Title: Space-Time, Quantum Mechanics and Scattering Amplitudes

Date: Jan 26, 2011  02:00 PM

URL: http://pirsa.org/11010111

Abstract: Scattering amplitudes in gauge theories and gravity have extraordinary properties that are completely invisible in the textbook formulation of quantum field theory using Feynman diagrams. In the standard approach--going back to the birth of quantum field theory--space-time locality and quantum-mechanical unitarity are made manifest at the cost of introducing huge gauge redundancies in our description of physics. As a consequence, apart from the very simplest processes, Feynman diagram calculations are enormously complicated, while the final results turn out to be amazingly simple, exhibiting hidden infinite-dimensional symmetries. This strongly suggests the existence of a new formulation of quantum field theory where locality and unitarity are derived concepts, while other physical principles are made more manifest. Rapid advances have been made towards uncovering this new picture, especially for the maximally supersymmetric gauge theory in four dimensions. These developments have interwoven and exposed connections between a remarkable collection of ideas from string theory, twistor theory and integrable systems, as well as a number of new mathematical structures in algebraic geometry. In this talk I will review the current state of this subject and describe a number of ongoing directions of research.
Space-Time, Quantum Mechanics + Scattering Amplitudes
with

F. Cachazo
C. Cheung
J. Kaplan
J. Bourjaily
J. Trnka
S. Cahn-Haft

also

E. Witten
L. Dolan
P. Goddard
M. Spradlin
A. Volovich
S. Gouchnoy

J. Maldacena
F. Alday
D. Gaiotto
P. Vieira
A. Sever
N. Beisert
M. Staudacher

Z. Bern
L. Dixon
D. Kosower
G. Korchemsky
E. Sokatchev
J. Henn
J. Drummond

R. Penrose
A. Hodges
L. Mason
D. Skinner
M. Baulinor
Motivations
Gravity + QM

"Space-time is Doomed"
$\Delta E \sim \frac{1}{\Delta t}$ $\rightarrow$ eventually make Black Hole!

No Operational meaning to distance $< 10^{-33}$ cm, times $< 10^{-43}$ s, ....

End of Space-Time
Our theories just break down when gravity is strong and quantum gravity effects are dominant.
Exact Quantum Predictions

Infinitely many measurements with an Infinitely large measuring apparatus!
No Local Observables!
\[(\text{Quantum Gravity})_{D+1} = (\text{Quantum Field Theory})_D\] (?!)

Emergent Space, Gravity, Strings ... 

"Anti-de Sitter Space"

String Theory = Particle Physics

[Weakly interacting] [strongly interacting]
Emergent Space-time?

What are the correct observables??

$10^{10}$ light years
\[ \ddot{x} = -\frac{\partial V}{\partial x} \]

Manifestly Deterministic

\[ S = \int dt \left[ \frac{1}{2} m \dot{x}^2 - V(x) \right] \]

Not manifestly deterministic

\[ x(t) \text{ minimizes action} \]
Quantum Mechanics

All paths are taken.

\[ \text{Amp} = \sum e^{i S/\hbar} \text{ paths} \]

\[ \hbar \to 0 \text{ limit of QM = Least action principle,} \]

\[ \not mc^2 \]

\[ F = ma \]
Feynman Follies

220 Diagrams

10's of thousands of terms...
\[ \text{Amp}(1^+2^-3^+4^-5^+6^+) = \frac{\langle 24 \rangle^4}{\langle 12 \rangle\langle 23 \rangle\langle 34 \rangle\langle 45 \rangle\langle 56 \rangle} \]

"MHV Amplitudes": \( \bar{ij} \), rest plus

\[ \langle i^4 \rangle \]

\[ \frac{\langle 12 \rangle\langle 23 \rangle \cdots \langle n1 \rangle}{\langle i^4 \rangle} \]
Q. What makes Feynman Diagrams so complicated, obscuring simplicity of answer?

A. Insistence on Manifest Locality + Unitarity!
$2$ helicities $\pm 1$.

Photon

Locality $\Rightarrow$ Field $A_\mu(x) = \epsilon_\mu \ e^{i p \cdot x}$

4 components!

