Title: The "in-in" Formalism and Cosmology: Inflation at Large N
Date: May 21, 2009  03:00 AM
URL: http://pirsa.org/09050061
Abstract: TBA
The “in-in” Formalism and Cosmology: Inflation at Large N

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arXiv: 0904:4207 [hep-th]

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Effective Field Theories In Inflation,
Perimeter Institute
May 21, 2009
Inflation at Zeroth Order

Zeroth order scalar field inflation very successful, solves all of the classic cosmological problems;

- Horizon
- Monopole
- Flatness
- Entropy

Quantum fluctuations about the classical trajectory provide the (nearly) scale invariant spectrum of perturbations that seed structure/ CMB anisotropies.

Unfortunately, implementation is not unique;

- Many ways of implementing an inflationary scenario,
- Nearly scale invariant spectrum of (almost) gaussian fluctuations is generic.
Inflationary Models at Lowest Order.

At lowest order, in field theory language, we think of the power spectrum, or 2-pt correlation function as the propagator:

$$P(k) \sim \ldots + \ldots$$

- Generated by QM fluctuations of inflaton during inflation
- Amplitude and shape constrained by CMB data

Gravity couples to all forms of energy density
- Beyond lowest order, modes will couple, evolve non-linearly...
Diagrammatically:

__________  +...

1980’s
Diagrammatically:

1990's - 2002 (Maldacena, ...)

+ ...
Diagrammatically:

2006 (Seery, Sloth, Lidsey, ...)

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Diagrammatically:

(Seery, Sloth, Weinberg...)

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Diagrammatically:
Outline

1. The ADM Formulation of GR and the “in-in” Formalism
   - Operator Formalism

2. Loop Corrections in N-Field Inflation: Bounds on N?
   - N-Field Inflation
   - Radiative Stability and Loop Corrections
   - Inflation with N-Spectator Fields
   - Coherent Field Description

3. Conclusions
The ADM Formulation of GR and the “in-in” Formalism
The ADM Formulation of GR and the “in-in” Formalism

The ADM Formulation of GR

Perturbing fields in the ADM metric:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$N$ and $N^i$ Lagrange multipliers, $h_{ij}$ metric on spatial hypersurface

- Not all $\{h_{ij}, N^i, N\}$ lead to unique field configurations
- Specify a gauge, i.e. a spatial slicing and a threading

Spatially flat gauge:

$$h_{ij} = a^2(t)(\delta_{ij} + \gamma_{ij}), \quad \phi(x, t) = \bar{\phi}(t) + \delta\phi(x, t)$$

$a(t)$ scale factor.
ADM action:

\[ S = \int d^3x dt \sqrt{h} N \left[ R^{(3)} - 2N V_1(\phi_I) + N^{-1}(E_{ij} E^{ij} - E^2 + \pi^l \pi_l) \right. \]

\[ \left. + h^{ij} (\partial_i \phi_I \partial_j \phi_I) \right] , \]

‘Gravitational momentum:’

\[ E_{ij} = \dot{h}_{ij} - \nabla_{(i} N_{j)} \]

Field momentum:

\[ \pi^l = \dot{\phi}^l - N^i \partial_i \phi^l \]

- \( N \) and \( N^i \) have no dynamics; they do not propagate, and are constraints.

- Once known, substituted back into the action.

- Action contains only dynamical degrees of freedom.
Calculation of cosmological correlation functions differs from usual QFT:

- Not interested in elements of a S-matrix, or transition amplitudes, but in expectation values of fields at fixed times,
- Conditions are imposed on the fields at very early times - only have “in-states,”

Can formulate as a path integral (Seery, Collins, Holman) or using operators (Weinberg).
Use the operator formulation of the “in-in” formalism of Schwinger;

Set up:

- Expand the action in powers of the fluctuations $\delta \phi$ and $\gamma_{ij}$ and discard the zeroth and first order pieces.
- Define conjugate momenta, e.g. $\pi_{\delta \phi} = \frac{\partial L}{\partial \delta \phi}$, and construct the Hamiltonian.
- Work in an interaction picture, divide the Hamiltonian into a quadratic piece, $H_0$ and a higher order piece, $H_{\text{int}}$.
- $H_0$ evolves the fields.
- $H_I$ evolves the states.
The interaction picture fields are free fields;

\[ \delta \phi_I(x, \tau) = \int d^3k \, e^{ik \cdot x} \left[ a_k U_k(\tau) + a_{-k}^\dagger U_k^*(\tau) \right] \]

\( U_k(\tau) \) are solutions to the equation of motion:

\[ \partial^2_\tau (a U_k) + \left[ k^2 - a^2 H^2 \left( 2 + \epsilon - m'^2 \right) \right] a U_k = 0 \]

\[ m' = \frac{V''}{H^2} \sim \eta \]

- de-Sitter limit (and taking the fields to be massless):

\[ U_k = \sqrt{\frac{H^2}{2(2\pi)^3 k^3}} (1 + ik\tau) e^{-ik\tau} \]
Quantization of Theories with Derivative Interactions

