Title: From Bohr to Bayes: Causality, Probability, and Statistics in Quantum Theory.

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Abstract: This paper critically examines the view of quantum mechanics that emerged shortly after the introduction of quantum mechanics and that has been widespread ever since. Although N. Bohr, P. A. M. Dirac, and W. Heisenberg advanced this view earlier, it is best exemplified by J. von Neumannâ€™s argument in Mathematical Foundations of Quantum Mechanics (1932) that the transformation of \'a [quantum] state ... under the action of an energy operator . . . is purely causal,\' while, \'on the other hand, the state ... which may measure a [given] quantity ... undergoes in a measurement a non-casual change.\' Accordingly, while the paper discusses all four of these arguments, it will especially focus on that of von Neumann. The paper also offers an alternative, radically noncausal, view of the quantum-mechanical situation and considers the differences between the ensemble and the Bayesian understanding quantum mechanics. It will also discuss the Bayesian approach to quantum information theory in this set of contexts.

From Bohr to Bayes: Causality, Probability, and Statistics in Quantum Theory

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Abstract. This paper critically examines the view of quantum mechanics that emerged shortly after the introduction of quantum mechanics and that has been widespread ever since. Although N. Bohr, P. A. M. Dirac, and W. Heisenberg advanced this view earlier, it is best exemplified by J. von Neumann's argument in *Mathematical Foundations of Quantum Mechanics* (1932) that the transformation of "a [quantum] state … under the action of an energy operator … is *purely causal*," while, "on the other hand, the state … which may measure a [given] quantity … undergoes in a measurement a *non-casual* change." Accordingly, while the paper discusses all four of these arguments, it will especially focus on that of von Neumann. The paper also offers an alternative, fundamentally noncausal, view of the quantum-mechanical situation and considers the differences between the ensemble and the Bayesian approaches to quantum mechanics from this perspective.

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[We] must revise our ideas of causality. Causality applies to a system which is left undisturbed. If a system is [quantum-level] small, we cannot observe it without producing a serious disturbance and hence we cannot expect to find any causal connexion between the results of our observations. Causality will still apply to undisturbed [quantum] systems and the equations [those of quantum mechanics] which will be set up to describe an undisturbed system will be differential equations expressing a causal connection between conditions at one time and conditions at a latter time. These equations will be in close correspondence with the equations of classical mechanics, but they will be connected only indirectly with the results of observations. There is an unavoidable indeterminacy in the calculation of observational results, the theory enabling us to calculate in general only the probability of our obtaining a particular result when we make an observation.

P. A. M. Dirac, *The Principles of Quantum Mechanics*, Clarendon, Oxford, 1930, p. 4

One can suppose that the initial state of a system determines definitively the state of the system at any subsequent time. … The notion of does not enter into the ultimate description of mechanical processes; only when one given some information that involves a probability … can one deduce results that involve probabilities.

P. A. M. Dirac, "The Physical Interpretation of the Quantum Dynamics," 1927, *Proc. Roy. Soc. (London)* A113, 243-265. 1927

On one hand, the definition of the state of a physical system, as ordinarily understood [i.e. in classical physics], claims the elimination of all external disturbances. But in that case, according to the quantum postulate [of Planck], any observation will be impossible, and, above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, and there could be no question of causality in the ordinary sense of the word. The very nature of the quantum theory thus forces us to regard *the space-time co-ordination and the claim of causality, the union of which characterizes the classical theories, as complementary but exclusive features of the description*, symbolizing the idealization of observation and definition respectively.

–Niels Bohr (1927), *PWNB* 1, pp. 54-55 (emphasis added)

Complementarity refers to a mutual exclusivity of certain phenomena, entities, or conceptions, and yet the possibility of applying each one of them separately at any given point and the necessity of using all of them at different moments for a comprehensive account of the totality of phenomena that we must consider.

CLASSICAL THEORY [no complementarity]

Causal relationships of phenomena described in terms of space and time

QUANTUM THEORY

Either	*Or*
Phenomena described in terms of space and time	Causal relationships expressed by mathematical laws
But	*But*
Uncertainty Principle of phenomena	Physical description in space-time is impossible

These two alternatives are related statistically (Heisenberg 1929, p. 65)

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Complementarity refers to a mutual exclusivity of certain phenomena, entities, or conceptions, and yet the possibility of applying each one of them separately at any given point and the necessity of using all of them at different moments for a comprehensive account of the totality of phenomena that we must consider.

