Abstract: I will describe antiferromagnets and superconductors near quantum phase transitions. There is a remarkable analogy between their dynamics and the holographic description of Hawking radiation from black holes. I will show how insights from this analogy have shed light on experiments on the cuprate high temperature superconductors.
Quantum Criticality and Black Holes

Talk online: mahbub.physics.harvard.edu
Particle theorists

Sean Hartnoll, KITP
Christopher Herzog, Princeton
Pavel Kovtun, Victoria
Dam Son, Washington

Condensed matter theorists

Markus Mueller, Harvard
Subir Sachdev, Harvard
Quantum Entanglement

Hydrogen atom:

Hydrogen molecule:

\[ = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \]

Superposition of two electron states leads to non-local correlations between spins
Quantum Phase Transition

Change in the nature of entanglement in a macroscopic quantum system.
Quantum Phase Transition

Change in the nature of entanglement in a macroscopic quantum system.

Familiar phase transitions, such as water boiling to steam, also involve macroscopic changes, but in thermal motion
Quantum Criticality

The complex and non-local entanglement at the critical point between two quantum phases
Outline

1. Entanglement of spins
   *Experiments on antiferromagnetic insulators*

2. Black Hole Thermodynamics
   *Connections to quantum criticality*

3. Nernst effect in the cuprate superconductors
   *Quantum criticality and dyonic black holes*

4. Quantum criticality in graphene
   *Hydrodynamic cyclotron resonance and Nernst effect*
Outline

1. Entanglement of spins
   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics
   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene
   Hydrodynamic cyclotron resonance and Nernst effect
The cuprate superconductors
Antiferromagnetic (Neel) order in the insulator
Antiferromagnetic (Neel) order in the insulator

No entanglement of spins
Antiferromagnetic (Neel) order in the insulator

Excitations: 2 spin waves (Goldstone modes)
Weaken some bonds to induce spin entanglement in a new quantum phase
Antiferromagnetic (Neel) order in the insulator

Excitations: 2 spin waves (Goldstone modes)
Weaken some bonds to induce spin entanglement in a new quantum phase
Antiferromagnetic (Neel) order in the insulator

Excitations: 2 spin waves (Goldstone modes)
Weaken some bonds to induce spin entanglement in a new quantum phase
Ground state is a product of pairs of entangled spins.
Excitations: 3 $S=1$ triplons

$= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
Excitations: 3 $S=1$ triplons

$$= \frac{1}{\sqrt{2}} \left( |\uparrow \downarrow \rangle - |\downarrow \uparrow \rangle \right)$$
Excitations: 3 $S=1$ triplons

$$= \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle)$$
Phase diagram as a function of the ratio of exchange interactions, $\lambda$
Phase diagram as a function of the ratio of exchange interactions, $\lambda$

Quantum critical point with non-local entanglement in spin wavefunction
Phase diagram as a function of the ratio of exchange interactions, $\lambda$
Phase diagram as a function of the ratio of exchange interactions, $\lambda$

量子临界点与非局部纠缠在自旋波函数中的关系

Quantum critical point with non-local entanglement in spin wavefunction
TlCuCl$_3$
Phase diagram as a function of the ratio of exchange interactions, $\lambda$

Pressure in TlCuCl$_3$
Phase diagram as a function of the ratio of exchange interactions, $\lambda$

Pressure in TlCuCl$_3$
TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0.0)$, $ii=(0.0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u.]. The spectrum at $T=1.5$ K

TlCuCl$_3$ with varying pressure

Observation of $3 \rightarrow 2$ low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point

TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0.0)$, $ii=(0.0,3.15)$ [r.l.u.]. The spectrum at $T=1.5$ K

Phase diagram as a function of the ratio of exchange interactions, $\lambda$

Pressure in TlCuCl$_3$
TlCuCl$_3$ at ambient pressure

![Graph showing neutron profiles](image)

**FIG. 1.** Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u.]. The spectrum at $T=1.5$ K.

Observation of $3 \rightarrow 2$ low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point.
TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0,0), ii=(0,0,3.15)$ [r.l.u.]. The spectrum at $T=1.5$ K.

TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$ for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

TlCuCl$_3$ with varying pressure

Observation of $3 \rightarrow 2$ low energy modes, emergence of new longitudinal mode in Néel phase, and vanishing of Néel temperature at the quantum critical point

Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j ; \quad \vec{S}_i \Rightarrow \text{spin operator with } S = 1/2 \]
\textbf{TlCuCl}_3 \textbf{at ambient pressure}

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$
for $i = (1.35, 0.0)$, $ii = (0, 0, 0.15)$ [r.l.u]. The spectrum at $T = 1.5 \, \text{K}$

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer
Phase diagram as a function of the ratio of exchange interactions, $\lambda$

Pressure in TlCuCl$_3$
TlCuCl$_3$ at ambient pressure

FIG. 1. Measured neutron profiles in the $a^*c^*$ plane of TlCuCl$_3$
for $i=(1.35,0,0)$, $ii=(0,0,3.15)$ [r.l.u]. The spectrum at $T=1.5$ K

N. Cavadini, G. Heigold, W. Henggeler, A. Furrer, H.-U. Güdel, K. Krämer
Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j ; \quad \vec{S}_i \Rightarrow \text{spin operator with } S = 1/2 \]
Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle i, j \rangle} \vec{S}_i \cdot \vec{S}_j ; \quad \vec{S}_i \Rightarrow \text{spin operator with } S = 1/2 \]
Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum \text{four spin exchange} \]
Quantum phase transition with full square lattice symmetry

Neel order

Valence Bond Solid (VBS) order

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum \text{four spin exchange} \]

Why should we care about the entanglement at an isolated critical point in the parameter space?
Temperature, $T$

Quantum criticality

Neel

VBS

$K/J$
Temperature, $T$

Quantum criticality

Conformal field theory (CFT) at $T>0$

Neel, VBS

$K/J$
Outline

1. Entanglement of spins
   *Experiments on antiferromagnetic insulators*

2. Black Hole Thermodynamics
   *Connections to quantum criticality*

3. Nernst effect in the cuprate superconductors
   *Quantum criticality and dyonic black holes*

4. Quantum criticality in graphene
   *Hydrodynamic cyclotron resonance and Nernst effect*
Outline

1. Entanglement of spins
   *Experiments on antiferromagnetic insulators*

2. Black Hole Thermodynamics
   *Connections to quantum criticality*

3. Nernst effect in the cuprate superconductors
   *Quantum criticality and dyonic black holes*

4. Quantum criticality in graphene
   *Hydrodynamic cyclotron resonance and Nernst effect*
Black Holes

Objects so massive that light is gravitationally bound to them.
Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole horizon is causally disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)
Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics.
**Black Hole Thermodynamics**

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics.

Entropy of a black hole

\[
S = \frac{k_B A}{4 \ell_P^2}
\]

where \( A \) is the area of the horizon, and

\[
\ell_P = \sqrt{\frac{G \hbar}{c^3}}
\]

is the Planck length.
Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics

Entropy of a black hole $S = \frac{k_B A}{4 \ell_P^2}$

where $A$ is the area of the horizon, and

$\ell_P = \sqrt{\frac{G \hbar}{c^3}}$ is the Planck length.

The Second Law: $dA \geq 0$
Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics.

Horizon temperature: \(4\pi k_B T = \frac{\hbar^2}{2M \ell_P^2}\)
**AdS/CFT correspondence**

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
AdS/CFT correspondence
The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
AdS/CFT correspondence
The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
Outline

1. Entanglement of spins
   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics
   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene
   Hydrodynamic cyclotron resonance and Nernst effect
Black Holes

Objects so massive that light is gravitationally bound to them.

The region inside the black hole horizon is causally disconnected from the rest of the universe.

Horizon radius \( R = \frac{2GM}{c^2} \)
Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum \text{four spin exchange} \]

Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum \text{four spin exchange} \]

Temperature, $T$

Quantum criticality

Neel

VBS

$K/J$
Outline

1. Entanglement of spins
   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics
   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene
   Hydrodynamic cyclotron resonance and Nernst effect
Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics.

Entropy of a black hole \( S = \frac{k_B A}{4\ell_P^2} \)

where \( A \) is the area of the horizon, and \( \ell_P = \sqrt{\frac{G\hbar}{c^3}} \) is the Planck length.