The interactions generically contain derivatives of the fields

Schematically:

\[ \mathcal{L} = \frac{1}{2} \delta \phi^2 - V(\delta \phi) + (\sqrt{\epsilon \delta \phi^2 + \delta \phi^3}) \delta \phi + \frac{1}{2} \left( \sqrt{\epsilon \delta \phi + \delta \phi^2} \right) \dot{\delta \phi}^2 + \frac{1}{3} \delta \phi \ddot{\delta \phi}^3 + O(\epsilon \delta \phi^3) + O(\epsilon \delta \phi^4) + O(\delta \phi^5) \]

What is \( \mathcal{H} \)? Is \( \mathcal{H}_{\text{int}} = -\mathcal{L}_{\text{int}} \)?

Recall:

\[ \mathcal{H}(\pi, \delta \phi) = \dot{\delta \phi}(\pi)\pi - \mathcal{L}(\pi, \delta \phi). \]

But, \( \pi = \dot{\delta \phi} + O(\sqrt{\epsilon \delta \phi^2}) + O(\delta \phi^2) \dot{\delta \phi} + ... \)

So,

\[ \mathcal{H} = \mathcal{H}_0 - \mathcal{L}_{\text{int}} + O(\epsilon \delta \phi^4) + O(\delta \phi^5). \]
Correlation functions

\[ \langle Q(t) \rangle = \left\langle \left[ T e^{-i \int_{t_0}^{t} H_{\text{int}}(t') dt'} \right]^\dagger Q_l(t) \left[ T e^{-i \int_{t_0}^{t} H_{\text{int}}(t'') dt''} \right] \right\rangle, \]

- \( Q_l(t) \) is some product of fields.

Nothing mysterious about “in-in,”

\[ = \int \, d\alpha \, d\beta \langle 0 | \left( T e^{-i \int_{t_0}^{t} H_{\text{int}}(t') dt'} \right)^\dagger |\alpha\rangle \langle \alpha | Q(t) |\beta\rangle \langle \beta | \left( T e^{-i \int_{t_0}^{t} H_{\text{int}}(t'') dt''} \right) |0\rangle \]

\[ = \int \, d\alpha \, d\beta \langle \alpha | Q(t) |\beta\rangle \langle \beta | T e^{-i \int_{t_0}^{t} H_{\text{int}}(t'') dt''} |0\rangle \left( \langle \alpha | T e^{-i \int_{t_0}^{t} H_{\text{int}}(t'') dt''} |0\rangle \right)^\dagger \]

\( N \)-pt function \( \langle \delta \phi^N \rangle \) is simply the sum over ways of obtaining a final state with \( \alpha + \beta = N \)
Time Path Interpretation:

\[
\langle Q(t^*) \rangle = \left\langle \left[ T e^{-i \int_{t_0}^{t^*} H_{\text{int}}(t') dt'} \right]^\dagger Q_I(t^*) \left[ T e^{-i \int_{t_0}^{t^*} H_{\text{int}}(t'') dt''} \right] \right\rangle ,
\]
At second order: $\langle Q(t^*) \rangle_2 =$
Rather than inserting states explicitly, use the Dyson solution;

\[
Te^{-i \int_{t_0}^{t} H(t'')dt''} = \sum_{N=0}^{\infty} (-i)^N \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{N-1}} dt_N H(t_1)H(t_2)\cdots H(t_N)
\]

\[
\left( Te^{-i \int_{t_0}^{t} H(t'')dt''} \right)^\dagger = \sum_{N=0}^{\infty} (i)^N \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{N-1}} dt_N H(t_N)\cdots H(t_2)H(t_1)
\]

Then, expanding

\[
\langle Q(t) \rangle = \langle Q(t) \rangle_0 + \langle Q(t) \rangle_1 + \langle Q(t) \rangle_2 + \ldots,
\]

where

\[
\langle Q(t) \rangle_1 = -2i \mathfrak{S} \int_{t_0}^{t} dt_1 \langle H_{\text{int}}(t_1)Q(t) \rangle,
\]

\[
\langle Q(t) \rangle_2 = -2\mathfrak{R} \left[ \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \langle H_{\text{int}}(t_2)H_{\text{int}}(t_1)Q(t) \rangle \right]
\]

\[
+ \left\langle \int_{t_0}^{t} dt_1 H_{\text{int}}(t_1)Q(t) \int_{t_0}^{t} dt_2 H_{\text{int}}(t_2) \right\rangle.
\]

Time ordering has been taken care of!
At tree level, the two point correlation function is:

\[ \langle \delta \phi_i \delta \phi_j \rangle = U_k U^*_k \delta(k - k') \]

- Contraction of two fields:
  \[ \delta \phi^I_k \delta \phi^J_p = \delta \phi^I_k \delta \phi^J_p - : \delta \phi^I_k \delta \phi^J_p : \]

