A_a^i = \Gamma^i_a (\mathcal{e}) + \frac{1}{G} \frac{\partial}{\partial x^i} \frac{E^a}{\det \mathcal{e}} \)
\[ A^a_i = \Gamma^a_{ik}(\xi) + i \frac{\partial}{\partial \xi^k} E^b_i + \frac{1}{\text{length}} \]
\[ G = SU(2) \]

\[ \{ A_i \} \rightarrow E_i \]

\[ \varepsilon \rightarrow -i \]

\[ \{ A_{ij} \} \rightarrow E_{ij} \]

\[ = 3 \text{ Constraints} \]

\[ \text{Theory Constraints} \]

\[ E^n = \varepsilon E \]

\[ \forall \chi \in G \]
\[ \tilde{E}^a \tilde{E}^b = (\det(e)) Z^{ab} \]

\[ A^i_a = \Gamma^i_a (\xi) + i \pi \frac{\tilde{E}^b_i}{\delta \det \xi} \]
compact
no boundary
$\phi = 3$ constant

no metric

$S = \int \mathcal{L} B_1 F$

$g^{-1} = (\xi D_a \xi^i)$

$E_i^i = F_{ab} (A)$
\[ G = SU(2) \]

\[ \{ A^a, E_i^{\lambda}(x) \} = g^a_{\lambda \beta} S^\beta_i(x, \xi) \]

\[ \Pi^i \rightarrow E \]

\[ = 3 \text{ constraints} \]

\[ \text{Theory} \text{ constraints} \]

\[ E^0 = c \]

\[ (\mathcal{D} E) \]
\[ A_i^a = \Gamma^i_a(\xi) + \frac{\partial \xi^i}{\partial \xi^a} \]

Euclidean signature \( \Theta = 1 \)
\[ \tilde{E}_i^a \tilde{E}_i^b = (\text{det} \tilde{E}) Z_{ab} \]

\[ A_a = \Gamma_a^{i(x)} + \frac{i}{\hbar} \int \frac{\partial \Pi_{ab} \tilde{E}^{bi}}{\partial \tilde{E}_{aj}} \]

Euclidean signature $\Omega = 1$  

$G = SU(2)$ real
Euclidean signature $\Omega = 1$.

Lorentzian $A \in \mathcal{L}(\mathbb{R})$.
Lorentzian reality conditions

\[ q_{\mu \nu} = g_{\mu \nu} \quad E_\alpha^\mu = E^\mu \]
Lorentzian reality conditions

\[ g_{\mu \nu} = g_{\mu \nu} \quad E^\mu v^\nu = E^\mu \]

\[ A + \bar{A} = \]
Lorentzian reality conditions

\[ q_{\mu \nu} = \bar{q}_{\mu \nu} \quad E^{\mu \nu} = \bar{E}^{\mu \nu} \]

\[ A + A = \Gamma (\varphi) \]
Lorentzian reality conditions

\[ q_{\mu \nu} = q_{\nu \mu} \quad E^{\nu \mu} = E^{\mu \nu} \]

\[ A + \overline{A} = \Gamma (\chi) \]
\[ E_i E^i = (\det g) g_{ab} \]

\[ A^a = \Gamma^a_i (x) + \frac{\delta A}{\delta x^i} \frac{\partial \Gamma^a_i}{\partial x^j} \]

Euclidean signature $\Theta = 1$

Lorentzian $A \in SL(2)$
Lorentzian reality conditions

\[ q_{\mu} = q_{\nu} \]
\[ E_{\mu} = E_{\nu} \]
\[ A + A = \Gamma(\mathbb{C}) \]

Ashtekar- Biclero

\[ A = \Gamma + \mathbb{C} \Pi \mathbb{E} \]
Lorentzian reality conditions

\[ q_{\mu \nu} = \mathcal{R}_{\mu \nu} \quad E_\mu = E_\mu^* \]

\[ A + \mathcal{A} = \Gamma (\mathcal{U}) \]

Ashtekar-Barbero

\[ A = \Gamma + \mathcal{G} \Pi E \quad A^\mu = A^\mu \quad \mathcal{G} = SU(2) \]
\( \{ A_{a_i} (x) , \mathcal{A} \} \hat{=} 0 \)
\[ A = \Gamma + \text{GTE} \]

\[ C = C_{\text{Ashtekar}} + \text{new term} \]

\[ \Delta \xi = A \]
\[ \{ A^i_a(x), A^i_b(y) \} \hat{=} 0 \]

\[ A^i_{\alpha} = A^i_{\alpha} \quad \Rightarrow \quad \{ A^i_{\alpha}(x) \} = \mathcal{S}(z) \quad \forall \alpha \in \mathcal{D} \]
\( \left\{ A^a_a(x), \ H^a(x) \right\} = 0 \)

\( \forall \text{ Immirzi} \)

\( A^{a+}_a = A^a_a \quad c = \mathbb{SU}(2) \quad \forall \in \mathbb{C} \)
\[ \text{proj of } S \text{ into } S \]
Phase space itself is closed submanifold

$$f(A, t)$$
Phase space action $C$ could substitute $f(A,t) \rightarrow A \xrightarrow{rep} \mathcal{A}$ operators on $\mathcal{H}$
Phase space elements can lead to subshift in $\Lambda^\text{kin} \rightarrow \Lambda^\text{kin}$ operates on $\mathcal{H}$.