The Second Law: \( dA \geq 0 \)
Black Hole Thermodynamics

Bekenstein and Hawking discovered astonishing connections between the Einstein theory of black holes and the laws of thermodynamics.

Horizon temperature: \[ 4\pi k_B T = \frac{\hbar^2}{2M \ell_P^2} \]
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
**AdS/CFT correspondence**

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.
AdS/CFT correspondence

The quantum theory of a black hole in a 3+1-dimensional negatively curved AdS universe is holographically represented by a CFT (the theory of a quantum critical point) in 2+1 dimensions.

3+1 dimensional AdS space

Quantum critical dynamics = waves in curved space

Quantum criticality in 2+1 dimensions
Outline

1. Entanglement of spins
   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics
   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene
   Hydrodynamic cyclotron resonance and Nernst effect
Outline

1. Entanglement of spins
   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics
   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene
   Hydrodynamic cyclotron resonance and Nernst effect
Dope the antiferromagnets with charge carriers of density $x$ by applying a chemical potential $\mu$.
Superconductor
Scanning tunnelling microscopy

Superconductor
STM studies of the underdoped superconductor

$\text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2$

$\text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y$

$a_0 = 3.9\text{Å}$

$a_0 = 5.4\text{Å}$
Topograph

$\text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2$

$\text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y$
Intense Tunneling-Asymmetry (TA) variation are highly similar

Topograph

\[ \text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2 \]

\[ \text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y \]
\[ \text{dI/dV Spectra} \]

\[ \text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2 \]

\[ \text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y \]

Intense Tunneling-Asymmetry (TA) variation are highly similar

Topograph

$\text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2$  \hspace{1cm}  $\text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y$
Intense Tunneling-Asymmetry (TA) variation are highly similar

Topograph

$\text{Ca}_{1.90}\text{Na}_{0.10}\text{CuO}_2\text{Cl}_2$

$\text{Bi}_{2.2}\text{Sr}_{1.8}\text{Ca}_{0.8}\text{Dy}_{0.2}\text{Cu}_2\text{O}_y$
Tunneling Asymmetry (TA)-map at $E=150\text{meV}$

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$  \hspace{1cm}  $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$
Tunneling Asymmetry (TA)-map at $E=150\text{meV}$

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$  $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$
Tunneling Asymmetry (TA)-map at $E=150\text{meV}$

$Ca_{1.90}Na_{0.10}CuO_2Cl_2$ \quad $Bi_{2.2}Sr_{1.8}Ca_{0.8}Dy_{0.2}Cu_2O_y$

Indistinguishable bond-centered TA contrast with disperse 4$a_0$-wide nanodomains

TA Contrast is at oxygen site (Cu-O-Cu bond-centered)
TA Contrast is at oxygen site (Cu-O-Cu bond-centered)
TA Contrast is at oxygen site (Cu-O-Cu bond-centered)

Evidence for a predicted valence bond supersolid

Superconductor

Scanning tunnelling microscopy
Use coupling $g$ to induce a transition to a VBS insulator.
Use coupling $g$ to induce a transition to a VBS insulator.

- **Superconductor**
- **QCP**
- **Insulator $x = 1/8$**
Proposed generalized phase diagram

Superconductor

Insulator $x=1/8$
Nernst experiment
Non-zero temperature phase diagram

- VBS Supersolid
- Superfluid
- Quantum critical
- VBS Insulator

Coulomb interactions
Non-zero temperature phase diagram

VBS Supersolid

Quantum critical

Superfluid

VBS Insulator

Coulomb interactions
Non-zero temperature phase diagram

- VBS Supersolid
- Quantum-critical dynamics in a magnetic field, at generic density, and with impurities
- Superfluid
- VBS Insulator
- Coulomb interactions

Pirsa: 08040006
To the CFT of the quantum critical point, we add

- A chemical potential $\mu$
- A magnetic field $B$

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

To the CFT of the quantum critical point, we add

• A chemical potential $\mu$

• A magnetic field $B$

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

• An electric charge

• A magnetic charge

A precise correspondence is found between general hydrodynamics of vortices near quantum critical points and solvable models of black holes with electric and magnetic charges
In the hydrodynamic regime, $\hbar \omega \ll k_B T$, we can use classical principles involving relaxation to local equilibrium to understand these perturbations.