- Propagator:
  \[ \langle \delta \phi^I_k(\tau) \delta \phi^J_p(\tau') \rangle = U_k(\tau) U^*_p(\tau') \delta^{IJ} \delta(k + p) \]

- Operator ordering matters.
- Wightman functions instead of Feynman propagators
- Wick’s theorem follows in the usual way.
  - Disconnected diagrams cancel by unitarity:
    \[ \left\langle \left[ T e^{-i \int_{t_0}^t H_{\text{int}}(t) dt} \right]^\dagger \left[ T e^{-i \int_{t_0}^t H_{\text{int}}(t) dt} \right] \right\rangle = 1 \]
Subtleties:

\[ \langle Q(t) \rangle = \left\langle \left[ Te^{-i \int_{t_0}^{t} H_{\text{int}}(t) dt} \right]^\dagger Q_I(t) \left[ Te^{-i \int_{t_0}^{t} H_{\text{int}}(t) dt} \right] \right\rangle, \]

To calculate:

- Assume the initial (infinite past) conditions are adiabatic vacuum,
- Computationally this amounts to allowing a small amount of evolution in imaginary time in the far past: \(-\infty \rightarrow -\infty (1 + i\epsilon)\)
- Left and right time integrations (vertices) no longer equivalent, but conjugates of each other.
- Implementation:
  - Active: Redefine integrations to run over a complex interval
  - Passive: Analytically continue the time variable to include a small imaginary piece.
Subtleties:

Temptation: use

\[ \langle Q(t) \rangle = \sum_{N=0}^{\infty} i^N \int_{t_0}^{t} dt_N \int_{t_0}^{t_{N-1}} dt_{N-1} \ldots \int_{t_0}^{t_2} dt_1 \langle [H_{\text{int}}(t_1), [H_{\text{int}}(t_2), \ldots [H_{\text{int}}(t_N), Q(t)] \ldots ]] \rangle. \]

- Physical terms are broken up into unphysical pieces

At 2nd order:

\[ \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \langle H_{\text{int}}(t'') Q(t) H_{\text{int}}(t') \rangle \equiv \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \int d\alpha d\beta \langle 0 | H_{\text{int}}(t'') | \alpha \rangle \langle \alpha | Q(t) | \beta \rangle \langle \beta | H_{\text{int}}(t') \rangle \]

\[ \rightarrow \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' (H_{\text{int}}(t') Q(t) H_{\text{int}}(t'') + H_{\text{int}}(t'') Q(t) H_{\text{int}}(t')) \]

\[ = 2 \Re \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \langle H_{\text{int}}(t'') Q(t) H_{\text{int}}(t') \rangle \]
Subtleties:

\[
\langle Q(t) \rangle = \Bigg\langle \left[ Te^{-i \int_{r_0}^{t} H_{\text{int}}(t) dt} \right]^\dagger Q_I(t) \left[ Te^{-i \int_{r_0}^{t} H_{\text{int}}(t) dt} \right] \Bigg\rangle,
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\]

\[\rightarrow \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \langle H_{\text{int}}(t') Q(t) H_{\text{int}}(t'') + H_{\text{int}}(t'') Q(t) H_{\text{int}}(t') \rangle
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\[ \rightarrow \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \langle H_{\text{int}}(t') Q(t) H_{\text{int}}(t'') + H_{\text{int}}(t'') Q(t) H_{\text{int}}(t') \rangle \]

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\[ \langle Q(t) \rangle = \left\langle \left[ Te^{-i \int_{r_0}^r H_{\text{int}}(t)dt} \right]^\dagger Q_1(t) \left[ Te^{-i \int_{r_0}^r H_{\text{int}}(t)dt} \right] \right\rangle, \]

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\[ \rightarrow \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' (H_{\text{int}}(t')Q(t)H_{\text{int}}(t'') + H_{\text{int}}(t'')Q(t)H_{\text{int}}(t')) \]

\[ = 2R \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \langle H_{\text{int}}(t'')Q(t)H_{\text{int}}(t') \rangle \]

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\[ \delta \phi_k \delta \phi_p = \delta \phi_k \delta \phi_p - : \delta \phi_k \delta \phi_p : \]

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At 2nd order:

$$\int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \langle H_{\text{int}}(t'')Q(t)H_{\text{int}}(t') \rangle = \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \int d\alpha d\beta \langle 0 | H_{\text{int}}(t'') | \alpha \rangle \langle \alpha | Q(t) | \beta \rangle \langle \beta |$$

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\[
\to \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' (H_{\text{int}}(t')Q(t)H_{\text{int}}(t'') + H_{\text{int}}(t'')Q(t)H_{\text{int}}(t'))
\]

\[
= 2R \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \langle H_{\text{int}}(t'')Q(t)H_{\text{int}}(t') \rangle
\]
Summary - Operator Formalism

- Nothing mysterious about “in-in” formalism:
  - Simple interpretation via transition amplitudes.
  - Just ordinary QFT rigged to compute correlation functions.

