Title: Enroute to a Quantum Dynamics for Causal Sets.

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Abstract: In causal set quantum gravity, spacetime is assumed to have a fundamental atomicity or discreteness, and is replaced by a locally finite poset, the causal set. After giving a brief review of causal sets, I will discuss two distinct approaches to constructing a quantum dynamics for causal sets. In the first approach one borrows heavily from the continuum to construct a partition function for causal sets. This is illustrated in a 2-d model of causal sets, in which typicality replaces quantum probabilities. The second approach is intrinsic, and uses the quantum measure formulation in which dynamics is described in the language of measure theory and observables are measurable sets in an event algebra. Using the example of complex percolation dynamics I will show that naive attempts to carry out this process for the quantum measure may not work. I will end by discussing possible ways to address this question.
EN ROUTE TO A QUANTUM DYNAMICS
FOR CAUSAL SETS

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OUTLINE:

- REVIEW OF CAUSAL SET THEORY
- A 2D MODEL FOR CAUSAL SETS
- A QUANTUM MEASURE FORMULATION
- COMPLEX PERCOLATION
- OPEN QUESTIONS
OUTLINE:

- REVIEW OF CAUSAL SET THEORY
- A 2D MODEL FOR CAUSAL SETS
- A QUANTUM MEASURE FORMULATION
- COMPLEX PERCOLATION
- OPEN QUESTIONS
CAUSAL SET THEORY

\[(M, g) \rightarrow (M, \prec)\]

CAUSAL SPACETIME CAUSAL STRUCTURE

\[(M, \prec)\] is a POSET:

(i) \(x \nless x\) : IRREFLEXIVE

(ii) \(x \less y, y \less z \Rightarrow x \less z\) : TRANSITIVE.

\[(M, \prec)^*\] determines conformal class [97].

* (given certain causality order) - Mccarthy, King, Hawking - Malament.

CAUSAL STRUCTURE + VOL. ELEMENT = GEOMETRY.

\(\uparrow\) CAUSAL SET HYPOTHESIS: SPACETIME IS REPLACED BY A LOCALLY FINE POSET, THE CAUSAL SET

(iii) \(|I(x, y)| < \infty, I(x, y) = \{ z | x \less z \less y \}\)

\(\Downarrow\) HYPOTHESIS OF FUNDAMENTAL DISCRETENESS

ORDER + CARDINALITY = GEOMETRY.
CAUSAL SPACETIME  CAUSAL STRUCTURE

\((M, \leq)\) is a POSET:

1. \(x \leq x\) : IRREFLEXIVE
2. \(x \leq y, y \leq z \Rightarrow x \leq z\) : TRANSITIVE.

\((M, \leq)\) determines CONFORMAL CLASS \(\mathfrak{g}\).

* (GIVEN CERTAIN CAUSALITY CONDITIONS) - Mccarthy, King, Hawking, MacClellan.

CAUSAL STRUCTURE + VOL. ELEMENT = GEOMETRY.

\[\text{CAUSAL SET HYPOTHESIS: SPACETIME IS REPLACED BY A LOCALLY FINE}
\text{POSET, THE CAUSAL SET}\]

\[\text{III:} \quad |I(x, y)| < \infty, I(x, y) = \{z \mid x < z < y\}\]

LOCAL FINITENESS

HYPOTHESIS OF FUNDAMENTAL DISCRETENESS

ORDER + CARDINALITY \sim GEOMETRY.
REGULAR DISCRETISATION

\[ N = \frac{V}{V_0} \Rightarrow \text{FUNDAMENTAL VOLUME CUT-OFF} \]

\[ |I(x, y)| = \pi \]

\[ |I(x, z)| = 2 \]
\[ N = \left( \frac{V}{V_0} \right) \]
REGULAR DISCRETISATION

\[ |I(x,y)| = \pm \]
\[ |I(x,\bar{z})| = 2 \]

\( N = \frac{V}{V_0} \)

\( \Rightarrow \text{FUNDAMENTAL VOLUME CUT-OFF} \)
Causal discretisation

Random lattice

\[ p_n(N) = \left( \frac{V}{V_0} \right)^n e^{-\frac{N}{V_0}} n! \]

\[ \langle N \rangle = \frac{V}{V_0} \]

This preserves Lorentz invariance

- Bombelli, Henson & Sorkin.

