Law without Law: Entropic Dynamics

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Laws of Nature Workshop
Perimeter Institute 05/2010
Question:
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Do the laws of Physics reflect Laws of Nature?
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Do the laws of Physics reflect Laws of Nature? Or...

Are they rules for processing information about Nature?
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Our objective:

To derive Quantum Theory as Entropic Dynamics
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Do the laws of Physics reflect Laws of Nature? Or...

Are they rules for processing information about Nature?

Our objective:

To derive Quantum Theory as Entropic Dynamics and discuss the implications for the theory of time.
Entropic Inference
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prior

$q$

constra
Entropic Inference

\[ S[p, q] = -\int dx \ p(x) \log \frac{p(x)}{q(x)} \]
Entropic Inference

$$S[p, q] = - \int dx \ p(x) \log \frac{p(x)}{q(x)}$$

Maximize $S[p, q]$ subject to the appropriate constraints.
Entropic Inference

\[ S[p, q] = -\int dx \, p(x) \log \frac{p(x)}{q(x)} \]

Maximize \( S[p, q] \) subject to the appropriate constraints.

(MaxEnt, Bayes' rule and Large Deviations are special cases.)
Step 1: The Statistical Model
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configuration space with metric

\[ v_{ab} = \frac{\delta_{ab}}{\sigma^2} \]
Step 1: The Statistical Model

hidden variables

configuration space with metric

\[ \gamma_{ab} = \frac{\delta_{ab}}{\sigma^2} \]
Step 1: The Statistical Model

hidden variables

$y$

$p(y|\mathbf{x})$

configuration space with metric

$\gamma_{ab} = \frac{\delta_{ab}}{\sigma^2}$
Step 1: The Statistical Model

\[ \mathcal{M} \]

\[ X \]

\[ p(y \mid x) \]

statistical manifo

configuration space

with metric

\[ \gamma_{ab} = \frac{s_{ab}}{\sigma^2} \]
Step 2: Entropic Dynamics
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Changes happen gradually.

Maximize

$$S_J[P, Q] = - \int dx' dy' P(x', y' \mid x) \log \frac{P(x', y' \mid x)}{Q(x', y' \mid x)}$$
Step 2: Entropic Dynamics

Changes happen gradually.

Maximize

\[ S_f[P, Q] = -\int dx' dy' P(x', y' | x) \log \frac{P(x', y' | x)}{Q(x', y' | x)} \]

uniform
Step 2: Entropic Dynamics

Changes happen gradually.

Maximize

$$S_J[P, Q] = -\int dx' dy' P(x', y' | x) \log \frac{P(x', y' | x)}{Q(x', y' | x)}$$

$$P(x' | x) p(y' | x') \in \mathcal{M}$$

Short steps:  $$\langle \Delta l^2 \rangle = \langle \gamma_{ab} \Delta x^a \Delta x^b \rangle = \lambda^2(x)$$
The result:
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\[ P(x' | x) = \frac{1}{\zeta} \exp \left[ S(x') - \frac{1}{2} \alpha(x) \Delta \ell^2 \right] \]
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The result:

\[ P(x' \mid x) = \frac{1}{\zeta} \exp \left[ S(x') - \frac{1}{2} \alpha(x) \Delta l^2 \right] \]

where \[ S(x') = -\int dy' p(y' \mid x') \log \frac{p(y' \mid x')}{q(y')} \]

Displacement:

\[ \Delta x = \Delta \bar{x} + \Delta w \]

Expected drift:

\[ \Delta \bar{x}^a = \frac{1}{\alpha(x)} \gamma^{ab} \partial_b S(x) \]
Step 2: Entropic Dynamics

Changes happen gradually.

Maximize

$$S_{\gamma}[P, Q] = -\int dx' dy' P(x', y' | x) \log \frac{P(x', y' | x)}{Q(x', y' | x)}$$

Uniform

$$P(x' | x) \ p(y' | x') \in \mathcal{M}$$

Short steps: $$\langle \Delta \ell^2 \rangle = \langle \gamma_{ab} \Delta x^a \Delta x^b \rangle = \lambda^2 (x)$$
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\[ P(x' \mid x) = \frac{1}{\zeta} \exp \left[ S(x') - \frac{1}{2} \alpha(x) \Delta l^2 \right] \]

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\[ S(x') = -\int dy' p(y' \mid x') \log \frac{p(y' \mid x')}{q(y')} \]

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Expected drift:

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Fluctuations:

\[ \langle \Delta w^a \Delta w^b \rangle = \frac{1}{\alpha(x)} \gamma^{ab} \]
Step 3: Entropic Time
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Time is introduced to keep track of the accumulation of many small changes.
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Time is introduced to keep track of the accumulation of many small changes.