- Operator Formalism:
  - Fast, transparent way of doing “in-in” calculations.
  - Only one contraction.
  - One must be careful with derivative couplings.
  - One should avoid artificially splitting up diagrams.

Powerful technique for calculating correlation functions.
Subtleties:

Temptation: use

\[
\langle Q(t) \rangle = \sum_{N=0}^{\infty} i^N \int_{t_0}^{t} dt_N \int_{t_0}^{t_{N-1}} dt_{N-1} \ldots \int_{t_0}^{t_2} dt_1 \langle [H_{\text{int}}(t_1), [H_{\text{int}}(t_2), \ldots [H_{\text{int}}(t_N), Q_i(t)]\ldots] \rangle.
\]

- Physical terms are broken up into unphysical pieces

At 2nd order:

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\int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' (H_{\text{int}}(t'') Q(t) H_{\text{int}}(t')) \equiv \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \int d\alpha d\beta \langle 0 | H_{\text{int}}(t'') | \alpha \rangle \langle \alpha | Q(t) | \beta \rangle \langle \beta |
\]

\[
\rightarrow \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' (H_{\text{int}}(t') Q(t) H_{\text{int}}(t'') + H_{\text{int}}(t'') Q(t) H_{\text{int}}(t'))
\]

\[
= 2 \Re \int_{t_0}^{t} dt' \int_{t_0}^{t'} dt'' \langle H_{\text{int}}(t'') Q(t) H_{\text{int}}(t') \rangle
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Nothing mysterious about “in-in” formalism:
- Simple interpretation via transition amplitudes.
- Just ordinary QFT rigged to compute correlation functions.

Operator Formalism:
- Fast, transparent way of doing “in-in” calculations.
- Only one contraction.
- One must be careful with derivative couplings.
- One should avoid artificially splitting up diagrams.

Powerful technique for calculating correlation functions.
Gravitationally Induced Loop Corrections in N-Field Inflation: Bounds on N?
Consider an action of the form with $N$ scalar fields (participator fields), $M$ massless scalars (spectator fields):

$$S = \frac{1}{2} \int d^4x \sqrt{g} \left[ M_{\text{pl}}^2 \mathcal{R} + \sum_{l=1}^{N} ((\partial \phi_l)^2 - 2V(\phi_l)) + \sum_{J=1}^{M} (\partial \sigma_J)^2 \right],$$

Potential:

$$V(\phi_l) = \sum_{l=1}^{N} V_l(\phi_l)$$

Each $V_l$ depends on a single $\phi_l$.

(Canonical example, considered here $N$ copies of $m^2 \phi^2$.)
**W-Participators: W-Field Inflation**

Friedmann equation:

\[ 3H^2 = \sum_I \left( \frac{1}{2} \dot{\phi}_I^2 + V_I(\phi_I) \right) \]

Homogeneous Klein-Gordon equation:

\[ \ddot{\phi}_I + 3H \dot{\phi}_I + \frac{dV(\phi)}{d\phi_I} = 0 \]

- Each field feels gradient of its own potential.
- Feels the Hubble friction of all fields.
- Obtain inflation from a collection of potentials for which inflation cannot occur individually.

**Slow Roll Params:**

\[ \epsilon = 2M_{pl}^2 \left( \frac{H'}{H} \right)^2 = \frac{1}{2} \sum_{I=1}^{N} \left( \frac{\dot{\phi}_I}{HM_{pl}^2} \right)^2 = \sum_{I=1}^{N} \epsilon_I \]
Why N Fields?

- Many candidate theories of the early universe contain many additional degrees of freedom, e.g. string theory
- $N$-field inflation provides (theoretically!) a way of realizing chaotic inflation consistently within an effective field theory.
  - i.e. It is a way of side-stepping the problem of Planckian vevs,
    - $\epsilon \rightarrow \epsilon_l = \epsilon/N$,
    - $\Delta \phi \rightarrow \Delta \phi/\sqrt{N}$.
  - Get significant gravity waves while respecting the Lyth bound.
- $N$-copies of the Standard Model might solve the hierarchy problem
  - Novel solution to hierarchy problem if $N \sim 10^{32}$ (Dvali)
Simple Bounds on $N$

All approximately massless fields fluctuate with an amplitude set by the Hubble scale:

$$\delta \phi_i \sim \frac{H}{2\pi}$$

- Fluctuations freeze out on scales larger than $1/H$,
- Each field contributes gradient energy, $(\nabla \phi)^2/2$.

Gradient energy scales like

$$\frac{N}{2} \left( \frac{\delta \phi}{\delta x} \right)^2 \sim N \frac{H^4}{8\pi^2}$$

Given $H$, $\rho = 3M_{\text{pl}}^2 H^2$. For self consistency:

$$N \ll \frac{M_{\text{pl}}^2}{H^2}$$
Radiative Stability and Loop Corrections

Assume the form of the potential is radiatively stable for this work. What about gravitationally induced loop corrections?