The variables entering the hydrodynamic theory are

- the external magnetic field $F^{\mu \nu}$,

$$F^{\mu \nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & B \\ 0 & -B & 0 \end{pmatrix},$$

- $T^{\mu \nu}$, the stress energy tensor,

- $\rho$, the local number density,

- $P$, the local pressure,

- $\sigma_Q$, a universal conductivity, which is the single transport co-efficient.

- $J^\mu$, the current,

- $\varepsilon$, the local energy density,

- $u^\mu$, the local velocity, and

The dependence of $\varepsilon$, $P$, $\sigma_Q$ on $T$ and $v$ follows from simple scaling arguments.
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[ \partial_{\mu} J^\mu = 0 \]
\[ \partial_{\mu} T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} \left( \delta^\mu_{\nu} + u^\mu u_\nu \right) T^{\nu\gamma} u_\gamma \]
\[ T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \]
\[ J^\mu = \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[ (-\partial_\nu \mu + F_\nu^\lambda u_\lambda) + \mu \frac{\partial_{\mu} T}{T} \right] \]

Momentum relaxation from impurities

From these relations, we obtained results for the transport co-efficients expressed in terms of a “cyclotron” frequency and damping:

\[ \omega_c = \frac{2eB \rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2 v^2}{c^2(\varepsilon + P)} \]

Transverse thermoelectric co-efficient

\[
\left( \frac{h}{2ek_B} \right) \alpha_{xy} = \Phi_s \overline{B} (k_B T)^2 \left( \frac{2\pi \tau_{\text{imp}}}{\hbar} \right)^2 \frac{\overline{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon+P} (k_B T)^3 \hbar/2\pi \tau_{\text{imp}}}{\Phi_{\varepsilon+P} (k_B T)^6 + \overline{B}^2 \overline{\rho}^2 \left( 2\pi \tau_{\text{imp}} / \hbar \right)^2}
\]

where

\[ B = \overline{B} \phi_0 / (\hbar v)^2 \quad ; \quad \rho = \overline{\rho} / (\hbar v)^2. \]
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[ \partial_\mu J^\mu = 0 \]

\[ \partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} (\delta^{\mu}_{\nu} + u^\mu u_\nu) T^{\nu\gamma} u_\gamma \]

\[ T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} \]

\[ J^\mu = \rho u^\mu + \sigma_Q (g^{\mu\nu} + u^\mu u^\nu) \left[ (\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right] \]

**Momentum relaxation from impurities**

Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[
\begin{align*}
\partial_\mu J^\mu &= 0 \\
\partial_\mu T^{\mu\nu} &= F^{\mu\nu} J_\nu \\
T^{\mu\nu} &= (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu} \\
J^\mu &= \rho u^\mu + \sigma Q (g^{\mu\nu} + u^\mu u^\nu) \left[ (-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]
\end{align*}
\]

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[ \partial_\mu J^\mu = 0 \]
\[ \partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu \]
\[ T^{\mu\nu} = (\varepsilon + P) u^\mu u^\nu + Pg^{\mu\nu} \]
\[ J^\mu = \rho u^\mu \]

Constitutive relations which follow from Lorentz transformation to moving frame
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[ \partial_\mu J^\mu = 0 \]

\[ \partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu \]

Conservation laws/equations of motion
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[ \partial_{\mu} J^{\mu} = 0 \]
\[ \partial_{\mu} T^{\mu\nu} = F^{\mu\nu}\pi_{\nu} \]
\[ T^{\mu\nu} = (\varepsilon + P)u^{\mu}u^{\nu} + Pg^{\mu\nu} \]
\[ J^{\mu} = \rho u^{\mu} \]

Constitutive relations which follow from Lorentz transformation to moving frame
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

$$\partial_\mu J^\mu = 0$$
$$\partial_\mu T^{\mu \nu} = F^{\mu \nu} J_\nu$$
$$T^{\mu \nu} = (\varepsilon + P)u^\mu u^\nu + Pg^{\mu \nu}$$
$$J^\mu = \rho u^\mu + \sigma_Q (g^{\mu \nu} + u^\mu u^\nu) \left[ (-\partial_\nu \mu + F_{\nu \lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right]$$