$f(A, E)$
\[ \text{det} + 9 > 0 \]
A canonical $f(A, \varepsilon) \rightarrow \text{Fock space}$

$Igshum \ \det \varphi > 0$

Polyakov:
A canonical

\[ \text{Fock space} \rightarrow \text{rep} \]

by choice

\[ \det g > 0 \]

Ishum

Polyakov: right observables

Eckert/coordinate

Field
$T[A, \lambda] = \text{Tr} P e^{\lambda A - d}$
A canonical \[ \frac{\text{Fock space}}{\text{rep}} \] choice

Ishum \[ \det G > 0 \]

Polyakov: right observables fields/coordinates
\[ \text{on the metric} \]

\[ \text{time evolution} \]

\[ \int_{\text{in}}^{\text{out}} \int_{\text{in}}^{\text{out}} \]

\[ T(A, \Phi) = \text{Tr} P e^{-iH_{\phi}} \]
"Little Tarsa"
"Little T also"
Little Traxels
"Little T algebra"  "T-one"  

\[ \text{Tr}[\tilde{E}^a(P)U_y] = T^a[y] \]

\[ \gamma_{10} = \rho \]
\[ \{ T'[\alpha], T'[\beta] \} = \int ds \sigma(x, s) S^3(x(s), p) \]
\[
\left\{ \mathbf{1} \right\}_{\alpha}, \mathbf{T}^{\alpha}_{\beta} = \int d^4 x (5) \Delta (x, y, z, \mu) \frac{1}{2} \text{Tr}(U_{\alpha}(x) U_{\beta}(y)) \text{Tr}(U_{\beta}(y))
\]
\[
\text{Tr}(A^T i) \text{Tr}(B^T i) = \text{Tr}(AB) - \text{Tr}(AB^T) \\
\text{check}
\]
\[ \text{Tr}(A^2) \text{Tr}(B^2) \]

\[ h_0 = \text{Tr}(AB) - \text{Tr}(A^2 B^2) \]

(check)
$\mathcal{F} T^a \equiv \mathcal{F} T^p = \mathcal{T} \mathcal{S} \mathcal{S}$

check
\[ E^{(y)} \{ T[A] \} = \int ds \quad \tilde{\mathcal{A}}^{-1} (s) \quad \tilde{\mathcal{F}}(x(s), y) \quad e^{iH(x) - \sum_{a \neq b} \epsilon_{abc} \bar{\mathcal{F}}_a(x) \mathcal{F}_b(s) \mathcal{F}_c(y)} \]

\[ \text{Tr} \quad \mathcal{U} = \mathcal{T} \]

\[ \{ T[A], T[B] \} = 0 \]
\[ E'(y) = \int_\mathcal{Y} \delta s \ x'(s) S^3(x(s), y) \]

\[ \text{Tr} \ U_t U_t^* \]

\[ \{ T[X], T[B] \} = 0 \]
\( SU(2) \text{ real} \)

\[ \Psi[A] = \text{(some equation and notes on the board)} \]
$S(U(2))$ real

$\Psi \left[ A \right] = \sum_{x} \frac{\alpha \Phi}{x}$

Moreover, ADIA variables
computation sum
\[ \prod \left[ x \right] \]

SU(2) lattice gauge theory on \( L \)
$\chi_1 \sum \left[ a^2 \right]$

$SU(2)$ lattice gauge theory on $L$

$c = \sum_{i=1}^{N} \left[ SU(2) \right]_i$ element to each segment
\[ L \rightarrow C \cap f(u_i) \]

\[ \lambda = \prod S_{\Delta m(u)} \]
\[ S_\mathcal{H}(U(H)) \in \mathcal{H}(U(H)) \]
Check

\[ N \leq \chi^2 \leq n \]

M Table 5

\[ \sum \]

8
$$\hat{T}(x, y) \mathcal{A}(x, y) = T \mathcal{A}(x, y)$$

$$\sum_{i} \int \mathcal{A}(x, y) = \sum_{i}$$

$$T(x, y) \mathcal{A}(x, y) = T \mathcal{A}(x, y)$$
$SU(2)$ lattice gauge theory on $L$

$C = \{ u \}$ $SU(2)$ element to each segment

$\forall [x] \in C \quad \exists \in (y)$

$1411^2 = 14 \times 41$
theory on $L$

$n_2$ elements to each segment

$$\langle \psi | \chi \rangle = \Psi^* X[A]$$
orthonormal basis
orthonormal basis

\[ C = \{ u_i \} \]
\[ Y_s^j = P \epsilon \int_{s}^{T} \int_{j}^{T} \int_{j}^{T} \]
\[ T[kV]^{x \ldots} = \prod_s (U_s S_i)^{i_s} \]
$T[k]_{\Delta}^{a} = \prod_{s} (U_{s} \delta_{s})^{i_{g_{s}}} \prod_{i \neq s} \delta_{i_{s}}$

Spin-network basis