- Graviton couples to everything
- Loop corrections from the potential $\rightarrow$ radiative corrections to the slow roll parameters
- Gravitationally induced loop corrections $\rightarrow$ radiative corrections to the power spectrum.
  - $N$-degrees of freedom to run round the loops.
Simple Bounds on $N$

All approximately massless fields fluctuate with an amplitude set by the Hubble scale;

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Gradient energy scales like

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- Gravitationally induced loop corrections $\rightarrow$ radiative corrections to the power spectrum.
  - $N$-degrees of freedom to run round the loops.
Density fluctuations:

\[ P_k = \frac{1}{N^2} \sum_{l=1}^{N} \left( \frac{H}{\dot{\phi}_l} \right)^2 \langle \delta \phi_l \delta \phi_l \rangle \]

- Can bounds be put on \( N \) from loop corrections to the power spectrum?
- One might expect an \( m \)-loop correction to scale like \( N^m \).

To one loop order

\[ \langle \delta \phi_n \delta \phi_n \rangle \sim \frac{H^2}{2(2\pi)^3 M_{\text{pl}}^2} \left( 1 + N \frac{H^2}{M_{\text{pl}}^2} \right) \]

So might expect \( N \ll \frac{M_{\text{pl}}^2}{H^2} \).
Leading order third and fourth order actions are, respectively,

\[ S^{(3)} = -\int dt d^3x \left[ \frac{a^3}{4} \sqrt{2\epsilon_1 \delta \phi^I \dot{\delta \phi}^J \dot{\delta \phi}^J} + \frac{a^3}{2} \sqrt{2\epsilon_1 \partial^{-2} \delta \phi^I \dot{\delta \phi}^J \partial^2 \delta \phi^J} \right], \]

- **Coupling:** \( \epsilon_1 \equiv \frac{j^2}{2H^2} \),

\[ S^{(4)} = \int dt d^3x a^3 \left[ \frac{1}{4Ha^2} \partial_i \delta \phi^J \partial_i \delta \phi^J \partial^{-2} (\partial_j \dot{\delta \phi}^I \partial_j \delta \phi^I + \dot{\delta \phi}^I \partial^2 \delta \phi^I) \\
+ \frac{1}{4H} \delta \phi^J \dot{\delta \phi}^J \partial^{-2} (\partial_i \dot{\delta \phi}^I \partial_i \delta \phi^I + \dot{\delta \phi}^I \partial^2 \delta \phi^I) \\
+ \frac{3}{4H} \partial^{-2} (\partial_j \dot{\delta \phi}^J \partial_j \delta \phi^J + \dot{\delta \phi}^J \partial^2 \delta \phi^J) \partial^{-2} (\partial_j \dot{\delta \phi}^I \partial_j \delta \phi^I + \dot{\delta \phi}^I \partial^2 \delta \phi^I) \\
+ \frac{1}{4} \beta_{2,j} \partial^2 \delta \phi^I \partial_j \delta \phi^I + \dot{\delta \phi}^I \beta_{2,i} \partial_i \delta \phi^I \right], \]

\[ \frac{1}{2} \beta_{2,j} \approx \partial^{-4} \left( \partial_j \partial_k \dot{\delta \phi}^I \partial_k \delta \phi^I + \partial_j \dot{\delta \phi}^I \partial^2 \delta \phi^I - \partial^2 \dot{\delta \phi}^I \partial_j \delta \phi^I - \partial_m \dot{\delta \phi}^I \partial_j \partial_m \delta \phi^I \right). \]
Interactions

- Four point interaction:
  \[ H^{(4)}(t) \sim \int d^3x \frac{1}{aH^2} \partial^{-n}(\delta \phi^I \delta \phi^I) \partial^{-m}(\delta \phi^J \delta \phi^J) \]

- Three point interaction
  \[ H^{(3)}(t) \sim \int d^3x \frac{1}{aH} \sqrt{2\epsilon_1} \delta \phi^I \delta \phi^J \delta \phi^J \]

- Loop corrections given by:
  \[ \langle \delta \phi^I(t) \delta \phi^I(t) \rangle_{1L,1V} = -2\Im \int_{-\infty}^t dt_1 \langle H^{(4)}(t_1) \phi^I(t) \delta \phi^I(t) \rangle, \]
  and
  \[ \langle \delta \phi^I(t) \delta \phi^I(t) \rangle_{1L,2V} = -2\Re \left[ \langle \int_{-\infty}^t dt_2 \int_{-\infty}^{t_2} dt_1 H^{(3)}(t_1) H^{(3)}(t_2) \delta \phi^I(t) \delta \phi^I(t) \rangle \right] \]
  \[ + \langle \int_{-\infty}^t dt_1 H^{(3)}_f(t_1) \delta \phi^I(t) \delta \phi^I(t) \int_{-\infty}^t dt_2 H^{(3)}_f(t_2) \rangle. \]
Diagrams:

- Not typical Feynman diagrams.
- Time doesn’t flow through the diagrams - propagators have only 3-momenta.
- Times associated with vertices.
- Diagrams useful for visualization.
- Feynman rules can be constructed, but are cumbersome.
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- Diagrams useful for visualization.
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N-Field Inflation: One Vertex One Loop

