Revenge of the S-Matrix

or

What is the simplest QFT?

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with Jared Kaplan

+ in progress with Freddy Cachazo
Biggest Crises/Opportunity in fundamental physics:

- The Landscape
- Vacuum Selection
- Fundamental issues of QM, Gravity, Cosmology

MUST BE DEALT WITH.
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  - QM
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MUST BE DEALT WITH.
Something goes wrong with locality + gravity, not just @ LPL.

Information Paradox

Infinities in eternal inflation.

Can we talk about local things?
E.g. infalling observer into BH.

In cosmology, we are like those guys!

Local physics $\rightarrow$ Flat Space
Can we talk about ordinary QFT in a different way, not manifest local?

Analogy:

\[ m \dot{x} = -V'(x) \quad \text{manifest deterministic} \]

\[ S[A, B] \text{ extremized not manifest: det.} \rightarrow \text{better jumping eff point to QM} \]
2000's "top down" attempt: Witten's
twistor formulation of SYM

\[ \text{boundary theory for flat space.} \]

CSW recursion relations

Very special to 4D, MHV amp.
play special role, related to \( F = \pm i \tilde{F} \)
solutions of \( \mathcal{T} \).

\[ \text{led to BCF-W recursion relations, which can be described from "bottom up", and are much more general.} \]
BCFW Redux

Complexify 2 momenta $p_j,k$, keeping them on shell:

$$p_j' = p_j + q' \cdot z = p_j(z)$$

$$p_k' = p_k - q' \cdot z = p_k(z)$$

$$0 = p_j^2(z) \Rightarrow q \cdot p_j = 0, \quad q^2 = 0$$
\( q^2 = 0, \quad q \cdot p_i^k = 0 \)

\[
p_j = (1, \pm 1, 0, 0, 0, \ldots 0), \quad q = \frac{1}{\sqrt{2}} (0, 0, 1, i, 0, \ldots, 0)
\]

[Or keep momenta real, \((D-2, 2)\) sig.]

Pol vectors:

\( z = 0: \quad E_j^- = E_k^+ = q, \quad E_j^+ = E_k^- = \overline{q}, \quad E_T = (0, 0, 0, 0; \ldots 1) \)

\( E_j^- (z) = E_k^+ (z) = q, \quad E_T (z) = (0, 0, 0, 0; \ldots 1) \)

\( E_j^+ (z) = \overline{q} - z p_k, \quad E_k^- = \overline{q} + z p_j \)

[\[ P_j^k \quad E_j^k (z) = 0, \quad E_j^- E_j^+ = E_k^- E_k^+ = 1, \ldots \]]

\( M(p_i, h_i) \rightarrow M^{h_j h_k} (z) \)
$M(z)$: only simple poles

\[ P_j(z) = \begin{cases} P_j^2 & j,k \in \mathbb{Z} \\ P_j + 2q & j \in \mathbb{Z} \\ P_j - 2q & k \in \mathbb{Z} \end{cases} \]

\[ P_j^2(z) = P_j^2 + 2P_j \cdot q \cdot z \text{, poles at } z \to z_j = -\frac{P_j^2}{2P_j \cdot q} \]

\[ \text{res } M(z \to z_j) = \sum_{h} \left( \begin{array}{c} j \\ h \end{array} \right) \times \left( \begin{array}{c} -h \\ k \end{array} \right) \text{ amp} \]
If \( M(z \to \infty) \to 0 \),

\[
0 = \frac{1}{2\pi i} \int \frac{dz}{z} M(z) = M(0) + \text{other res.}
\]

\[ \downarrow \]

\[
\sum_{j, h} \frac{1}{P_i^2 - \frac{1}{P_j^2}}
\]

On-shell BCFW recursion relations.

[Sufficient \( M^{-\text{any}}(z \to \infty) \to 0 \text{ Gauge} \)]

[Sufficient \( M^{-\text{any}}(z \to \infty) \to 0 \text{ Univ} \)]
Can recursively reduce all amplitudes to

\[ 1 \rightarrow 3 \]

which normally can't be on-shell, but can be for complex momenta (or in (D-2, 2) signature).
Remarkable Object [Casas, Benincasa]

Completely determined by Lorentz Invariance.

In 4D, $P_{\alpha i} = 2\alpha\bar{\tau}_i$,

$\mathcal{M}_{123} = \frac{s_1 + s_2 - s_3}{2} \left\langle 23 \right\rangle^2 \left\langle 31 \right\rangle^2$

$+$ \begin{array}{c}
1 \\
2 \\
3 \\
\end{array}

$+$ \begin{array}{c}
2 \\
3 \\
\end{array}

$\frac{4}{3} \cdot \frac{1}{2} - c_3$

Spin 4

$f_{abc} \left\langle 12 \right\rangle \left\langle 23 \right\rangle$

$\left[ \frac{\left\langle 12 \right\rangle^3}{\left\langle 13 \right\rangle^2 \left\langle 23 \right\rangle} \right]^2$

Spin 2
Behavior of $M(z \to \infty)$ is surprising

Naively, $M(z \to \infty) \to 0$ is never true! e.g. $\phi^4$-theory

$M_{\phi^4}(z) \to z^0$

Gauge / Gravity is worse!

