Abstract: It is an important task to embed inflation in a fundamental microphysical theory such as string theory. Since string theory possesses a vast landscape of 4-dimensional theories, we would like to know which portions contain inflation and which do not. I prove a no-go theorem that inflation and de Sitter vacua are forbidden in an exponentially large number of infinite families of simple and well understood compactifications of type IIA string theory. I also mention more complicated and less well understood compactifications, which may have the ingredients for our cosmology.
Inflationary Constraints on String Theory

PASCOS 08

Mark Hertzberg, MIT

Cosmology & String Theory
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- Cosmologists are asking questions like:
  What is the inflaton potential?
  What is the inflaton/s?
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  Where in the space of string models is our universe?
  What can be predicted?
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Cosmologists and String theorists can learn from each other!
# Stringlish to English

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Approximate meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha')</td>
<td>Regge parameter</td>
<td>Inverse string tension</td>
</tr>
<tr>
<td>(l_s)</td>
<td>String length</td>
<td>(= 2\pi \sqrt{\alpha'}) (in our convention)</td>
</tr>
<tr>
<td>(\kappa_{10})</td>
<td>10-d gravitational strength</td>
<td>(= \sqrt{8\pi G_{10}} = l_s^4/\sqrt{4\pi}), gravitational strength in 10 dims</td>
</tr>
<tr>
<td>(m_P)</td>
<td>(Reduced) Planck mass</td>
<td>(= 1/\sqrt{8\pi G}), mass scale of quantum gravity in 4 dims</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Dilaton</td>
<td>Scalar field that rescales the strength of gravity</td>
</tr>
<tr>
<td>(a_i)</td>
<td>Axions</td>
<td>Pseudo-scalars that appear in the 4-d theory</td>
</tr>
<tr>
<td>(b_i)</td>
<td>Geometric moduli</td>
<td>Scalar fields describing (\phi) and the size &amp; shape of the compact space</td>
</tr>
<tr>
<td></td>
<td>- Dilaton modulus(^a)</td>
<td>(\sim e^{-\phi}) (explicit form is model dependent)</td>
</tr>
<tr>
<td></td>
<td>- Kähler moduli</td>
<td>Scalar fields that specify the size of the compact space</td>
</tr>
<tr>
<td></td>
<td>- Complex structure moduli</td>
<td>Scalar fields that specify the shape of the compact space</td>
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<tr>
<td>(\psi_i)</td>
<td>Complex moduli</td>
<td>(= a_i + i b_i)</td>
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<tr>
<td>(\psi)</td>
<td>Complex inflaton vector</td>
<td>((\psi_1, ..., \psi_n)), the complex moduli-vector that can evolve during inflation</td>
</tr>
<tr>
<td>(\phi)</td>
<td>Real inflaton vector</td>
<td>((a_1, b_1, ..., a_n, b_n)), the real moduli-vector that can evolve during inflation</td>
</tr>
<tr>
<td>(g_s)</td>
<td>String coupling</td>
<td>(= e^\phi), the string loop expansion parameter</td>
</tr>
<tr>
<td>(F_p)</td>
<td>p-form field strength</td>
<td>Generalized electromagnetic field strength carrying (p)-indices</td>
</tr>
<tr>
<td>(f_p)</td>
<td>Flux</td>
<td>(\propto \int F_p), (normally integer valued) equivalent to a generalized electric or magnetic charge, but can arise purely due to non-trivial topology</td>
</tr>
<tr>
<td>(g_{10}/R_{10})</td>
<td>10-d string metric/Ricci scalar</td>
<td>Metric/Ricci scalar in the fundamental 10-d action in string frame</td>
</tr>
<tr>
<td>(g_4/R_4)</td>
<td>4-d string metric/Ricci scalar</td>
<td>Metric/Ricci scalar in the effective 4-d action in string frame</td>
</tr>
<tr>
<td>(g_E/R_E)</td>
<td>4-d Einstein metric/Ricci scalar</td>
<td>Metric/Ricci scalar in the effective 4-d action after a conformal transformation to Einstein frame</td>
</tr>
<tr>
<td>(g_6)</td>
<td>Metric on compact space</td>
<td>2nd block of (g_{10} = \text{diag}(g_4, g_6)), describing the geometry of compact space</td>
</tr>
<tr>
<td>Vol</td>
<td>6-d volume of compact space</td>
<td>(= \int_{\text{cs}} d^6 x \sqrt{g_6}) ((\text{cs} \equiv \text{compact space}))</td>
</tr>
<tr>
<td>(T^{\phi})</td>
<td>6-d term</td>
<td>A 6-d manifold that is Riemann flat, defined by periodic identifications</td>
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Outline
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- Review Conditions for Inflation
- Type IIA String Theory
- Compactification
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**Inflation Constrains String Theory!**

- Outlook: Evading the No-Go Theorem
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- Introduce scalar field $\phi$

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S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left( R + 4(\partial_{\mu}\phi)^2 - \frac{1}{2}|H_3|^2 - e^{2\phi} \sum_p |F_p|^2 \right) - \mu_6 \int_{D6} d^7\xi \sqrt{-g} e^{-\phi} + 2\mu_6 \int_{O6} d^7\xi \sqrt{-g} e^{-\phi}
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- IIA with D6/O6 branes on CYs are "semi-realistic" e.g., can build MSSM
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- We proved 4D theory has the Lagrangian:

\[ \mathcal{L} = \frac{1}{16\pi G} R_E - \left( \frac{1}{2} (\partial_{\mu} \phi_d)^2 + \frac{1}{2} (\partial_{\mu} \phi_v)^2 + \ldots \right) - V(\phi_i) \]
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Einstein gravity
Dilaton
6D Volume

All other "moduli"
size & shape

Fluxes, Branes & Planes
Naively, why might this approach fail?
Naively, why **might** this approach fail?

- Potential typically needs some fine tuning

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**In Summary:** we explore an **exponentially large number of infinite** ($10^{500}$) families of 4D models
The Potential $V$
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e.g.,

Maxwell 2-form

$$V_2 = |E|^2 = \frac{q^2}{R^4}$$

p-form

$$\frac{q^2}{R^{2p}}$$
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p-form

$$\frac{q^2}{R^{2p}}$$

- We proved rigorously (using $K$ and $W$, or direct dimensional reduction) that in the Einstein frame: $(R \leftrightarrow \psi_v)$

$$V = V_3 + \sum_p V_p + V_{D6} + V_{O6}$$

$$= \frac{A_3(\phi_j)}{\psi_d^2 \psi_v^3} + \sum_p \frac{A_p(\phi_j)}{\psi_d^p \psi_v^{p-3}} + \frac{A_{D6}(\phi_j)}{\psi_d^3} - \frac{A_{O6}(\phi_j)}{\psi_d^3}$$
No-Go Theorem
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Fact:

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Outlook: Evading the No-Go Theorem
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