- Biggest possible effect from $I$ propagator corrected by $J$ other fields.
- Contribution of a loop of this form is given by:

$$\langle \delta \phi^I_q(t) \delta \phi^I_{q'}(t) \rangle_{1L,1V} \supset \xi \int_{-\infty}^t \frac{dt_1}{aH^2} \langle \partial^{-m}(\delta \phi^I_p(t_1) \delta \phi^I_{p'}(t_1))\delta \phi^I_q(t)\delta \phi^I_{q'}(t) \rangle \times \sum_{J=1}^N \int d^3k \int d^3k' \langle \partial^{-n}(\delta \phi^J_k(t_1) \delta \phi^J_{k'}(t_1)) \rangle$$

- Loop integral scale free - independent of the external momentum: does not make a physical contribution.

Can any of the one-loop one vertex loops contribute?
Unlike $\lambda \phi^4$, can sneak the external scale into the integral:

In Fourier space:

$$\partial^{-n}(\delta \phi^J(t_1)\delta \phi^J(t_1)) \sim \frac{1}{(k+p)^n} \delta \phi^J_k(t_1)\delta \phi^J_p(t_1)$$

Contract $I$ fields with $J$ fields, obtain

$$\langle \delta \phi^I_q(t)\delta \phi^I_q(t) \rangle_{1L,1V} \sim \sum_{J=1}^{N} \mathcal{G} \int_{-\infty}^{t} \frac{dt_1}{aH^2} \langle \delta \phi^I_p(t_1)\delta \phi^I_k(t_1)\delta \phi^I_q(t)\delta \phi^I_q(t) \rangle$$

$$\times \int d^3k \int d^3p \frac{1}{(k+k')^n} \frac{1}{(p+p')^m} \langle \delta \phi^J_k(t_1)\delta \phi^J_p(t_1) \rangle$$

$$\sim \sum_{J=1}^{N} \delta^{IJ} \mathcal{G} \left( \frac{H^2}{M^2_{pl}} \right)^2 \int d^3k \frac{1}{k^3(k+q)^{n+m}}$$

$\partial^{-n}$ contracted across two fields yields an integral with a scale.
Interactions

- Four point interaction:

\[ H^{(4)}(t) \sim \int d^3x \frac{1}{aH^2} \partial^{-n}(\delta \phi^I \delta \phi^I) \partial^{-m}(\delta \phi^J \delta \phi^J) \]

- Three point interaction

\[ H^{(3)}(t) \sim \int d^3x \frac{1}{aH} \sqrt{2\epsilon_1} \delta \phi^I \delta \phi^J \delta \phi^J \]

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\[ \langle \delta \phi^I(t) \delta \phi^I(t) \rangle_{1L,1V} = -2\Im \int_{-\infty}^{t} dt_1 \langle H^{(4)}(t_1) \phi'(t) \delta \phi'(t) \rangle, \]

and

\[ \langle \delta \phi^I(t) \delta \phi^I(t) \rangle_{1L,2V} = -2\Re \left[ \left\langle \int_{-\infty}^{t} dt_2 \int_{-\infty}^{t_2} dt_1 H^{(3)}(t_1)H^{(3)}(t_2) \delta \phi'(t) \delta \phi'(t) \right\rangle \right] \]

\[ + \left\langle \int_{-\infty}^{t} dt_1 H_i^{(3)}(t_1)\delta \phi'(t) \int_{-\infty}^{t} dt_2 H_i^{(3)}(t_2) \right\rangle. \]
• Unlike $\lambda \phi^4$, can sneak the external scale into the integral:

$$\partial^{-n}(\delta \phi^J(t_1)\delta \phi^J(t_1)) \sim \frac{1}{(k+p)^n} \delta \phi^J_k(t_1)\delta \phi^J_p(t_1)$$

• In Fourier space:

$$\langle \delta \phi^l_q(t) \delta \phi^l_q(t) \rangle_{1L,1V} \sim \sum_{J=1}^N \int_{-\infty}^{t} \frac{dt_1}{aH^2} \langle \delta \phi^l_{p'}(t_1) \delta \phi^J_{k'}(t_1) \delta \phi^l_q(t) \delta \phi^l_q(t) \rangle$$

$$\times \int d^3k \int d^3p \frac{1}{(k+k')^n} \frac{1}{(p+p')^m} \langle \delta \phi^J_k(t_1) \delta \phi^l_p(t_1) \rangle$$

$$\sim \sum_{J=1}^N \delta^l_J \langle \frac{H^2}{M_{pl}^2} \rangle^2 \int d^3k \frac{1}{k^3(k+q)^{n+m}}$$

• $\partial^{-n}$ contracted across two fields yields an integral with a scale.
Hidden Gravitons

Non-appearance of the diagrams scaling with $N$ can be understood clearly as follows:

- The one loop, one vertex diagrams considered above really have gravitons secretly hidden inside them:
- The four point interaction:

\[
\begin{array}{c}
\text{J} \\
\downarrow \\
\text{I} \\
\uparrow \\
\text{J} \\
\end{array}
\]

is really mediated by a graviton:

\[
\begin{array}{c}
\text{J} \\
\downarrow \\
\text{I} \\
\uparrow \\
\text{J} \\
\end{array}
\]
In this gauge, the two one-loop one-vertex diagrams we drew above look like:

- Diagram that might scale like $N^2$, is a “balloon” diagram
- The propagator can’t change species in the 2nd diagram.
What about the two vertex loop?