$\not{\epsilon} \cdot p = 0$, $\epsilon_\mu \sim \epsilon_\mu + \alpha \rho_\mu$

$A_\mu \sim A_\mu + \partial_\mu \Lambda$

Gauge Redundancy $\Rightarrow$ All the trouble!
Sitting Under our Noses for 60 yrs

String Theory

Scattering without Spacetime - emergent locality + unitarity

Twistor Theory

Algebraic Geometry

Integrable systems
Cast of Characters
Gluon Scattering Amplitudes

\[ i \quad j \]

\[ N_c \text{ "colors".} \]

\[ h_1 \quad h_2 \quad h_3 \]

\[ + \quad + \quad + \quad + \quad + \]

\[ \text{"trees"} \quad \text{"Loops"} \]

\[ \text{Restrict to} \]

\[ \text{planar diagrams} \]

\[ \text{[dominate as} \quad N_c \to \infty \text{]} \]

\[ \text{Important: Planar Loop} \]

\[ \text{Integrand is well-defined} \]
\[ M \left[ p_A, h_1, \ldots, h_n \right] \]

\[ \mathcal{E}_i : \text{loop momenta} \]
More Kinematics

\[ p^\mu = \left( p^0, \vec{p} \right) \quad \leftrightarrow \quad P_{A\bar{A}} = \begin{pmatrix} p^0 + p^3 & p^1 + i p^2 \\ p^1 - i p^2 & p^0 - p^3 \end{pmatrix} \]

\[ \det P = p^2 = 0 \]

\[ \Rightarrow \quad P_{A\bar{A}} = \lambda_A \bar{\lambda}_{\bar{A}} \cdot \text{Lorentz: } SL(2) \times SL(2) \]

Invariants

\[ \langle \lambda_1, \lambda_2 \rangle = \varepsilon^{AB} \lambda_{1A} \lambda_{2B} \]

\[ [\lambda_1, \lambda_2] = \varepsilon^{AB} \lambda_{1A} \lambda_{2B} \]
Manifest Little Group Transt.

\[ M_{\lambda_a, \lambda_a, h_a} \cdot M_{(t_a \lambda_a, t_a^{-1} \lambda_a, h_a)} = t_a^{-2h_a} \cdot M_n(\lambda_a, \lambda_a) \]

e.g.

\[ M_{6}(1^2 2^3 3^4 5^5 6^6) = \frac{\langle 2^4 \rangle^4}{\langle 2^2 \rangle \langle 2^3 \rangle \langle 3^4 \rangle \langle 4^5 \rangle \langle 5^6 \rangle} \]

\[ M_{6}(t_2 \lambda_2) = t_2^a \cdot M_{6}(\lambda_2) \Leftrightarrow \text{helicity of 2 is } -1. \]
More Kinematics

\[ p^\mu = (p^0, \vec{p}) \leftrightarrow P_{\lambda\lambda'} = \begin{pmatrix} p^0 + p^3 & p^0 + ip^2 \\ p^0 - ip^2 & p^0 - p^3 \end{pmatrix} \]

\[ \det P = p^2 = 0 \]

\[ \Rightarrow P_{\lambda\lambda'} = \lambda^\lambda_A \lambda^\lambda_A \cdot \text{Lorentz: } SL(2) \times SL(2) \]

Invariants

\[ \langle \lambda_1, \lambda_2 \rangle = E^{AB} \lambda_1^A \lambda_2^B \]

\[ [\tilde{\lambda}_1, \tilde{\lambda}_2] = E^{AB} \tilde{\lambda}_1^A \tilde{\lambda}_2^B \]
\[ M \left[ p^\alpha, h_{\alpha j} \right] \]

\[ p_1^\alpha \rightarrow h_1 \]

\[ p_2^\alpha \rightarrow h_2 \]

\[ p_1^\alpha + \ldots + p_n^\alpha = 0 \]

"loop momenta"
More Kinematics

\[ p'^{\mu} = (p', \vec{p}) \quad \leftrightarrow \quad P_{\text{A} \bar{A}} = \begin{pmatrix} p^0 + p^3 & p^1 + ip^2 \\ p^1 - ip^2 & p^0 - p^3 \end{pmatrix} \]

\[ \det p = p^2 = 0 \]

\[ \Rightarrow \quad P_{\text{A} \bar{A}} = \lambda^\dagger_A \lambda^A \quad \text{Lorenz: } SL(2) \times SL(2) \]

Invariants
\[ \langle \lambda_1, \lambda_2 \rangle = \varepsilon^{AB} \lambda^{A}_1 \lambda^{A}_2 \]
\[ [\lambda^\dagger_1, \lambda^\dagger_2] = \varepsilon^{AB} \lambda^A_1 \lambda^B_2 \]
Manifest Little Group Transform

\[ M(\lambda_a, \lambda_b, \lambda_c), M(t_{a} \lambda_a, t_{a} \lambda_a, \lambda_a) \]

\[ = t_{-2a}^{2} M_{n}(\lambda_{a}^{2}) \]

