Abstract: Basic epistemological considerations suggest that the laws of nature should be scale invariant and no fundamental length scale should exist in nature. Indeed, the standard model action contains only two terms that break scale invariance: the Einstein-Hilbert term and the Higgs mass term. We give a simple introduction to Weyl's 1918 scale invariant gravity based on basic epistemology and discuss the three main objections put forth by Einstein: 1) the hydrogen spectrum depends on their previous history of the atom (something which is empirically ruled out to a high precision), 2) there is no account for proper time in Weyl's theory, and 3) field equations are 4th order leading to Ostrogradsky-type instabilities. We show that the first two objections can readily be answered. In particular the second objection is answered by developing a physical model of an ideal clock from which proper time is identified as the reading of the clock. We then outline an attempt to tackle the third objection by breaking foliation invariance and so introduce a preferred simultaneity. We show that Lorentz invariance can still be maintained if only the gravitational sector is sensitive to the preferred foliation. We impose the restrictions I) the new theory should contain general relativity in the limit of zero scale curvature, II) no fundamental length scales should appear, III) the field equations should be of second order.
Scale Invariance, Weyl Gravity and Einstein’s Three Objections

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Outline

Introduction
- Weyl’s 1918 theory and basic epistemology
- Mathematical implementation
- Further motivation from Standard model

Einstein’s Three Objections
(1) Size of hydrogen atoms
(2) No account for proper time
(3) 4th order field equations and Ostrogradsky type instabilities

Resolving the Two First Objections
- Why the Weyl field is not the electromagnetic field
- Physical clocks modelled as light clocks

Attempt to Tackle the 3rd Objection
- A family of scale invariant actions
- Scale invariance trivialized
Disclaimer:
All ideas, arguments, and objections have been altered beyond all recognition!
GR as gauge theory of rotations and boosts

Parallel transport preserves angles and lengths:

\[
\frac{\mathcal{D}}{\mathcal{D}t}(U^a U^b g_{mr}) = \frac{\mathcal{D}}{\mathcal{D}t}(U^a U^b g_{mv}) = \frac{\mathcal{D}}{\mathcal{D}t}(U^2 U^2 g_{mr}) = 0 \Rightarrow \nabla_c g_{ab} = 0
\]
Basic Epistemology

But lengths cannot be directly observed: only ratios of lengths are epistemologically accessible!

Weyl’s idea: Only ratios of lengths and angles should be preserved under a parallel transport.

\[ \frac{D}{DT} \left( \frac{U^a U_2^b \Gamma^{ab}}{|U_1| |U_2|} \right) = 0 \]

\[ |U_{1,2}| = \sqrt{g_{ab} U_1^a U_2^b} \]
Unification of gravity and EM

Conformal transformations:

\[ g_{ab} \rightarrow \tilde{g}_{ab} = e^{-\phi} g_{ab} \]
\[ \xi_a \rightarrow \tilde{\xi}_a = \xi_a + \partial_a \phi \]

Gravity unified with electromagnetism!

Einstein: "... a stroke of genius of first rank!"

Reasons why \( \xi_a \) cannot be the EM field:

- We assumed above that the rescaling was universal so that all physical systems would be affected in the same way. But EM field discriminates between positive and negative charge.
- Gauge group manifold is \( \mathbb{R} \) not the unit circle \( S^1 \).
Further Arguments for Scale Invariance

Apart from epistemological arguments we can add two more arguments for scale invariance.

(i) Standard model is already almost scale invariant!

If we include the Weyl field in the right places there are only two terms in the standard model that are not scale invariant:

\[ \sqrt{g} (R - 2\Lambda) \]

Higgs mass term: \(-\sqrt{g} m^2 \Phi^2\)

(ii) Removing singularities

Singularities in GR emerge because the expansion "blows up". But if we turn scale (and therefore expansion) into a gauge degree of freedom, this will be prevented. Thus, we should expect a scale invariant theory to be singularities free.
Einstein’s Three Objections

(1) Hydrogen Spectra depends on history
(2) No Account of proper time
(3) 4th order field equations

Objection (1): Hydrogen Spectra
**Objection (2): Proper Time**

The usual expression $d\tau^2 = -g_{ab} dx^a dx^b$ is gauge-dependent:

$$d\tau^2 = -g_{ab} dx^a dx^b \rightarrow -\tilde{g}_{ab} dx^a dx^b = e^{-\phi} g_{ab} dx^a dx^b = e^{\phi} dt^2 = d\tilde{t}^2$$

So which proper time does a clock show?

$$\tau = \int \frac{\lambda}{\sqrt{-g_{ab} x^a x^b}} d\lambda \quad \text{or} \quad \tilde{\tau} = \int \frac{\lambda}{\sqrt{-\tilde{g}_{ab} x^a x^b}} d\lambda$$

Weyl's first (desperate?) proposal:
Physical Model of Light-Clock

Mathematically we can represent the light clock as a spatial vector $S^a$ where $|S| = \sqrt{g_{ab}S^aS^b}$ is the size of the clock.