Single dissipative term allowed by requirement of positive entropy production. There is only one independent transport co-efficient
Lorentz invariance and positivity of entropy production lead to the hydrodynamic equations of motion and constitutive relations:

\[ \partial_\mu J^\mu = 0 \]

\[ \partial_\mu T^{\mu\nu} = F^{\mu\nu} J_\nu + \frac{1}{\tau_{\text{imp}}} (\delta^{\mu}_\nu + u^\mu u_\nu) T^{\nu\gamma} u_\gamma \]

\[ T^{\mu\nu} = (\varepsilon + P) u^\mu u_\nu + P g^{\mu\nu} \]

\[ J^\mu = \rho u^\mu + \sigma_Q(g^{\mu\nu} + u^\mu u^\nu) \left[ (-\partial_\nu \mu + F_{\nu\lambda} u^\lambda) + \mu \frac{\partial_\mu T}{T} \right] \]

Momentum relaxation from impurities
From these relations, we obtained results for the transport co-efficients expressed in terms of a “cyclotron” frequency and damping:

\[
\omega_c = \frac{2eB \rho v^2}{c(\varepsilon + P)}, \quad \gamma = \frac{B^2 v^2}{c^2(\varepsilon + P)}
\]

Transverse thermoelectric co-efficient

\[
\left(\frac{\hbar}{2ek_B}\right) \alpha_{xy} = \Phi_s \overline{B} (k_B T)^2 \left(\frac{2\pi \tau_{\text{imp}}}{\hbar}\right)^2 \frac{\overline{\rho}^2 + \Phi_\sigma \Phi_{\varepsilon + P}(k_B T)^3 \hbar / 2\pi \tau_{\text{imp}}}{\Phi_{\varepsilon + P}(k_B T)^6 + \overline{B}^2 \overline{\rho}^2 (2\pi \tau_{\text{imp}} / \hbar)^2}
\]

where

\[
B = \overline{B} \phi_0 / (\hbar v)^2 \quad ; \quad \rho = \overline{\rho} / (\hbar v)^2.
\]
**LSCO Experiments**

Measurement of $\alpha_{xy} \approx \sigma_{xy} e_N$

\[ \alpha_{xy} \propto \frac{1}{T^4} \]

\[ \alpha_{xy} \propto \frac{BT^2 (\# \rho \tau_{imp} + \# T^3)}{T^6 + B^2 \rho \tau_{imp}^2} \]

(T small)

\[ \frac{\alpha_{xy}}{B} (B \to 0) \approx \left( \frac{2e k_B}{h \Phi_0} \right) \frac{\Phi_s}{\Phi_{s+P}} \left( \frac{2 \pi \tau_{imp}}{\hbar} \right)^2 \rho^2 (\hbar \nu) (k_B T)^4 \]

*Y. Wang et al., Phys. Rev. B 73, 024510 (2006).*
From these relations, we obtained results for the transport co-efficients expressed in terms of a “cyclotron” frequency and damping:

\[
\omega_c = \frac{2eB\rho v^2}{c(\varepsilon + P)} \quad , \quad \gamma = \frac{B^2v^2}{c^2(\varepsilon + P)}
\]

Transverse thermoelectric co-efficient

\[
\left(\frac{\hbar}{2ek_B}\right) \alpha_{xy} = \Phi_s \overline{B} (k_B T)^2 \left(\frac{2\pi \tau_{\text{imp}}}{\hbar}\right)^2 \frac{\rho^2 + \Phi_\sigma \Phi_{\varepsilon+P}(k_B T)^3 \hbar/2\pi \tau_{\text{imp}}}{\Phi_{\varepsilon+P}(k_B T)^6 + \overline{B}^2 \rho^2(2\pi \tau_{\text{imp}}/\hbar)^2}
\]

where

\[
B = \overline{B} \phi_0/(\hbar v)^2 \quad ; \quad \rho = \overline{\rho}/(\hbar v)^2.
\]
**LSCO Experiments**

Measurement of $\alpha_{xy} \approx \sigma_{xx} e_N$

\[ \alpha_{xy} \propto \frac{1}{T^4} \]

\[ \alpha_{xy} \propto \frac{B T^2 (\# \rho^{2} \tau_{imp} + \# T^3)}{T^6 + \# B^2 \rho^{2} \tau_{imp}^2} \]

