Title: A real ensemble interpretation of quantum mechanics

Date: May 03, 2011 04:00 PM

URL: http://pirsa.org/11050022

Abstract: A new ensemble interpretation of quantum mechanics is proposed according to which the ensemble associated to a quantum state really exists: it is the ensemble of all the systems in the same quantum state in the universe. Individual systems within the ensemble have microscopic states, described by beables. The probabilities of quantum theory turn out to be just ordinary relative frequencies probabilities in these ensembles. Laws for the evolution of the beables of individual systems are given such that their ensemble relative frequencies evolve in a way that reproduces the predictions of quantum mechanics. These laws are highly non-local and involve a new kind of interaction between the members of an ensemble that define a quantum state. These include a stochastic process by which individual systems copy the beables of other systems in the ensembles of which they are a member. The probabilities for these copy processes do not depend on where the systems are in space, but do depend on the distribution of beables in the ensemble. Macroscopic systems then are distinguished by being large and complex enough that they have no copies in the universe. They then cannot evolve by the copy law, and hence do not evolve stochastically according to quantum dynamics. This implies novel departures from quantum mechanics for systems in quantum states that can be expected to have few copies in the universe. At the same time, we are able to argue that the centre of masses of large macroscopic systems do satisfy Newton's laws.
A real ensemble interpretation of quantum mechanics

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May 3 2011

arxiv: 1104.2822 and work in progress


Thanks to Saint Clair Cemin, Cohl Furey, Sean Gryb, Lucien Hardy, Adrian Kent, Aaron Lanier, Michael Neilsen, Simon Saunders, Rob Spekkens, Antony Valentini, ... for comments and encouragement.
1. Motivation
2. The real ensemble framework
3. Recovery of the Schroedinger dynamics
4. A model for phase alignment
5. Issues:
   - The subsystem problem
   - The classical limit for macroscopic bodies
   - The getting stuck at nodes problem
6. Further issues
7. Conclusions
Some general aspirations:

Leibniz’s two great principles guide the search for deeper laws of physics:

- **The principle of sufficient reason:** there must be a rational answer to every question that can be imposed of the form of “Why is the universe like X and not otherwise?”
  
  - No fundamental symmetries.
  - Space and time are completely relational.
  - Perhaps space is emergent from a network of relations.
  - If space is emergent than so is locality. Perhaps non-local entanglement is a clue to the real nature of relations in the world before space?
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• **The principle of explanatory closure:** Everything that causally influences the behavior of a physical system within the universe must be another physical system within the universe.
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• The principle of explanatory closure: Everything that causally influences the behavior of a physical system within the universe must be another physical system within the universe.

• The principle of reciprocity: There is nothing in nature that acts without being acted upon in return.
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**The principle of the identity of the indiscernible:** If two systems have the same properties, i.e., the same relations to the rest of the universe, they are the same system. There cannot be two identical systems.
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How does this apply to the quantum world?

Quantum statistics: Two identical particles have wavefunctions which are symmetrized or antisymmetrized. ie you don’t know which particle is where.

Does this apply to bound systems? What if you have a hydrogen atom in the ground state a box. It should be indistinguishable from every hydrogen atom in the ground state in the universe. The Hamiltonian just depends on relative position, not centre of mass position.

How do you know which hydrogen atom you get when you open the box?

Can quantum theory dispense with the notion that the hydrogen atom in our box is somehow intrinsically distinct just because of where it is? All the hydrogen atoms in the ground state are identical Quantum theory is non-categorical, maybe which atom is where is inessential? Maybe all you know is that when you open the box you get one of the ensemble of all the atoms in the universe?
Aspirations for quantum theory:

Solve the measurement problem. One clean way is with a theory of beables. Bohm-deBroglie is a proof of concept that this can work. But it has awkward features; it would be interesting to understand if they can be avoided.

Don’t tie the theory to the assumption that beables are coordinates in space or spacetime or configuration space. Equivalence of bases in QM too important if a clue to drop. (See Vink).

Eliminate double ontology. Too extravagant to think that every individual macro system has a whole wavefunction in addition to its other beables. This also involves an reciprocated action. Better to try to keep the wavefunction as a property of an nsemble.