Transport without torque or stretching is mathematically represented by a Fermi-Walker transport:
The number of counts is inversely proportional to the size of the tube!

\[ du \propto \frac{dt}{s} \]
Thus the expression for the proper time is given by:

\[
\tau = \int \frac{1}{\sqrt{g_{00}}} \exp \left[ -\frac{1}{2} \int \sqrt{-g} \, \eta_{\mu \nu} \, dx^\mu \, dx^\nu \right] \, d\lambda
\]

The proper time \( \tau \) was also discovered by Perlick by requiring that four-velocity \( u^a \) should be orthogonal to the four velocity \( a^a \):

*Reparametrization covariant*

\[
\frac{d\tau}{du} = \frac{dx^a}{du} \quad a^a = - D^2 x^a
\]

\( g_{ab} u^a u^b = 0 \Rightarrow du = \exp \left[ -\frac{1}{2} \sqrt{-g} \, dx^\mu \, dx^\nu \right] \, d\lambda
\]

*Not reparametrization covariant*

Second clock effect: the rate of ticking is path dependent

\( \dot{t}_1 \neq \dot{t}_2 \)
**Objection (3): 4th order field equations**

(Apparently) There are no actions which are
(1) Seriously conformally invariant
(2) Foliation invariant
(3) Yields 2nd order field equations

Idea: give up foliation invariance to save conformal invariance. Introduce a preferred time \( t \) and kill the extra degree of freedom by imposing conformal invariance. A family of these actions, written down on "best-matching" form, has been written down in Westman '09:

\[
L_{\text{W}} = \sqrt{-g} \sqrt{(\alpha R + \alpha K_c)(\beta H^2 + \gamma R^2 + \delta R \gamma + R_{ij} H^{ij})}
\]

\[
T_\alpha = g^{ijk} (\dot{g} + \Lambda \dot{g} + \phi g_{ij})(\dot{g}_{ne} + \Lambda \dot{g} + \phi g_{ne})
\]

\[
T_c = g^{ij} (\partial_i + \Lambda \partial_i)(\partial_j + \Lambda \partial_j)(\partial_i + \Lambda \partial_i)
\]

\[
H^2 = H_{ij} H^{ij} \quad H_{ij} = \partial_i \partial_j - \partial_j \partial_i \quad \sim \frac{\partial}{\partial t}
\]

Tom Zlosnik (DI) managed to these actions on standard form...
Three reasons for not wanting foliation invariance

- Gives rise to the infamous problem of time including that "damned" Wheeler-DeWitt equation.
- Breaking Lorentz invariance improves power counting renormalizability (Horava).
- All generally covariant field theories will have a conformal anomaly, i.e. even though the classical equations of motion are conformally invariant that symmetry does not survive quantization. Thus, if we demand general covariance then the quantized theory will not be conformally invariant.

But what about Lorentz invariance?

\[
L = L_{\text{matter}} + \sqrt{g} F^2 + \sqrt{g} F^2 + \sqrt{g} Tr (g^{-1} \nabla \phi^2) + \sqrt{g} i \bar{\psi} \gamma^\mu \gamma^\nu \gamma^\lambda \nabla_\lambda \phi + \text{...}
\]

Lorentz invariance is recovered as long as matter part of the action is Lorentz invariant, i.e. matter as long as matter is insensitive to the preferred foliation.
Mathematical criteria for serious scale invariance

Scale Invariance Trivialized:
- It is trivial to find formally scale invariant actions. Just take the Einstein-Hilbert action and perform a general conformal transformation: \[ g_{ab} \rightarrow e^{-\phi} g_{ab} \]

\[ \mathcal{L}_{EH}(g) \rightarrow \mathcal{L}_{EH}(\phi, g) = \mathcal{R} e^{\phi} R + \mathcal{R} e^{\phi} \ldots \]

- But of course nothing interesting has been achieved by this conformalization. It is the same theory with conformally invariant "make-up"!

Any seriously scale invariant theory should not contain scalar fields that picks out a preferred length scale. Furthermore, a scale invariant theory should not contain constants that determines a preferred length scale.

Absence of fundamental length scales:
Getting rid of cosmological constant and Planck scale.

\[ g_{ab} + \Lambda g_{ab} = 0 \quad n_a = -N \nabla a t \quad n^a n^b g_{ab} = -1 \]

\[ \Rightarrow g_{ab} n^a n^b + \Lambda n^a n^b g_{ab} = 0 \Rightarrow \Lambda = g_{ab} n^a n^b = -\frac{1}{3} (\mathcal{R}_{ab} \mathcal{R}^{ab} - \frac{2}{3} \mathcal{R}^2) \]

\text{gravitational energy.}

\text{Ideas: } \Lambda \text{ is not really a constant. It is made constant by imposing a conformal gauge. Undoing the conformal transformation yields:}

\[ L_N = \frac{\sqrt{g}}{k} \left[ T^{(g)} - (\Box R)^2 \right] = \frac{1}{\Lambda} \left( e^{2\phi} |g^{ij} (6 g^{ab} \nabla_a \phi \nabla_b \phi + 4 n^a \nabla_a n^b \nabla_b \phi) \right) \]

\text{Nasty terms...}

- Yields 2nd order field equations
- reproduces GR when Weyl field strength is zero
- is an example of the family of actions obtained by Tom Zlosnik.
The Fundamental Problem

In nature we find objects of identical internal compositions pointing in all kinds of directions and with all kinds of velocities. We call this rotational and Lorentz symmetry.

However, we do not find objects with the same internal compositions (e.g. a collection of hydrogen atoms) in all kinds of sizes.

Symmetry breaking?

Symmetry breaking without Higgs mass term
Conclusion and Outlook

Physics is hard... very hard!
Why scale invariance?
- Epidemiology
- Singularities
- Standard model Lagrangian

Why absolute simultaneity?
- Problem of time (comes from local relabelling of simultaneity)
- Improve renormalizability
- Quantum non-locality
- Conformal anomaly

What is scale invariance?
- Absence of fundamental length scales
- No local metrics
- Absence of scale fields! against conformal invariance

Talk outline:
- Why scale invariance?
- Why gravity?
- Einstein's four equations
- Resolution of the two first
- What is scale invariance?