Faithful embedding

\[ \Phi: (\mathbb{C}, \prec) \rightarrow (M, g) \]

(Fundamental) (Approximation)

(i) \( a \prec b \iff \Phi(a) \prec \Phi(b) \) [Order preserving]

(ii) \( \Phi(c) \subset M \) is a Poisson sprinkling

\[ \Psi \]

Is not spacetime interval in \( M^2 \)
RANDOM LATTICE

\[ P_n(N) = \left( \frac{V}{V_0} \right)^n e^{-\frac{V}{V_0}} \frac{n!}{n!} \]

\[ \langle N \rangle = \frac{V}{V_0} \]

THIS PRESERVES LORENTZ INVARINCE

-_BOMBEII, HENSON & SOLOKIN_

FAITHFUL EMBEDDING

\[ \Phi : (\mathcal{C}, \leq) \to (M, g) \]

(FUNDAMENTAL APPROXIMATION)

(i) \( a \leq b \iff \Phi(a) \leq \Phi(b) \) [ORDER-PRESERVING]

(ii) \( \Phi(c) \in M \) IS A POISSON SPRINKLING

\[ \Psi \]

SPACE TIME INTERVAL IN M²

IS NOT A FAITHFUL EMBEDDING.
KINEMATICS - WHAT WE KNOW

- DIMENSION : D.A. MYERS, J. MYRHEIM
- TOPOLOGY : S. MAJOR, D. RIDEOUT & S. BUCKA
- DISTANCE : C. BRIGHTWELL, R. GREGORY, R. RIDEOUT & P. WALDON
- D'ALGEBERTIAN : R.D. SORKIN
- QUANTUM FIELD THEORY : S. JOHNSTON
- ACTION : D. BENINCASA & F. DOWKER
CAUSAL SET DYNAMICS

0 HISTORIES FRAMEWORK MOST SUITABLE

\[ \{ (M, g) \} \rightarrow \{ (G, \mathcal{C}) \} \]

_\underline{\text{COLLECTION OF ALL CONTINUUM SPACETIMES}}  _\underline{\text{COLLECTION OF ALL CAUSAL SETS}}

MOST CAUSAL SETS ARE NOT MANIFOLD-LIKE:

\[ \frac{N}{4}, \frac{N}{2}, \frac{N}{4} \]

\_\underline{\text{SUCH CAUSAL SETS DOMINATE } \mathcal{C}_{1,\frac{3}{2}} \text{ AS } N \rightarrow \infty.}\]

DYNAMICS SHOULD OVERCOME THIS "ENTROPY"

STOCHASTIC CLASSICAL/GROWTH MODELS OF CAUSAL SETS DO OVERCOME THIS ENTROPY.

- Rideout & Sorkin.
CAUSAL DISCRETISATION

RANDOM LATTICE

\[ P_n(N) = \left( \frac{N}{V_0} \right)^n e^{-\frac{n(V_0)}{n!}} \]

\[ \langle N \rangle = \frac{V}{V_0} \]

THIS PRESERVES

LORENTZ INVARIANCE

- BOMBELLI, HENSON & SOREKIN.

FAITHFUL EMBEDDING

\[ \Phi : (\mathcal{L}, \prec) \to (M, g) \]

(FUNDAMENTAL) (APPROXIMATION)

(i) \( a \prec b \iff \Phi(a) \prec \Phi(b) \) [ORDER PRESERVING]

(ii) \( \Phi(c) \in M \) IS A POISSON SPRINKLING

\[ \psi \]

IS NOT

A FAITHFUL EMBEDDING

SPACE TIME INTERVAL IN \( M^2 \)
A 2D toy model:

Class of 2D interval spacetimes

\((\mathbb{I}, g)\), \(g\) is conformally flat.