(T small)

\[ \frac{\alpha_{xy}}{B} (B \to 0) \approx \left( \frac{2e k_B}{h \phi_0} \right) \frac{\Phi_s}{\Phi_{\varepsilon+P}} \left( \frac{2 \pi \tau_{imp}}{\hbar} \right)^2 \rho^2 (\hbar \nu) \left( k_B T \right)^4 \]

*Y. Wang et al., Phys. Rev. B 73, 024510 (2006).*
LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xy} e_N$

$\alpha_{xy} \propto \frac{1}{T^4}$

$\alpha_{xy} \propto \frac{BT^2(\# \rho^2 \tau_{imp} + \# T^3)}{T^6 + \# B^2 \rho^2 \tau_{imp}^2}$

(T small)

$\frac{\alpha_{xy}}{B} (B \to 0) \approx \left( \frac{2ek_B}{h \phi_0} \right) \frac{\Phi_s}{\Phi_{e+p}} \left( \frac{2\pi \tau_{imp}}{\hbar} \right)^2 \rho^2(\hbar \nu) \left( \frac{1}{k_B T} \right)^4$

$\hbar \nu \approx 47 \text{meV} \AA$

$v \approx 2.5 \times 10^{-5} \text{c}$

$\tau_{imp} \approx 10^{-12} \text{s}$

LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xy} e_N$

\[ \alpha_{xy} \propto \frac{1}{T^4} \]

\[ \alpha_{xy} \propto \frac{B T^2 (\# \rho \tau_{imp}^2 + \# T^3)}{T^6 + \# B^2 \rho \tau_{imp}^2} \]

(T small)

\[ \frac{\alpha_{xy}(B \to 0)}{B} \approx \left( \frac{2 e k_B}{h \phi_0} \right) \Phi_s \left( \frac{2 \pi \tau_{imp}}{\hbar} \right)^2 \rho^2(\hbar \nu) \left( k_B T \right)^4 \]

$\nu \approx 47 \text{meV A}^0$

$\nu \approx 2.5 \times 10^{-5} c$

$\tau_{imp} \approx 10^{-12} s$

\[ \omega_c = 6.2 \text{ GHz} \frac{B}{1 T} \left( \frac{35 \text{ K}}{T} \right)^3 \]

→ Prediction for $\omega_c$:

- T-dependent cyclotron frequency!
- 0.035 times smaller than the cyclotron frequency of free electrons (at T=35 K)
- Only observable in ultra-pure samples where $\tau_{imp}^{-1} \leq \omega_c$.
LSCO Experiments

$B, T$-dependence

Theory for $\alpha_{xy} \approx \sigma_{xx} N$

LSCO Experiments

Measurement of \( \alpha_{xy} \approx \sigma_{xy} e_N \)

\[ \alpha_{xy} \propto \frac{1}{T^4} \]

\[ \alpha_{xy} \propto \frac{BT^2 (\# \rho_{\text{imp}}^2 \tau_{\text{imp}}^2 + \# T^3)}{T^6 + \# B^2 \rho_{\text{imp}}^2 \tau_{\text{imp}}^2} \]

\( (T \text{ small}) \)

\[ \frac{\alpha_{xy}}{B} (B \to 0) \approx \left( \frac{2ek_B}{h\Phi_0} \right) \frac{\Phi_s}{\Phi_{\varepsilon+p}} \left( \frac{2\pi\tau_{\text{imp}}}{\hbar} \right)^2 \rho_{\text{imp}} \frac{(\hbar\nu)}{(k_B T)^{1/2}} \]

\( \hbar\nu \approx 47 \text{meV} \AA \)

\( \nu \approx 2.5 \times 10^{-5} c \)

\( \tau_{\text{imp}} \approx 10^{-12} \text{s} \)

---


\( \rightarrow \) Prediction for \( \omega_c \):

\[ \omega_c = 6.2 \text{ GHz} \frac{B}{1 \text{ T}} \left( \frac{35 \text{ K}}{T} \right)^3 \]

