Abstract: The simulation of systems of anyons offers a significant challenge to the condensed matter physicist. These systems are presently of substantial theoretical and experimental interest due to their potential for universal quantum computation, but due to their non-trivial exchange statistics, the tools available for their study have been limited. In this talk, I will present a formalism whereby any existing tensor network algorithm may be adapted for use with both Abelian and non-Abelian anyons, culminating in our recent simulations of infinite 1-D chains of interacting anyons using the Multi-scale Entanglement Renormalisation Ansatz, or MERA, demonstrating that tensor network algorithms may be effectively employed in the study of anyonic systems.
Anyonic Tensor Network Algorithms

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Anyons

- Non-bosonic, non-fermionic exchange statistics
- 2-D and quasi-1-D systems
- Currently a hot topic...
Tensor Network Algorithms

- Tensor network ansatz
- Update algorithm

\[ |\psi\rangle = \sum c_{i_1 i_2 i_3 \ldots} |i_1, i_2, i_3, \ldots\rangle = \sum \Gamma^{(1)}_{i_1 j_1} \lambda^{(1)}_{j_1} \Gamma^{(2)}_{i_2 j_2 j_1} \lambda^{(2)}_{j_2} \Gamma^{(3)}_{i_3 j_3 j_2 j_1} \ldots |i_1, i_2, i_3, \ldots\rangle \]
Tensor Network Algorithms

- Tensor network ansatz

![Diagram of tensor network ansatz]
Tensor Network Algorithms

- Tensor network ansatz
Tensor Network Algorithms

- Tensor network ansatz
- Update algorithm
  - To construct optimised representation of the ground state
  - Time evolution
  - ...
Tensor Network Algorithms

- Tensor network ansatz

\[ C_1 i_2 i_3 \ldots \]
Tensor Network Algorithms

- Tensor network ansatz
Tensor Network Algorithms

- Tensor network ansatz
Tensor Network Algorithms

- Tensor network ansatz
- Update algorithm
  - To construct optimised representation of the ground state
  - Time evolution
  - ...

Pirsa: 11010110
Tensor Networks and Fermions

- Successful description of low-dimensional bosonic systems
- Recent extension to fermionic systems in 1-D and 2-D (e.g. Corboz et al., 2009)
Tensor Networks and Fermions
Tensor Networks and Fermions

- Successful description of low-dimensional bosonic systems
- Recent extension to fermionic systems in 1-D and 2-D (e.g. Corboz et al., 2009)
- No “sign problem”
- Can we generalise this success to anyons?
Tensor Networks and Anyons

\[ |\psi\rangle = \sum_{a_1 \ldots a_{10} \atop a_1 \ldots a_{10}} c_{a_1 \ldots a_{10} u_1 \ldots u_5} a_9 \]
\begin{align*}
\left| x \right| & \rightarrow 1 \\
\left| x \right| \times 2 & \rightarrow 2 \\
\tau \times 2 & \rightarrow \tau \\
\tau & \rightarrow 1 + \tau
\end{align*}
Tensor Networks and Anyons

\[ |\psi\rangle = \sum_{a_1 \ldots a_{10} u_1 \ldots u_5} c_{a_1 \ldots a_{10} u_1 \ldots u_5} a_9 a_{10} \]
Tensor Networks and Anyons

\[ |\psi\rangle = \]
Tensor Networks and Anyons

\[ |\psi\rangle = \left( \begin{array}{c} c \end{array} \right) \]

\[ \hat{M} = \left( \begin{array}{c} M \end{array} \right) \]

\[ c^\alpha \quad M^\alpha_\beta \]
\[ \frac{1}{2} \times 1 \rightarrow 1 \]
\[ \frac{1}{2} \times 2 \rightarrow 1 \]
\[ \frac{1}{2} \times \frac{1}{2} \rightarrow \frac{1}{4} \]
Tensor Networks and Anyons

\[ |\psi\rangle = \sum_{\alpha} c_\alpha \}

\[ \hat{M} = M \]
Tensor Networks and Anyons

\[ |\psi\rangle = \sum F \quad \text{and} \quad |\psi\rangle = \sum c^\alpha c'^\alpha \]

\[ \hat{M} = \text{Diagram} \]

\[ c'^\alpha = \sum_{\beta} c^\beta F^{\alpha}_{\beta} \]
Tensor Networks and Anyons

\[ |\psi\rangle = \quad \hat{M} = \quad \]

\[ M \]

\[ d \]

\[ d' \]
Tensor Networks and Anyons

\[ |\psi\rangle = \]

\[ \hat{M} |\psi\rangle = \]

\[ \hat{M} = M \]

\[ c \]

\[ M \]

\[ c' \]
Anyonic Tensor Networks

- Anyonic states and operators
  - Allows writing of an anyonic TN state
- Rules for manipulating anyonic trees
  - Allows translation of algorithms
Anyonic MERA!
Anyonic MERA!
Anyonic MERA!
Anyonic MERA!
Golden Chain

\[ 1 \times 1 \rightarrow 1 \]
\[ 1 \times \tau \rightarrow \tau \]
\[ \tau \times 1 \rightarrow \tau \]
\[ \tau \times \tau \rightarrow 1 + \tau \]
Scale-Invariant MERA
Scaling Operators
\[ 1 \times 1 \rightarrow 1 \]
\[ 1 \times 2 \rightarrow 2 \]
\[ 2 \times 1 \rightarrow 2 \]
\[ 2 \times 2 \rightarrow 4 \]
\[ 1 + 1 = 2 \]
\[ 2 + 2 = 4 \]
Scaling Operators
Results

AFM Golden Chain  
FM Golden Chain
Conclusions

- Anyonic tensor networks:
  - possible
  - useful
  - efficient
  - shown to work!
Conclusions

- The microscopic behaviours of anyonic systems are still largely unstudied.

Tensor networks for anyonic systems make their study possible.

With these tools, there is a great deal of research to be done!