Then fold in $\varepsilon$'s, $\to z^0 \text{ or } z^1 ...$
<table>
<thead>
<tr>
<th>Gauge</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>++</td>
<td>( \mathbb{Z} )</td>
</tr>
<tr>
<td>---/++</td>
<td>( \mathbb{Z}^2 )</td>
</tr>
<tr>
<td>++</td>
<td>( \mathbb{Z}^3 )</td>
</tr>
<tr>
<td>Grow.</td>
<td></td>
</tr>
<tr>
<td>---++</td>
<td>( \mathbb{Z}^{n-1} )</td>
</tr>
<tr>
<td>:</td>
<td></td>
</tr>
<tr>
<td>++,--</td>
<td>( \mathbb{Z}^{n+3} )</td>
</tr>
</tbody>
</table>
Gauge N = ve

\[
\begin{align*}
-+ & \rightarrow \mathbb{Z} \\
--+ & \rightarrow \mathbb{Z}^2 \\
+ - & \rightarrow \mathbb{Z}^3
\end{align*}
\]

---

Gauge

\[
\begin{align*}
-++ & \rightarrow \mathbb{Z}^{n-1} \\
+ - & \rightarrow \mathbb{Z}^{n+3}
\end{align*}
\]

---

Unexpectedly good behavior of \( M(2 \to 3\alpha) \) encapsulates heavy cancellations in explicit diagram calculations.
Understanding $M(z \to \infty)$

$$P_{k\ell} = (z) = P_{k\ell} \pm z q$$

$z \to \infty$: hard (complex) light-like particle blasting through soft background. Familiar for real momenta (eikonal). "Not much" scattering, "helicity concerned". We'll formalize and extend to future research.
Yang–Mills

\[ A_\mu = A_\mu + q_\mu. \quad \text{Usual} \quad g_\mu \quad \text{G-Fixing} \]

\[ Z = -\frac{1}{4} \text{tr} D_\mu a_a D_\nu a_b \eta^{ab} \]
\[ + \frac{i}{2} \text{tr} [a_a, a_b] F_{ab} \]

\[ \rightarrow \infty : \quad \text{"Spin Lorentz invariance".} \]

\[ M^{ab} = (c\xi + \ldots) \eta^{ab} + A^{ab} + \frac{1}{2} \mathcal{B}^{ab} + \ldots \]

Also Ward id.: \[ P^a \mathcal{E}_a M^{ab} \mathcal{E}_b = 0 \]

\[ \Rightarrow q \quad a_b \quad \mathcal{E}_b \quad \mathcal{E}_b \quad \mathcal{E}_b \]
Pure Gravity

\[ Z = \sqrt{g} \left[ \frac{1}{4} g^{\mu \nu} g_{\rho \sigma} V_{\mu} h_{\rho \sigma} V_{\nu} h_{\rho \sigma} - \frac{i}{2} h_{\rho \sigma} h_{\mu \nu} R^{\rho \mu \sigma \nu} \right] \]

(Bern-Grant trick used).

L, R h_{\rho \sigma} indices separately contracted.

\[ \Rightarrow 2 \text{ copies of spin L.I.} \]

\[ h_{\mu \nu} = e^a_{\mu} \tilde{e}^\alpha_{\nu} h_a \tilde{\alpha} \]

Gauge \[ \omega^+_a b = \tilde{\omega}^+_a \tilde{b} = 0 \]
What is simplest QFT?

$\phi^4 \rightarrow YM \rightarrow \text{Gravity}$

$N=4 \text{ SYM} \downarrow \quad N=8 \text{ SUGRA}$
Why? We've seen that amps of \( s \geq 1 \) particles are much nicer than scalars. But: **fundamentally discrete** objects. YM

\[ M \quad 1 \]

Graw 1

\[ +2 \quad + \frac{3}{2} \quad + \frac{1}{2} \quad 0 \quad + \frac{1}{2} \quad - \frac{1}{2} \quad - \frac{1}{2} \quad - \frac{3}{2} \quad -2 \]
Label states of man. \( (\vec{a}, \vec{\alpha}) \):

\[
|\eta\rangle = e^{-iQ} |\eta\rangle \quad \Rightarrow \quad \langle \omega | \langle \omega \rangle = 1
\]

\[
|\eta\rangle = e^{-iQ} \quad \Rightarrow \quad |\eta\rangle = e^{-iQ} |\eta\rangle \quad \langle \omega | \langle \omega \rangle = 1
\]

\[
Q_{\alpha I} |\eta\rangle = \alpha_{\alpha I} |\eta\rangle \quad Q_{\alpha I} |\tilde{\eta}\rangle = \tilde{\alpha}_{\alpha I} |\tilde{\eta}\rangle
\]

\[
|\eta\rangle \leftrightarrow |\bar{\eta}\rangle \quad \text{Complementary}
\]

\[
|\alpha\rangle \quad \text{MHV} \leftrightarrow \text{MHV} \quad \text{Complementary}
\]
Label states of $\eta (\tilde{\eta}, \tilde{\eta})$:

$$|\eta\rangle = e^{-} |\tilde{\eta}\rangle$$

or

$$|\tilde{\eta}\rangle = e^{+} |\eta\rangle$$

$$Q_{\tilde{\eta}} |\eta\rangle = \lambda_{\tilde{\eta}} |\eta\rangle$$

$$Q_{\tilde{\eta}} |\tilde{\eta}\rangle = \lambda_{\tilde{\eta}} |\tilde{\eta}\rangle$$

$$|\eta\rangle \rightarrow |\tilde{\eta}\rangle \text{ Complementary}$$

$$\eta \leftrightarrow \tilde{\eta} \text{ Complementary}$$
Using (trivially) SUSY, can show

Remarkable: SUSY transmits good properties of gravity amplitudes to lower-spin (e.g. scalar) amplitudes.
Fundamental Q:

What is the dual theory (weak-weak!) that explains these amazing properties? Should exist + be nicest for $N=4$, $N=8$. But since BCFW is true in any $D$, it shouldn't crucially rely on 4D.

To start with: is there an analog of twistor string theory that makes BCFW "obvious"?