\(\mathbb{R}^A\) (Lorentzian analogue of the disk.)

\[
\int_{\mathbb{I}} g_{ij} \left( \frac{1}{8\pi G} \right) \left( \frac{1}{2} \int_{\mathbb{I}} k dS - \frac{1}{\Theta} \int_{\mathbb{I}} \Theta j - \frac{1}{16\pi G} \right) \frac{1}{8\pi G} \\
\text{const. (Lorentzian Gauss-Bonnet) theory fixes} \ \nu_i.
\]

\(\mathbb{\hat{z}}_v = (\ ) \int [idg]\)

"Discretise"

\(\mathbb{\hat{z}}_v = (\ ) \leq 1\)

What causal sets correspond to conformally flat 2D geometries?
CLASS OF 2D INTERVAL SPACETIMES 
\[(I, g)\], \(g\) is conformally flat. 
\(\equiv\) (Lorentzian analogue of the disk.)

\[\Sigma g = \frac{1}{16\pi G} \left( \int R dv - \frac{1}{8\pi G} \int \left( \frac{\partial S}{\partial j} - 2j - \frac{\partial j}{\partial v} \right) \right) \]

\(\text{const. (Lorentzian Gauss-Bonnet) theory fixes } \psi_i.\)

\[\therefore \quad \xi_{\psi} = \left( 1 \right) \int d[\rho g] \]

"DISCRETISE"

\[Z^N = \left( \frac{\xi_{\psi}}{\left( \xi_{\psi} \right)^2} \right) \leq 1.\]

Causal sets correspond to conformally flat 2D geometries.
2D ORDERS

\[ \Phi: (C, \leq) \to (I, \geq) \]

\[ \begin{align*}
V &= (v_1, v_3, v_2) \\
U &= (u_1, u_3, u_2)
\end{align*} \]

\[ \text{each is linearly ordered.} \]

\[ \Phi(C) \]

\[ V \cap U \]

- IF \( \Phi \) IS FAITHFUL THEN \( \Phi(C) \) IS A 2D ORDER

\[ \therefore \mathcal{N} = (\leq) \leq 1 \text{ 2D-ORDERS.} \]

- (i) NOT ALL 2D-ORDERS ARE MANIFOLD LIKE

- (ii) 2D-ORDERS ARE "SPATIALLY" TOPOLOGICALLY TRIVIAL.
IF $\tilde{\Omega}(N)$ is the uniform distribution over 2D orders, then $\tilde{\Omega}(N) \to \Omega(N)$ in random order, as $N \to \infty$.

- El-Zahar, Sauber, Winkler.

$S = \{e_1, e_2, \ldots, e_N\}$ admits $N!$ linear orderings $U, V$: are chosen independently & randomly from these $N!$ orderings.

- Poisson sprinkling into $M^2$ is a random order!

Dominant contribution to $Z_N$ as $N \to \infty$ is flat spacetime.

"Typical" 2D order is faithfully embeddable into the flat interval as $N \to \infty$. 
**The Benincasa-Dowker Action**

\[
\begin{align*}
S^{(2)}[C] &= N - 2N_1 + 4N_2 - 2N_3 \\
S^{(4)}[C] &= N - N_1 + 9N_2 - 16N_3 + 8N_4
\end{align*}
\]

- \( N \) = \# of elements
- \( N_1 \) = \# of links
- \( N_2 \) = \# of 2-element inclusive orders
- \( N_3 \) = \# of 4-element inclusive orders

\[ N_4 = \]
THE QUANTUM MEASURE FORMULATION

- View quantum theory as a generalized stochastic theory.
- Quantum dynamics is described as a quantum measure space.

\[
\mu(A) \neq \mu(A) + \mu(B).
\]

\[
\mu(A \cup B \cup C) = \mu(A \cup B) + \mu(A \cup C) + \mu(B \cup C) - \mu(A) - \mu(B) - \mu(C).
\]

- R. Sorkin.