- Expect to scale as $N$ due to $N$ species which can appear in the loop.
- Cannot be cheated out of this loop, due to topology the external momenta must flow through the loop.
- Six distinct diagrams which must be summed:

$$\langle \delta \phi^I(t) \delta \phi^J(t) \rangle_{1L,2V} = \frac{H^2}{2(2\pi)^3 q^3} N \epsilon_I \left[ \frac{2017}{120} \ln(q) \right]$$

- Note: $\epsilon_I$ is the slow roll parameter of one of the fields.
- The global slow roll parameter is:

$$\epsilon = N \epsilon_I$$

The two vertex loop also yields no bound on $N$. 
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$$\langle \delta \phi'(t) \delta \phi'(t) \rangle_{1L,2V}^{1L,2V} = \frac{H^2}{2(2\pi)^3 q^3} N \epsilon_1 \left[ \frac{2017}{120} \ln(q) \right]$$

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What about more loops?

No matter how many loops one goes to, no factors of $N$;

- Leading order 4-pt interaction is only non-zero for self interactions.
- Coupling in the 3-pt interaction has a $1/\sqrt{N}$ hidden inside of it.
- 3-pt interactions must occur in pairs
What about higher order terms?

- Interactions must appear in the action as scalars with respect to the field indices.
- With flat target space, fields pair with other fields (same index) or with background fields, i.e. $\dot{\phi}^I \delta \phi^I \delta \phi^J \delta \phi^J$ or $\dot{\phi}^I \phi^J \phi^K \delta \phi^I \delta \phi^J \delta \phi^K$.
- Interaction like $\delta \phi^I \delta \phi^J \delta \phi^K$, scaling like $N^3$ is forbidden.
- In terms of the background properties, $\ddot{\phi}^I, \dot{\phi}^I \sim 1/\sqrt{N}$.

We can’t do any better than the leading order scaling.
What about the other extreme:

- **Four point interaction:**

  \[
  H_4(t) \sim \int d^3x \frac{1}{aH^2} \left[ \partial^{-n}(\delta \phi \delta \phi + \sum_{J=1}^{M} \delta \sigma^J \delta \sigma^J) \partial^{-m}(\delta \phi \delta \phi + \sum_{K=1}^{M} \delta \sigma^K \delta \sigma^K) \right]
  \]

- **Three point interaction**

  \[
  H_3(t) \sim \int d^3x \frac{1}{aH} \sqrt{2\epsilon} \delta \phi \sum_{J=1}^{M} \delta \sigma^J \delta \sigma^J
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- **4-pt generates only one loop**
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Inflation with M-Spectator Fields: Loop Corrections

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What about the two vertex loop?

- One finds (Weinberg)

\[ P_k \sim \frac{1}{\epsilon} \frac{H^2}{M_{pl}^2} \left( 1 + M \epsilon \frac{\pi}{10} \frac{H^2}{M_{pl}^2} \ln(k) \right) \]

- Gives a bound:

\[ M < \frac{M_{pl}^2}{H^2} \frac{1}{\epsilon} \]

- Weaker than the gradient energy bound by \( \epsilon \)
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A Coherent Field?

- Non appearance of any scaling of $N$ in $N$-field inflation; really only one effective degree of freedom.
- Suggests that, effective degree of freedom: $\psi^2 = \sum_{j=1}^{N} \phi_j^2$
- For $m^2 \phi^2$ potentials; Lagrangian is:

$$\mathcal{L} = \frac{1}{2} (\partial \psi)^2 - \frac{1}{2} m^2 \psi^2 + \frac{1}{2} \psi^2 (\partial \Omega)^2,$$

- Looks like one inflaton, $\psi$, and $N - 1$ massless scalars, $\Omega$.
- Why don’t we recover Weinberg’s result?

$$P_k \sim \frac{1}{\epsilon} \frac{H^2}{M_{pl}^2} \left( 1 + N \epsilon \frac{\pi}{10} \frac{H^2}{M_{pl}^2} \ln(k) \right)$$
Short answer: this isn’t quite the same case as Weinberg

The fields $\Omega_i$ are not completely free; they satisfy

$$\sum_{i=1}^{N-1} \Omega_i = 1$$

$$\langle \psi^2 \rangle \sim \left( N\epsilon_1 \right)^{-1} \left( H^2 / M_{pl}^2 \right)$$