E.g.

\[ M_{b}(1, 2, 3, 4, 5, 6) = \frac{\langle 24 \rangle^{4}}{\langle 2 \rangle \langle 2 \rangle \langle 3 \rangle \langle 3 \rangle \langle 45 \rangle \langle 56 \rangle} \]

\[ M_{b}(t_{a} \lambda_{a}) = t_{a}^{2} M_{b}(\lambda_{a}) \Leftrightarrow \text{helicity of 2 is -1.} \]
Finally - simplest gauge theory of all is “maximally supersymmetric, \( N=4 \) Super Yang-Mills”:
“Harmonic Oscillator of the 21st Century”

Unifies helions.

\[
\begin{align*}
Q_{1,2,3,4} & \quad + \frac{1}{2} \\
\downarrow & \quad 0 \\
Q_3 & \quad - \frac{1}{2} \\
\downarrow & \quad -1
\end{align*}
\]

\[
\begin{align*}
\{ & \quad \text{“Supermultilet”} \\
\begin{array}{c}
\eta_{\bar{1}} \\
\eta_{\bar{2}} \\
\eta_{\bar{4}}
\end{array} & = & e^{Q_{\bar{2}} \bar{2}} |+\bar{1}\rangle
\end{align*}
\]

\[
= |+\bar{1}\rangle + \eta_{\bar{2}} |+\bar{2}\rangle + \cdots + \eta_{\bar{4}} |+\bar{4}\rangle
\]
\[
M_n (\tilde{\lambda}_a, \tilde{\lambda}_a, \tilde{\eta}_a) = \sum_{k=0}^{n} M_{n,k} (\tilde{\lambda}_a, \tilde{\lambda}_a, \tilde{\eta}_a)
\]

Turns out \( M_{n,0} = M_{n,1} = 0 \)

\( M_{n,2} \) = "MHV" amplitude

\( M_{n,3} \) = "NMHV" amplitude

contains amps with \( k \) - helicity gluons.
Summary: We are after a theory for 

\[ M_{n,k}[\lambda, \lambda', \lambda''] \]

Without Unitary evolution through Spacetime

\{Emergent Space-time, Emergent QM\}
Tree Amplitudes:

Gathering "Data"
"BCFW Recursion"

Deform $\lambda_n \rightarrow \lambda_n + \pm \lambda_1$
$\tau_1 \rightarrow \tau_1 \pm \pm \lambda_n$

Pole:

Lower point trees
Cauchy:

\[ \sum_{\text{interm.}} \left\{ \begin{array}{c} n \\ \overline{n} \end{array} \right\} \rightarrow \frac{1}{p^2} \rightarrow \left\{ \begin{array}{c} n \\ \overline{n} \end{array} \right\} \]

\text{On-Shell recursion relation!}
BCFW $\mathcal{G}_{\text{pt}}$

\[
\begin{align*}
\langle 46 \rangle^4 [13]^4 & = \\
\frac{1}{[10][23] \langle 45 \rangle \langle 56 \rangle} \frac{1}{(p_1 + p_2 + p_3)^2} & \times \frac{1}{\langle 6| 5 + 4 | 3 \rangle} \frac{1}{\langle 4| 5 + 6 | 1 \rangle} \lessgtr \text{"Spurious Poles: Don't occur in local theories!"
}
\end{align*}
\]

+ \{i \rightarrow i+2\} + \{i \rightarrow i+4\}
**Remarkable 6-term Id**

\[
\frac{\langle 46 \rangle^4 \langle 13 \rangle^4}{\langle 6 \rangle \langle 13 \rangle \langle 45 \times 56 \rangle (p_1 + p_2 + p_3)} = \frac{\langle 31 \rangle^4 (2 + 3) \langle 6 \rangle^4}{\langle 22 \rangle \langle 34 \rangle \langle 56 \langle \bar{6} \rangle \rangle (p_5 + p_6 + p_7)^2}
\]

\[\times \frac{1}{\langle 61 \rangle^5} \frac{1}{\langle 45 + 61 \rangle^2}
\]

\[\times \{i \rightarrow i+2\} \{i \rightarrow i+4\}
\]