- T-dependent cyclotron frequency!
- 0.035 times smaller than the cyclotron frequency of free electrons (at \( T=35 \text{ K} \))
- Only observable in ultra-pure samples where \( \tau_{\text{imp}}^{-1} \leq \omega_c \).
LSCO Experiments

LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xy} e_N$

\[ \alpha_{xy} \propto \frac{1}{T^4} \]

\[ \alpha_{xy} \propto \frac{BT^2 \left( \# \rho \tau_{imp}^2 + \#T^3 \right)}{T^6 + \#B^2 \rho \tau_{imp}^2} \]

(T small)

\[ \frac{\alpha_{xy}}{B} (B \to 0) \approx \left( \frac{2ek_B}{h \phi_0} \right) \Phi_s \left( \frac{2\pi \tau_{imp}}{\hbar} \right)^2 \rho^2 \left( \frac{\hbar}{k_BT} \right)^4 \]

$\hbar \nu \approx 47\text{meV} \ \AA$

$\nu \approx 2.5 \times 10^{-5}c$

$\tau_{imp} \approx 10^{-12}\text{s}$


→ Prediction for $\omega_c$:

\[ \omega_c = 6.2 \text{GHz} \frac{B}{1/T} \left( \frac{35}{T} \right)^3 \]

- T-dependent cyclotron frequency!
- 0.035 times smaller than the cyclotron frequency of free electrons (at $T=35$ K)
- Only observable in ultra-pure samples where $\tau_{imp}^{-1} \leq \omega_c$.
LSCO Experiments

Theory for $N$

$B(T)$

$T(K)$

$\mu_0 H(T)$

$LSCO-0.12$  Nernst signal $N$

$T(K)$

LSCO Experiments

Measurement of $\alpha_{xy} \approx \sigma_{xy} e_N$

\[ \alpha_{xy} \propto \frac{1}{T^4} \]

\[ \alpha_{xy} \propto \frac{BT^2 (\# F_{\tau_{imp}}^2 + \# T^3)}{T^6 + \# B^2 \rho^2 \tau_{imp}^2} \]

\[ B T^2 (\# \rho^2 \tau_{imp}^2 + \# T^3) \]

\[ \alpha_{xy} \propto \frac{1}{T^4} \]

\[ \frac{\alpha_{xy}}{B} (B \to 0) \approx \left( \frac{2ek_B}{h\phi_0} \right) \frac{\Phi_s}{\Phi_{s+p}} \left( \frac{2\pi \tau_{imp}}{\hbar} \right)^2 \rho^2(\hbar \nu) (k_B T)^4 \]

\[ \hbar \nu \approx 47 \text{meVÅ} \]

\[ \nu \approx 2.5 \times 10^{-5} c \]

\[ \tau_{imp} \approx 10^{-12} \text{s} \]


→ Prediction for $\omega_c$:

\[ \omega_c = 6.2 \text{ GHz} \frac{B}{1 \text{ T}} \left( \frac{35 \text{ K}}{T} \right)^3 \]

- T-dependent cyclotron frequency!
- 0.035 times smaller than the cyclotron frequency of free electrons (at T=35 K)
- Only observable in ultra-pure samples where $\tau_{imp}^{-1} \leq \omega_c$.
LSCO Experiments

$B, T$-dependence

Theory for $\alpha_{xy} \approx \sigma_{xx} N$

LSCO Experiments

Theory for $N$

LSCO Experiments

$B, T$-dependence

Theory for $\alpha_{xy} \approx \sigma_{xx} N$

LSCO Experiments

Theory for $N$

$B(T)$

$T(K)$

$LSCO-0.12$ $Nernst$ $signal$ $N$

$\mu_0 H(T)$

$T(K)$

To the CFT of the quantum critical point, we add

- A chemical potential $\mu$
- A magnetic field $B$

After the AdS/CFT mapping, we obtain the Einstein-Maxwell theory of a black hole with

- An electric charge
- A magnetic charge

A precise correspondence is found between general hydrodynamics of vortices near quantum critical points and solvable models of black holes with electric and magnetic charges

Outline

1. Entanglement of spins
   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics
   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene
   Hydrodynamic cyclotron resonance and Nernst effect
Outline