All concepts which can be used in classical theory for the description of a mechanical system can also be defined exactly in atomic processes in analogy to the classical concepts. The experiment which provides such definitions themselves suffers from an indeterminacy introduced purely by the observational procedures that we use when we ask of them the simultaneous determination of two canonically-conjugate quantities. The magnitude of this indeterminacy is given by the [uncertainty] relation [$\Delta q \Delta p \approx h$]

The presumption that behind the perceived statistical world [of quantum observations] there still hides a 'real' world in which causality holds… is [a] fruitless and senseless … speculation.

[Quantum] physics ought to describe only correlations of observations.

Quantum mechanics establishes the final failure of causality.

W. Heisenberg, "The Intuitable [*anschaulich*] Content of Quantum Kinematics and Mechanics, 1927 (*Quantum Theory and Measurement* p. 83)

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On the one hand, a state ϕ is transformed into the state ϕ' under the action of an energy operator H in the time interval $0 \leq \tau \leq t$:

$$\partial\phi_\tau/\partial t = -(2\pi i/h)H\phi_\tau \qquad (0 \leq \tau \leq t),$$

so if we write $\phi_0 = \phi, \phi_t = \phi'$, then

$$\phi' = e^{-(2\pi i/h)tH}\phi$$

which is *purely causal*. A mixture U [U is a statistical operator] is corresponding transformed into:

$$U' = e^{-(2\pi i/h)tH}\, U\, e^{(2\pi i/h)tH}$$

Therefore, *as a consequence of the causal change of ϕ into ϕ'*, the state $U = P_{[\phi]}$ go over into the states $U' = P_{[\phi']}$

$$U \rightarrow U_t = e^{-(2\pi i/h)tH}\, U\, e^{(2\pi i/h)tH}$$

On the other hand, the state ϕ -- which my measure a quantity with a pure discrete spectrum, distinct eigenvalues and eigenfunctions $\phi_1, \phi_2, \ldots$ -- undergoes in a measurement *a non-causal change* in which each of the states $\phi_1, \phi_2, \ldots$ can result, and in fact does result with the respective probabilities $|(\phi, \phi_1)|^2, |(\phi, \phi_2)|^2, \ldots$. That is, the mixture

$$U' = \sum_{n=1}^{\infty} |(\phi, \phi_n)|^2 P_{[\phi_n]}$$

obtains. More generally, the mixture U goes over into

$$U' = \sum_{n=1}^{\infty} |(U\phi_n, \phi_n)|^2 P_{[\phi_n]}$$

Since the states go over into mixtures, the process is not causal.

The differenced between these two processes $U \rightarrow U'$ is very fundamental one: aside from the different behaviors in regard to the principle of causality, they are also different in that the former is (thermodynamically) reversible, while that the latter is not.

--John von Neumann, *The Mathematical Foundations of Quantum Mechanics* (pp. 417–418)

The theory predicts only the *statistics* of the results of an experiment, when it is repeated under a given condition. Like an ultimate fact without any cause, the *individual* outcome of a measurement is, however, in general not comprehended by law.

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... if a (physical) classical state does not exist at any moment, it can hardly change causally.

--Erwin Schrödinger (1935)

the motion of particles follows the probability law but the probability itself propagates [in a wave-like manner] according to the law of causality

--Max Born (1926)

In [Heisenberg's] theory the attempt is made to transcribe every use of mechanical concepts in a way suited to the nature of the quantum theory, and such that in every stage of the computation only directly observable quantities enter. In contrast to ordinary mechanics, the new mechanics does not deal with a space-time description of the motion of atomic particles. It operates with manifolds of quantities which replace the harmonic oscillating components of the motion and symbolize the possibilities of transitions between stationary states in conformity with the correspondence principle. These quantities satisfy certain relations which take the place of the mechanical equations of motion and the quantization rules [of the old quantum theory].

--Bohr (1925) *PWNB* I, p. 48

An uncertainty relation such as [$\Delta q \Delta p \equiv h$] is not a statement about the accuracy of our measuring instruments. On the contrary, its derivation assumes the existence of *perfect* instruments (the experimental errors due to common laboratory hardware are usually much larger than these quantum uncertainties). The only [available?] correct interpretation of [$\Delta q \Delta p \equiv h$] is the following: If the *same* preparation procedure [defined by the classical control of measuring instruments] is repeated many times, and is followed either by a measurement of [q], or by a measurement of [p], the various results obtained for [q] and for [p] have standard deviations, [Δq] and [Δp], whose product cannot be less than [h]. There is never any question here that a measurement of [q] "disturbs" the value of [p] and vice-versa, as [is] sometimes claimed. These measurements are incompatible, but they are performed on *different* [*quantum objects*] (all of which are identically prepared) and therefore these measurements cannot disturb each other in any way. An uncertainty relation ... only reflects the randomness of the outcomes of quantum tests.

Asher Peres, *Quantum Theory: Concepts and Methods* (1993)

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