$\Omega_i$ are quickly damped to attractor; $\partial \Omega = 0$

What about loop corrections to perturbations?
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- Perturb:
  \[ \psi \rightarrow \bar{\psi} + Q \]
  \[ \Omega_i \rightarrow \bar{\Omega}_i + \omega_i \]

- Three new interactions generated: \( \bar{\Omega}_i QQ\partial \omega_1, QQ\partial \omega_1 \partial \omega_1 \) and \( \bar{\psi} Q\partial \omega_1 \partial \omega_1 \)
  - Choose, \( \bar{\Omega}_i = \{1, 0, \ldots, 0\} \); \( \bar{\Omega}_i QQ\partial \omega_1 \) gives at most one loop
  - \( QQ\partial \omega_1 \partial \omega_1 \) is scale free
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$$\mathcal{P}_k \sim \frac{1}{\epsilon} \frac{H^2}{M_{pl}^2} \left(1 + N \epsilon \frac{\pi}{10} \frac{H^2}{M_{pl}^2} \ln(k)\right)$$
What about the other extreme:

- **Four point interaction:**

\[
\mathcal{H}_4(t) \sim \int d^3x \frac{1}{aH^2} \left[ \partial^{-n} (\delta \phi \delta \phi + \sum_{J=1}^{M} \delta \sigma^J \delta \sigma^J) \partial^{-m} (\delta \phi \delta \phi + \sum_{K=1}^{M} \delta \sigma^K \delta \sigma^K) \right]
\]

- **Three point interaction**

\[
\mathcal{H}_3(t) \sim \int d^3x \frac{1}{aH} \sqrt{2\epsilon} \frac{\delta \phi}{\delta \phi} \sum_{J=1}^{M} \delta \sigma^J \delta \sigma^J
\]

- **4-pt generates only one loop**
A Coherent Field?

- Non appearance of any scaling of $N$ in $N$-field inflation; really only one effective degree of freedom.
- Suggests that, effective degree of freedom: $\psi^2 = \sum_{j=1}^{N} \phi_j^2$
- For $m^2 \phi^2$ potentials; Lagrangian is:

$$\mathcal{L} = \frac{1}{2} (\partial \psi)^2 - \frac{1}{2} m^2 \psi^2 + \frac{1}{2} \psi^2 (\partial \Omega)^2,$$

- Looks like one inflaton, $\psi$, and $N-1$ massless scalars, $\Omega$.
- Why don’t we recover Weinberg’s result?

$$\mathcal{P}_k \sim \frac{1}{\epsilon} \frac{H^2}{M_{pl}^2} \left( 1 + N \epsilon \frac{\pi}{10} \frac{H^2}{M_{pl}^2} \ln(k) \right)$$
Short answer: this isn’t quite the same case as Weinberg

- The fields $\Omega_i$ are not completely free; they satisfy

$$\sum_{i=1}^{N-1} \Omega_i = 1$$

- $\langle \psi^2 \rangle \sim (N\epsilon_I)^{-1} (H^2/M_{pl}^2)$

- $\Omega_i$ are quickly damped to attractor; $\partial \Omega = 0$

What about loop corrections to perturbations?
Loop Corrections

\[ \mathcal{L} = \frac{1}{2} (\partial \psi)^2 - \frac{1}{2} m^2 \psi^2 + \frac{1}{2} \psi^2 (\partial \Omega)^2, \]

- Perturb:
  
  \[ \psi \rightarrow \bar{\psi} + Q \]
  
  \[ \Omega_i \rightarrow \bar{\Omega}_i + \omega_i \]

- Three new interactions generated: \( \bar{\Omega}_i QQ \partial \omega_i \), \( QQ \partial \omega_1 \partial \omega_1 \) and \( \bar{\psi} Q \partial \omega_1 \partial \omega_1 \)

- Choose, \( \bar{\Omega}_i = \{1, 0, \ldots, 0\} \); \( \bar{\Omega}_i QQ \partial \omega_i \) gives at most one loop
- \( QQ \partial \omega_1 \partial \omega_1 \) is scale free
- Easily shown that \( \omega_i \propto a^{-3} \); loops quickly redshifted away
Summary

- **Bounds on $N$:**
  - Gradient energy bounds provide a constraint on the number of degrees of freedom in the early universe of:
    \[ N \ll \frac{M_{\text{pl}}^2}{H^2} \]
  - One loop quantum corrections to the power spectrum in $N$-flation provide no bound on $N$.
  - $N$-field inflation can be recast as a coherent single scalar field with one effective degree of freedom.
  - On the other extreme, single field field inflation with $N$ spectator fields yields a bound on $N$ which is weaker than the bound obtained from gradient energy considerations by $\epsilon$:
    \[ N \ll \frac{M_{\text{pl}}^2}{H^2} \frac{1}{\epsilon} \]
Acknowledgements

Many thanks to:

- Richard Easter and Eugene Lim
- Xingang Chen, Richard Holman, David Seery, Martin Sloth and Filippo Vernizzi.