1. Entanglement of spins
   Experiments on antiferromagnetic insulators

2. Black Hole Thermodynamics
   Connections to quantum criticality

3. Nernst effect in the cuprate superconductors
   Quantum criticality and dyonic black holes

4. Quantum criticality in graphene
   Hydrodynamic cyclotron resonance and Nernst effect
Graphene

\[ \varepsilon_{\vec{k}} = \hbar v_F \vec{k} \]
Graphene

Low energy theory has 4 two-component Dirac fermions, $\psi_\alpha$, $\alpha = 1 \ldots 4$, interacting with a $1/r$ Coulomb interaction

$$S = \int d^2r d\tau \psi_\alpha^\dagger \left( \partial_\tau - i v_F \vec{\sigma} \cdot \vec{\nabla} \right) \psi_\alpha$$

$$+ \frac{e^2}{2} \int d^2r d^2r' d\tau \psi_\alpha^\dagger \psi_\alpha(r) \frac{1}{|r - r'|} \psi_\beta^\dagger \psi_\beta(r')$$

Dimensionless “fine-structure” constant $\lambda = e^2/(4\hbar v_F)$.

RG flow of $\alpha$:

$$\frac{d\lambda}{d\ell} = -\lambda^2 + \ldots$$

Behavior is similar to a CFT3 with $\lambda \sim 1/\ln(\text{scale})$
Cyclotron resonance in graphene


\[ \omega = \pm \omega_c^{rel} - i\gamma - i/\tau \]

\( n = 1.1 \times 10^6 \, \text{m/s} \approx c / 300 \)

Conditions to observe resonance

- Negligible Landau quantization
- Hydrodynamic, collision-dominated regime
- Negligible broadening
- Relativistic, quantum critical regime

\[ E_{LL} = \hbar \nu \sqrt{\frac{2eB}{\hbar c}} < k_B T \]

\[ \hbar \omega_c^{rel} < k_B T \]

\[ \gamma \tau^{-1} < \omega_c^{rel} \]

\[ \rho \leq \rho_{th} = \frac{(k_B T)^2}{(\hbar \nu)^2} \]

\( T \approx 300 \, \text{K} \)
\( B \approx 0.1 \, \text{T} \)
\( \rho \approx 10^{11} \, \text{cm}^{-2} \)
\( \omega_c \approx 10^{13} \, \text{s}^{-1} \)
THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a ‘theory of everything’, might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007
Conclusions

- Quantum phase transitions in antiferromagnets
- Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.
- Theory of VBS order and Nernst effect in curpates.
- Quantum-critical transport in graphene.
Remarkable power of Einstein’s equation

In addition to describing gravitational phenomena (black holes, gravitational waves, etc.) it describes

- Renormalization group flow
- Hydrodynamics
- Quantum criticality
- Superconductivity of paired particles

Non-zero temperature phase diagram

- VBS Supersolid
- Quantum critical
- Superfluid
- VBS Insulator

Coulomb interactions
Conclusions

• Quantum phase transitions in antiferromagnets

• Exact solutions via black hole mapping have yielded first exact results for transport co-efficients in interacting many-body systems, and were valuable in determining general structure of hydrodynamics.

• Theory of VBS order and Nernst effect in curpates.

• Quantum-critical transport in graphene.
A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a ‘theory of everything’, might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007
THEORETICAL PHYSICS

A black hole full of answers

Jan Zaanen

A facet of string theory, the currently favoured route to a ‘theory of everything’, might help to explain some properties of exotic matter phases — such as some peculiarities of high-temperature superconductors.

NATURE|Vol 448|30 August 2007
Quantum Entanglement

Hydrogen atom: \( |\uparrow\rangle \)

Hydrogen molecule:

\[
\begin{align*}
| \quad \quad \quad \quad | \quad \quad \quad \quad | \\
|\uparrow\rangle & \quad \quad |\downarrow\rangle & \quad \quad - \\
| \quad \quad \quad \quad | \quad \quad \quad \quad | \\
= \frac{1}{\sqrt{2}} \left( |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) \\
\end{align*}
\]

Superposition of two electron states leads to non-local correlations between spins
diagram as a function of the ratio of exchange interactions, $\lambda$

Quantum critical point with non-local entanglement in spin wavefunction
Quantum phase transition with full square lattice symmetry

\[ H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + K \sum \text{four spin exchange} \]

\[ K/J \]