\((\Omega, \mathcal{A}, \mu) : \text{quantum measure space}\)

\[
\mu : \mathcal{A} \to [\mathbb{R}^+ \cup \{0\}]
\]
The Quantum Measure Formulation

- View Quantum Theory as a Generalised Stochastic Theory

- Quantum Dynamics is described as a Quantum Measure Space.
  \[ \mu(A) > 0 \]
  \[ \mu(A \cup B) \neq \mu(A) + \mu(B) \]
  \[ \mu(A \cup B \cup C) = \mu(A \cup B) + \mu(A \cup C) + \mu(B \cup C) - \mu(A) - \mu(B) - \mu(C) \]

  - R. Sorkin

\( (\Omega, A, \mu) : \text{Quantum Measure Space} \)

- Space of Histories \( \rightarrow \) Event Algebra
- \( \mu : A \rightarrow \mathbb{R}^+ \cup \{ 0 \} \)
THE QUANTUM MEASURE FORMULATION

- VIEW QUANTUM THEORY AS A GENERALISED STOCHASTIC THEORY

- QUANTUM DYNAMICS IS DESCRIBED AS A QUANTUM MEASURE SPACE.

\[ \mu(A) > 0 \]

\[ \mu(A \cup B) \neq \mu(A) + \mu(B) \]

\[ \mu(A \cup B \cup C) = \mu(A \cup B) + \mu(A \cup C) + \mu(B \cup C) - \mu(A) - \mu(B) - \mu(C) \]

-R. Sorkin

\[ (\Omega, A, \mu) : \text{QUANTUM MEASURE SPACE} \]

\[ \text{SPACE OF HISTORIES} \xrightarrow{\mu} \text{QUANTUM MEASURE} \]

\[ \mu : A \rightarrow [R^+ \cup 0] \]
STOCHASTIC THEORY

- Quantum dynamics is described as a quantum measure space.
  \[ \mu(A) \neq 0 \]
  \[ \mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B) \]
  \[ \mu(A \cup B \cup C) = \mu(A \cup B) + \mu(A \cup C) + \mu(B \cup C) - \mu(A) - \mu(B) - \mu(C) \]
  - R. Sorkin.

\((\Omega, A, \mu) : \text{quantum measure space}\)

- Space of histories \(\mathcal{H}\)
- Event algebra \(\mathcal{E}\)
- Quantum measure \(\mu: \mathcal{F} \rightarrow \mathbb{R}^+ \cup \{0\}\)
The Random Walk

\[ P(\gamma) = \frac{1}{8} \]

\( \Omega \) = set of \( \infty \) time paths.

\( \text{cyl}(\gamma) \) = set of paths in \( \Omega \) with \( \gamma \) as first 3 steps.

\[ P(\text{cyl}(\gamma)) = P(\gamma) = \frac{1}{8} \]

\{ \text{cyl}(\gamma) \} \) generates an event algebra

\( \uparrow \)

finite unions, finite intersections, & complementation.

\[ P : A \rightarrow [0,1] \] & \((\Omega, A, P)\) is a prob. space

all finite time events.

Infinite time qn: probability for return to origin.

\[ 0 \leq \lim_{n \to \infty} \mathbb{P}[S_n = \Omega] \]

\( \leq 1 \) \( \forall \omega \in \Omega \)

extended: \((\Omega, A, P) \rightarrow (\Omega, \mathcal{F}, \overline{P})\)

sigma algebra, carathéodory-extn. then

observables are measurable sets.
\[ N = \left( \frac{Y}{V_0} \right) \]

\[ \Omega \]

\[ Y \rightarrow P(cyl(\Theta)) \]
EXTENSION OF A MEASURE

$(\Sigma, A, \mu)$

$\Sigma$: COLLECTION OF SUBSETS OF $\Sigma$
Closed under finite unions, intersections & complementation.

$\Sigma(A)$: Smallest $\sigma$-algebra containing $\Sigma$. It is closed under countable set operations, $\bigcap, \bigcup$.

$\bar{\Sigma}$ $(\Sigma, \Sigma(A), \bar{\mu})$ is an extension of $(\Sigma, A, \mu)$ if then

$\bar{\mu}|_{A} = \mu$.