The foundation of any notion of time is dynamics.

\[ P(x') = \int dx' P(x', x) = \int dx' P(x' | x) P(x) \]

(1) Introduce the notion of an instant

\[ \rho(x', t') = \int dx' P(x' | x) \rho(x, t) \]
(2) Introduce the notion of *interval* between instants
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For large $\alpha$ the dynamics is all in the fluctuations:
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For large $\alpha$ the dynamics is all in the fluctuations:

$$\langle \Delta w^a \Delta w^b \rangle = \frac{1}{\alpha(x)} \gamma^{ab}$$

Define duration so that motion looks simple:
The result is a Fokker-Planck eq.:

$$\partial_t \rho = -\partial_a (\rho v^a)$$
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\[ v^a = \frac{\sigma^2}{\tau} \partial^a \phi \]

\[ \phi(x, t) = S(x) - \log \rho^{1/2}(x, t) \]

\[ v^a = b^a + u^a \]
The result is a Fokker-Planck eq.: \[ \partial_t \rho = -\partial_a (\rho v^a) \]

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\[ \phi(x, t) = S(x) - \log \rho^{1/2}(x, t) \]

\[ v^a = b^a + u^a \]

drift velocity: \[ b^a = \frac{\sigma^2}{\tau} \partial_a S \]

osmotic velocity: \[ u^a = -\frac{\sigma^2}{2\tau} \frac{\partial^a \rho}{\rho} \]
But this is just diffusion, not quantum mechanics!
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A wave function requires **two** degrees of freedom: \( \Psi(x, t) = \rho^{1/2} e^{i\phi(x, t)} \)
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\[ \Psi(x, t) = \rho^{1/2} e^{i\phi(x, t)} \]
Step 4: Manifold dynamics?
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Energy conservation [Nelson (1979), Smolin (2006)]
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Energy conservation \text{[Nelson (1979), Smolin (2006)]}

\[ E = \int d^3 x \rho \left[ \frac{1}{2} m v^2 + \frac{1}{2} m u^2 + V(x) \right] \]

where \[ m = \frac{2A}{\sigma^2} \]
The result: two coupled equations
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1) Fokker-Planck/diffusion equation

\[ \dot{\rho} = - \frac{\hbar}{m} \partial^a (\rho \partial_a \phi) \]
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1) Fokker-Planck/diffusion equation

\[ \dot{\rho} = - \frac{\hbar}{m} \partial^a (\rho \partial_a \phi) \]

2) energy conservation + diffusion

\[ \hbar \dot{\phi} + \frac{\hbar^2}{2m} (\partial_a \phi)^2 + V - \frac{\hbar^2}{2m} \nabla^2 \rho^{1/2} = 0 \]
Combine $\rho$ and $\phi$ into $\Psi = \rho^{1/2} e^{i\phi}$
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to get Quantum Mechanics, $i\hbar \dot{\Psi} = -\frac{\hbar^2}{2m} \nabla^2 \Psi + V \Psi$
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This includes the classical limit $F = ma$
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This includes the classical limit $F = ma$

Let $S_{HJ} = \hbar \phi$ then $\dot{S}_{HJ} + \frac{1}{2m} (\partial_a S_{HJ})^2 + V = 0$
More on entropic time
More on entropic time

A clock follows a classical trajectory.
More on entropic time
More on entropic time

For the composite system of particle and clock:
More on entropic time

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More on entropic time

For the composite system of particle and clock:
Entropic time vs "physical" time?
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We observe correlations at an instant.
Entropic time vs "physical" time?

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We do not observe the "absolute" order of the instants.

Entropic time is all we need.
There is an arrow of entropic time.
Conclusion:
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On Laws of Physics vs. Laws of Nature:
Conclusion:

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Entropic inference can lead to Laws of Physics.
Conclusion:

On Laws of Physics vs. Laws of Nature:

Entropic inference can lead to Laws of Physics.

It provides an alternative to Action Principles.
Conclusion:

On Laws of Physics vs. Laws of Nature:

Entropic inference can lead to Laws of Physics. It provides an alternative to Action Principles. The natural order/arrow of inference is entropic time...
The Big Picture:
Conclusion:

On Laws of Physics vs. Laws of Nature:

Entropic inference can lead to Laws of Physics.
It provides an alternative to Action Principles.
The natural order/arrow of inference is entropic time...
...and this is all one needs.
The Big Picture:
The Big Picture:

- inference
- society
- physics
- life
The Big Picture:

inference

society

physics

life