---

**Guarantees**

- Parity
- Cyclicity
- No Spurious Poles

---

7-pt: 12 terms
8-pt: 20 terms

SOME POWERFUL MATHEMATICAL STRUCTURE IS AT WORK!
Infinitely Many Hidden Symmetries
Theories of massless particles enjoy conformal invariance — the remarkable symmetry under inversions.

\[ X^\mu \rightarrow \frac{X^\mu}{X^2} \]
Twistor Space

\[ W = (\frac{x^A}{\lambda^A}) \]
\[ \det L = 1 \]

are conf.

transf.

\[ W_1 \]
\[ W_2 \]

Spacetime

\[ \tilde{x}_A = x_{\alpha} \tilde{x}^\alpha \]
null ray

\[ X = \frac{\mu_1 \gamma_2 - \mu_2 \gamma_1}{\langle 12 \rangle} \]

\[ (X-Y)^2 = 0 \]
Dual (Super) Conformal Symmetry

\[ P_a = x_{a+1} - x_a \]

"Experimental" observation
- amplitudes invariant under
  Conf. transf. on
  this \( x \) space!

[Term by term for BCFW form of trees]
\[
\left[ \frac{\langle 46 \rangle^4 \langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 45 \rangle \langle 56 \rangle} \frac{1}{(p_1 + p_2 + p_3)^2} \right] 
\times \frac{1}{\langle 61 \rangle \langle 54 \rangle \langle 3 \rangle} 
\times \frac{1}{\langle 14 \rangle \langle 56 \rangle \langle 1 \rangle} 
\]

Are there because these BCFW terms know about both spacetimes!
"Momentum" Twistor Space

\[ Z_a = \left( \begin{array}{c} \lambda^a \\ \eta^a \end{array} \right) , \quad \tilde{\chi}_a = \frac{\langle a-1 \ a \rangle a+1 + \text{cyclic}}{\langle a-1 \ a \rangle \langle a \ a+1 \rangle} \]

\[ \tilde{\eta}_a = \frac{\langle a+1 \ a \rangle z_{a+1} + \text{cyclic}}{\langle a-1 \ a \rangle \langle a \ a+1 \rangle} \]

[Invariants \[ \langle Z_1 Z_2 Z_3 Z_4 \rangle, \text{ Schouten-Skinner} \]

identity \[ \frac{\langle a b c d \rangle \langle e 1 2 3 \rangle}{\text{cyclic}} = 0 \]
(Super) Conformal + Dual (Super)Conformal

↓ generate

“Yangian Algebra”

Infinite Dimensional Symmetry

Completely Invisible In Z
Very striking connection with integrability

\[ H = \sum_i s_i \cdot s_{i+1} \]

\[ Q = \sum_{i < j} [s_i, s_j] \]

+ AdS/CFT, spectrum of anom.

dimensions in $N=4$ SYM, + amplitudes
In particular major breakthroughs in last ~ 5 yrs have solved the problem of determining anom. dimensions in N=4 SYM — again no Feynman diagrams! — extension to amplitudes more physical + expect richer structure ...
$(\text{Super}) \text{ Conformal} \ + \ \text{Dual (Super)Conformal}
\Downarrow \text{generate}

\"\text{Yangian Algebra}\"

\text{Infinite Dimensional Symmetry}

\text{Completely Invisible In } Z
Very striking connection with integrability

\[ H = \sum_i s_i \cdot s_{i+1} \]

\[ Q = \sum_{i<j} [s_i, s_j] \]

+ AdS/CFT, spectrum of anom.

dimensions in \( N=4 \) SYM, + amplitudes.
In particular major breakthroughs in last ~5 yrs have solved the problem of determining anom. dimensions in \( N=4 \) SYM — again no Feynman diagrams! — extension to amplitudes more physical + expect richer structure...
A New Formulation
Start by thinking about momentum conservation afresh!

\[ \lambda_A, \tilde{\lambda}_A \]

\[ \lambda^{(2)}_1 \]

\[ \lambda^{(2)} \]

\[ \lambda \cdot \tilde{\lambda} = 0. \]

Mom. conservation:
Note: parity invariant since
\[ \lambda \leftrightarrow \bar{\lambda} \]

K plane \( \leftrightarrow \) n-K plane

Note: impossible for \( k = 0, 1, n-1, n \).
Good!
Eqns:

\[ C = \left[ \begin{array}{c} u^i \\ \vdots \\ u^k \end{array} \right] = \alpha C \]

Invariance under \( GL(k) \): \( C_{\alpha a} \rightarrow L^\beta \alpha C_{\beta a} \).