If $\bar{\mu}$ is unique then $(\Sigma, A, \mu)$ has a unique extension $(\Sigma, \Sigma(A), \bar{\mu})$.
EXTENSION OF A MEASURE

\((\Omega, \mathcal{A}, \mu)\)

\(\mathcal{A}\): collection of subsets of \(\Omega\) closed under finite unions, intersections & complementation.

\(\mathcal{S}(\mathcal{A})\): smallest \(\sigma\)-algebra containing \(\mathcal{A}\). It is closed under countable set operations, \(\cap, \cup, \sigma\).

If \((\Omega, \mathcal{S}(\mathcal{A}), \overline{\mu})\) is an extension of \((\Omega, \mathcal{A}, \mu)\) then

\[ \overline{\mu} |_{\mathcal{A}} = \mu. \]

If \(\overline{\mu}\) is unique then \((\Omega, \mathcal{A}, \mu)\) has a unique extension \((\Omega, \mathcal{S}, \overline{\mu})\).
QUANTUM MEASURE

SHARED WITH CLASSICAL SYSTEMS

E4: DISCRETE TIME EVOLUTION FOR INFINITE UNITARY SYSTEMS

\[ T = (T_1, T_2, \ldots, T_n) \quad T_i \in \mathbb{C}^{d \times d}, \ldots, \mathbb{C}^{d \times d} \]

\[ \text{Cyl}(Y) \text{ GENERATES } A. \]

\[ M_Y(Y) = (U^*)^n \hat{C}_T \Psi_0 \]

\[ \hat{C}_T = \hat{P}_n \hat{P}_{n-1} \ldots \hat{P}_1 \]

FINITE TIME QUESTIONS CAN BE ANSWERED

BUT WHAT ABOUT INFINITE TIME?

IN OTHER WORDS DOES \((S_2, A, \mu)\) EXTEND TO \((\mathcal{A}, \Psi, \bar{\mu})\) ?

DECOHERENCE FUNCTIONAL:

1. \( D(\alpha \cup B, Y + B \mathcal{U}) = D(\alpha, Y) + D(\alpha, Y) + D(\alpha, \mathcal{B}) + D(\alpha, \mathcal{E}) \)

2. \( D(\alpha, B) = D^*(\alpha, B) \)
**Quantum Measure Space**

\((\Sigma, \lambda, \mu)\) 

\(\rightarrow\) Quantum Measure

Shared with Classical Systems

**Example:** Discrete Time Evolution for Infinite Unitary Systems

\(\tau = (\tau_1, \tau_2, \ldots, \tau_n) \quad \tau_i \in \{0, 1, \ldots, N\}^2\)

\(\{\text{Cyl}(\gamma)\}\) generates a class operator

\(M_\nu(\gamma) \equiv (U^\dagger)^n \hat{C}_\tau \nu_0\)

\(\hat{C}_\tau = \hat{p}_{\tau_1} \hat{p}_{\tau_2} \ldots \hat{p}_{\tau_n}\)

Finite Time Questions Can Be Answered But What About Infinite Time?

In other words, does \((\Sigma, \lambda, \mu)\) extend to \((\text{Arran}(\Sigma, \lambda, \overline{\mu})\) ?

**Decoherence Functional:**

\(D(x \cup y) \leq D(x, y) + D(x, \delta) + D(y, \delta) + D(\delta, \delta)\)

\(\delta = (\beta, \overline{\mu}) \rightarrow D^*(\beta, \overline{\mu})\)
THE DECOHERENCE FUNCTIONAL

D : \mathbb{A} \times \mathbb{A} \rightarrow \mathbb{C}.

(i) \ D(\alpha \mu \beta, \beta' \mu \tau) = D(\alpha, \beta') + D(\alpha, \tau) + D(\beta, \beta') + D(\beta, \tau)

(ii) \ D(\alpha, \beta) = D^{*}(\beta, \alpha) : \text{HERMITIAN}.

(iii) \text{STRONGLY POSITIVE} = M_{ij} = D(\alpha_i, \alpha_j)

\text{HAS & NON-NEGATIVE E. VALUES FOR ANY}

\text{FINITE COLLECTION } \mathcal{B}\{\alpha_i\}.

\text{STD. QM:}

D(\gamma_1, \gamma_2) = e^{i \Delta\gamma_1} e^{i \Delta\gamma_2} \delta(\gamma_1(t) - \gamma_2(t))

\mu(\alpha) = D(\alpha, \alpha) > 0.
THE DECOHERENCE FUNCTIONAL

\[ D : \mathbb{R}^d \times \mathbb{R}^d \rightarrow \mathbb{C}. \]

\text{(i)} \quad D(\alpha + \beta, \beta') = D(\alpha, \beta') + D(\alpha, \beta) + D(\beta, \beta') + D(\beta, \beta')

\text{(ii)} \quad D(\alpha, \beta) = D(\beta, \alpha) : \text{hermitian}.

\text{(iii)} \quad \text{Strongly positive} \Rightarrow M_{ij} = D(\alpha_i, \alpha_j)

\quad \text{has \& non-negative eigenvalues for any finite collection } \{\alpha_i\}.

\text{STD. QM:}

\[ D(\tau_1, \tau_2) = e^{iS[\tau_1] - iS[\tau_2]} \delta(\tau_1(t) - \tau_2(t)) \]

\[ \tau_1 \downarrow \tau_2 \]

\[ \mu(\alpha) = D(\alpha, \alpha) > 0. \]
THE QUANTUM VECTOR MEASURE

\( \mu_v: A \rightarrow \mathcal{H}_A \)

\( \mathcal{H}_A \text{ HISTORIES HILBERT SPACE} \)

- Dowker, Johnston & Sorkin

\( V: \text{VECTOR SPACE OF FNS. } U: A \rightarrow C \)

S.T. \( U(a) \neq 0 \text{ only for finite # of } a. \)

\( \langle u, v \rangle = \sum_{x \in A} D(x, a) u(x) v(a). \)

\( \mathcal{H}_A \) obtained after quotienting by zero norm vectors & Cauchy completing the space

\( \mu_v(\omega) = [x_\omega] \in \mathcal{H}_A \)

\( \Downarrow \) CHARACTERISTIC FUNCTION.

\( \| \mu_v(\omega) \|^2 = D(x, a). \)

\( \mu_v \text{ IS ADDITIVE.} \)

AND...

\( (\Omega, \Sigma, \mu_v) \text{ EXTENDS } (\Omega, \Sigma, \overline{\mu}) \)

\( \Downarrow \) \( \mu_v \) SATISFIES CERTAIN CONVERGENCE CONDNS.
$<u, v> = \sum_{i=1}^{n} u_i \overline{v_i}$

$\mathcal{H}_A$, obtained after quotienting by zero norm vectors & Cauchy completing the space

$\mu_v(x) = [x_v] \in \mathcal{H}_A$

$\mu_v$ characteristic function.

$\|\mu_v(x)\|_2^2 = D(x, x_v)$

$\mu_v$ is additive.

And...

$(\Omega, A, \mu_v) \xrightarrow{\text{extends}} (\Omega, \Sigma, \mu_v)$

If $\mu_v$ satisfies certain convergence conditions.
GROWTH MODELS FOR CAUSAL SETS

process generates labelled causal sets.

$3_{cyl}(C_n)$ generates $\Lambda$.

If $\mu$ is a probability measure, then

$(\Omega, \Sigma, \mu)$ is the extension.

set of labelled causal sets

covariant sets obtained from a quotient of $\Sigma$, not $\Lambda$.

extension of $\mu$ on $\Lambda$ to a $\overline{\mu}$ on $\Sigma$

crucial for covariant observables.
**Complex Percolation**

\[ P \cap Q \text{ complex } \quad P + Q = 1. \]

IF \( D(C_1, C_2) = A^c(C_1) \cup C_2 \) THEN \( \mathcal{A}_n \propto C \).