Space of \( k \)-planes in \( n \)-dim: Grassmannian \( G(k,n) \)

\[ \text{dim } G(k,n) = kn - k^2 = k(n-k) \]
\[ \int d^2 \alpha \delta^2 \left[ C \alpha \alpha \rho_{\alpha} - T_{\alpha} \right] \delta^3 \left[ C \alpha \alpha \tilde{\gamma}_{\alpha} \right] \delta^4 \left[ C \alpha \alpha \tilde{\gamma}_{\alpha} \right] \]

- \( C \) contains \( \lambda \)
- \( C \) orthogonal to \( \tilde{\gamma} \)

**Motivation:** preserve \( SL(C) \)
This object is very simple in Twistor Space:

$$\frac{k}{11} S_{4/4}^{\frac{4^4}{4}} \left[ C_{baW_a} \right]$$

$$\alpha = 1$$

Manifests (Super) conformal symmetry
\[ k = 0, 1, n-1, n : \text{no possible planes.} \]
\[ k = 2 \text{ unique: } C = 2 \text{ plane.} \]
\[ \text{General } k \text{ : integrate over all } k\text{-planes!} \]

\[ \int d^k x \sim \frac{C}{C_{12...k} C_{23...k+1} ... C_{n 1...k-1}} \]

\[ (m_1...m_k) : k \times k \text{ minor of } C \text{ made of columns } m_1,...,m_k. \]
$Z_{n,k} = \int \frac{k!}{c_{2\cdots k} \cdots c_{n+1 \cdots k+1}} \cdot \prod_{\alpha}^{\#} \left[ C_{\alpha \alpha} W_{\alpha} \right]$

simplest measure

simplest dependence on kinematics

All-Loop Scattering in $\mathcal{N}=4$ SYM!
Manifest Dual Superconformal Invariance

\[ C \text{ contains a plane:} \]
\[ (k-2) \text{ planes in } n \text{ dimensions!} \]

Natural linear transformation mapping bracket minors to \((k-2) \times (k-2)\) minors...
\[ Z_{n,k} \to \int \frac{d^{p \times (n-p)}}{(12\ldots p)\ldots (n1\ldots (p-1))} D_{\alpha a} \times \frac{P}{11} 8^{4/4} [ D_{\alpha a} Z_{\alpha}] \]

Identical Structure!

Dual superconformal symmetry manifest
The Grassmannian Formulation makes no mention of locality or Unitarity - but makes all symmetries - The Yangian - manifest.
Quick Example

First non-trivial $k = 3$, $n = 6$, NMHV, $(k-2)(n-k-2) = 1$ variable.

$$Z_{6,3} = \int \frac{2p}{(123)(12)(23)}$$

Each minor linear in $Z$.
Tree Amplitude

- residues: BCFW terms
- residues: $\mathcal{P}[\text{BCFW}]$ terms
- Cauchy: $\text{BCFW} = \mathcal{P}[\text{BCFW}] = \text{Remarkable 6-term identity}$

[Unique choices respecting cyclic symmetry]
Basic Operations on Yangian Invariants

$\int \frac{d^4 Z}{Z_{n+1}}$

BCFW terms composed from these

BCFW
Origin of Loops

\[ \int d^4 z_A \int d^4 z_B \]

"Entangled removal of a pair of particles"

Quantum Loop Corrections
All-Loop Recursion

\[ \sum_{n, k, l} = \text{Classical} + \text{Quantum} \]
Complete definition of theory, Yangian symmetry manifest.

The words "spacetime", "Lagrangian", "Path Integral", "Gauge Symmetry".... make no appearance.
In the dual space-time, this object is interpreted as a certain supersymmetric Wilson loop:

Perfect symmetry has been established between both descriptions.
B. Kissing double-box topologies

\[
-\frac{1}{4} \begin{bmatrix} a & a+1 & b-1 \\ b+1 & c-1 & c \end{bmatrix}
\]

\[\begin{align*}
&= -\frac{1}{4} \begin{bmatrix} a & a+1 & b-1 \\ b+1 & a-1 & a \end{bmatrix} + \frac{1}{4} \begin{bmatrix} a & a+1 \\ b-1 & b \end{bmatrix} \begin{bmatrix} b & b+1 \\ a-1 & a \end{bmatrix} \\
&\quad + \frac{1}{4} \left( x_{a,b+1}^2 - x_{a-1,b}^2 + x_{a,b+1}^2 - x_{a-1,b}^2 - x_{a,b}^2 - x_{a,b+1}^2 + x_{a-1,b}^2 - x_{a,b+1}^2 \right) \text{(54)}
\end{align*}\]
\[ A_{\text{2-loop}}^{\text{MHV}} = \frac{1}{2} \sum_{i < j < k < l < i} \]

\[ A_{\text{2-loop}}^{\text{NMHV}} = \sum_{i < j < k < l < i} \]

\[ \times [i, j, j + 1, k, k + 1] \]

\[ + \frac{1}{2} \sum_{i < j < k < l < i} \]

\[ \times \left\{ A_{\text{tree}}^{\text{NMHV}}(i, \ldots, k, l, \ldots, i) \right\} \]

\[ + A_{\text{tree}}^{\text{NMHV}}(i, j) \]

\[ + A_{\text{tree}}^{\text{NMHV}}(k, l) \]

\[ A_{\text{3-loop}}^{\text{MHV}} = \frac{1}{3} \sum_{h} \]

\[ A_{\text{3-loop}}^{\text{MHV}} = \frac{1}{2} \sum_{h} \]
Stunning Simplification

\[ R_6^{(2)}(u_1, u_2, u_3) = \sum_{i=1}^{3} \left( L_4(x_i^+, x_i^-) - \frac{1}{2} \text{Li}_4(1 - 1/u_i) \right) \]

\[ - \frac{1}{8} \left( \sum_{i=1}^{3} \text{Li}_2(1 - 1/u_i) \right)^2 + \frac{1}{24} J^4 + \frac{\pi^2}{12} J^2 + \frac{\pi^4}{72}. \]

[ Makes use of "theory of motives"]
New Integrals are Simple!

\[
\alpha^+ \alpha^- I_{\text{total}} = \frac{1}{\chi^+ \chi^-} \log \chi^+ \log \chi^- + \frac{2(\chi^+ - 1)}{\chi^+ \chi^- (1 + \chi^+)} \log \chi^- \log (1 + \chi^-) + \frac{2(\chi^- - 1)}{\chi^+ \chi^- (1 + \chi^-)} \log \chi^+ \log (1 + \chi^+) + \\
+ \frac{4(\chi^+ - 1)}{\chi^+ \chi^- (1 + \chi^+)} \text{Li}_2(-\chi^-) + \frac{4(\chi^- - 1)}{\chi^+ \chi^- (1 + \chi^-)} \text{Li}_2(-\chi^+) \]

(16)
This Picture

\[ A^2_{\text{MHV}} = \frac{1}{2} \sum_{i<j<k<l<i} \]

is telling a geometric story—once we understand it we'll just write down the answer...
In a specific sense, amplitudes are to be thought of as "the volume" of some polytope:

Different "triangulations" make different properties (Yangian, locality, unitarity...) manifest.
Our solution should be thought of as providing one class of triangulations — but we need to more deeply understand what the object is that is being triangulated!
\[ F_{j,n} = \sum_{i} \left( \begin{array}{c}
(j+1,j+1,i+1) \\
(j-1,j+1,i+1) \\
(j-1,j+1,j+2) \\
\end{array} \right) + \left( \begin{array}{c}
(j+1,i+1) \\
(j-1,i+1) \\
(j-1,j+1,j+2) \\
\end{array} \right) = \sum_{i,x=\pm1} \left( \begin{array}{c}
(j+1,i+1) \\
(j-1,i+1) \\
(j-1,j+1,j+2) \\
\end{array} \right) \]

\[ M_{n}^{NMHV} = \sum_{i,j,s=\pm1} \frac{\eta_{j}. \{j-1,j+1,j+2\i\}. \{j-1,j+1,i-s\i\}. \{j+j+i=1\i+i+1\}}{(j-1,j+1,j+2)(j-1,j-1,i). (j+j+i+1)(j+j+i-s)} \]

NEW LOCAL FORM!
String Theory

Integrability

Twistor theory

\[ T \]

Amazing new mathematical structures:
(Grassmannians, Polylogs, "Motivic Galois Theory", ...)
Still an enormous amount left to understand: what is the one-sentence "physical picture" behind all of this magic? What moral lesson should we extract from $N=4$ example?
But there is strong encouragement to try and fully eviscerate locality and unitarity from our language for describing all of standard physics.
STAY TUNED