Extension of \( \mu^o \) on \( A \) to \( \mu^o \) on \( \Xi \) requires that

\[ |\mu^o(\alpha)| = \sup \frac{\sum \|\mu^o(\alpha_i)\|}{n(\alpha)} < \infty \]

**Bounded Variation.**

**Eq:** \( |P| > 1 \):

\[ \Sigma = (n\text{-chain}) \cup (n\text{-chain})^c. \]

\[ \mu^o(\Sigma) = P^{n-1} \]

\[ |\mu^o(\Sigma)| > |P|^n - 1. \]

**Or:** \( |Q| > 1 \):

\[ \Sigma = (n\text{-antichain}) \cup (n\text{-antichain})^c. \]

\[ \mu_{P\cap Q}(\Sigma) = Q^{n(n-1)} \]

\[ |(\mu^o)(\Sigma)| > |Q|^n - 1. \]

**For:** \( |P|, |Q| < 1 \) as well \( \mu^o \) is not of bounded variation except for \( P \cap Q \) real.

Does this mean that complex percolation cannot be made cover?
\textbf{Comple\textbf{x}}

\[ p + q = 1. \]

If \( D(c_1, c_2) = A^\ast(c_1) \cdot A(c_2) \) then \( \mathbb{H}_{n-1} \subset \mathbb{C} \).

Extension of \( \mu_v \) on \( X \) to \( \overline{\mu_v} \) on \( \Xi \) requires that

\[ \forall \lambda \in \Xi, \| \mu_v(\lambda_i) \| < \infty \]

\[ \mu_v(\lambda) = \sup \| \mu_v(\lambda_i) \| \text{ or } \| \mu_v(\lambda_i) \| < \infty \]

\[ \text{bounded variation.} \]

**Example:** \( |p| > 1, \quad \lambda_i = (p\text{-chain}) \cup (\lambda\text{-chain})^c \cdot \mu_v(\lambda_i) = p^{n-1} \)

\[ \therefore |\mu_v(\lambda)| > |p|^{n-1}. \]

\[ \text{or: } \quad |q| > 1, \quad \lambda_i = (p\text{-antichain}) \cup (\lambda\text{-antichain})^c \cdot \mu_v(\lambda_i) = q^{n-1} \]

\[ \therefore |\mu_v(\lambda)| > |q|^{n-1}. \]

For \( |p|, |q| < 1 \) as well \( \mu_v \) is not of bounded variation. Except for \( p, q \text{ real} \).

Does this mean that complex percolation cannot be made covariant?
Some covariant observables can be found, even though there is no extension.

\[ \alpha = \text{set of causal sets in } \mathcal{S} \text{ with which are original} \]
\[ \mu_\nu(x) = \prod_{n=1}^{\infty} (1 - q^n) \quad \text{converges for } |q| < 1 \]

\[ \beta = \text{set of causal sets in } \mathcal{S} \text{ with a "stem" such that the maximal element of the stem is a post,} \]
\[ \lambda_\nu(y) = \prod_{n=1}^{\infty} p^n (1 - q^n) \]

- Perhaps bounded variation is too strong?
- Perhaps we can use the structure of cylinder sets to define a conditional convergence
GROWTH MODELS FOR CAUSAL SETS

RIDEOUT & SORKIN

W. MARTIN & O'CONNOR

PROCESS GENERATES LABELLED CAUSAL SETS.

$\mathcal{Cyl}(C_n)$ generates $A$.

If $\mu$ is a probability measure, then

$(\mathcal{L}, \Sigma, \mu)$ is the extension.

A set of labelled causal sets

Covariant sets obtained from a quotient of $\Sigma$, not $A$.

Extension of $\mu$ on $A$ to $\overline{\mu}$ on $\Sigma$.

Crucial for covariant observables.
Some covariant observables can be found, even though there is no extension.

\[ \alpha = \text{set of causal sets in } S_2 \text{ which are original} \]
\[ \mu_v(\alpha) = \prod_{i} (1 - q^n) \text{ converges for } 1 < q < 1. \]

\[ \beta = \text{set of causal sets in } S_2 \text{ with a "stem"} \]
\[ \mu_v(\beta) = p^2 q \times \prod_{i} (1 - q^n) \]

- Perhaps bounded variation is too strong

Suggestion:
- Perhaps we can use the structure of cylinder sets to define a conditional convergence.