Understand QM as an approximation to a cosmological theory which is not quantum mechanical.
Eliminate double ontology

In dBB every individual system has a wavefunction:

\[ \Psi(x, t) = \sqrt{\rho(x, t)} e^{i S(x, t)/\hbar} \]

- \( x(t) \) is a beable
- \( \rho(x) \) certainly looks like a property of an ensemble
- \( S(x, t)/\hbar = \varphi(t) \) could be a property of the individual system, i.e., the system at \( x \) at time \( t \) carries a phase \( e^{i \delta(t)} \).

The whole ensemble is needed to carry the information about the function \( S(x, t) \).

Hypotheses: The beables of an individual particle are the position \( x(t) \) and phase \( e^{i \varphi(t)} \). The Schrodinger equation reflects the interplay between the dynamics of an individual and an ensemble.
hypotheses:

The beables of an individual particle are the position $x(t)$ and phase $e^{i \varphi(t)}$. $\rho(x)$ and $S(x)$ represents information about an ensemble, reconstructable from knowledge of all the beables, $(x(t), e^{i \varphi(t)})$ for all members of the ensemble.

The Schrödinger equation reflects the interplay between the dynamics of an individual and an ensemble.

A cautionary lesson from dBB and Nelson: $x(t)$ for an individual particle depends on $\rho(x,t)$.

In essence, the Schrödinger equation reflects an influence of the ensemble on the dynamics of the beables of the individual systems. How can we understand this influence of an ensemble on an individual?
**Hypotheses:**

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A cautionary lesson from dBB and Nelson: $x(t)$ for an individual particle depends on $\rho(x,t)$.

Conversely, the Schrödinger equation reflects an influence of the ensemble on the dynamics of the beables of the individual systems. How can we understand this influence of an ensemble on an individual?

Embrace non-locality! Maybe the influence of the ensemble on the individual is due to the different atoms in the same state in different places really being indistinguishable-up to where they are and which one you get when you make a local measurement.
The principle of explanatory closure: Everything that causally influences the behavior of a physical system within the universe must be another physical system within the universe.

Hence:

- No influence of “potentialities” on “actualities”:
- No influence of epistemic ensembles which represent what might be true on the dynamics of a real particle.

If an ensemble influences an individual system, then every single member of that ensemble must be a real system somewhere in the universe.
Where in the universe are the members of the ensemble described by the wavefunction of a hydrogen atom in its ground state?

Could that ensemble be nothing but the collection of all the hydrogen atoms in their ground state in the universe? Might they somehow interact by virtue of their sharing the same state, leading to the influence of the ensemble on the individual?
ore questions:

Why do micro systems have indefinite values of some observables while macro systems have definite values?

Why do some properties of micro systems evolve stochastically?

Why are micro systems different from macro systems?

Hypothesis: because micro systems come in many copies in the universe.

Macro systems are then systems that are unique in the universe.

The members of the ensemble described by the wavefunction of a hydrogen atom in its ground state are all the other hydrogen atoms in the universe with similar preparation and environment.
Putting these ideas together:

The members of the ensemble described by the wavefunction of a hydrogen atom in its ground state are all the other hydrogen atoms in the universe with similar preparation and environment.

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\( \rho(x) \) and \( S(x) \) represents information about an ensemble, reconstructable from knowledge of all the beables, \((x(t), e^{i\Phi(t)})\) for all members of the ensemble.

The Schroedinger equation reflects the interplay between the dynamics of an individual and an ensemble.
The real ensemble framework:

Kinematics
Dynamics
Principles of real ensemble quantum theory: *kinematics*

Quantum mechanics describes a small subsystem $S$ of the universe.

$S$ has beables $(b(t), e^{i \varphi(t)})$
- $b(t)$ are the possible outcomes of some complete measurement.
- $e^{i \varphi(t)}$ are also beables, but not directly measurable.

$S$ is a member of an ensemble of similarly constituted and prepared subsystems in the universe, i.e. $S = S_l$ which is a member of $\{ S_l \}$, $l = l,...N$.

The total state of the ensemble is $\{(b_l(t), e^{i \varphi_l(t)}))\}$.
- $n(b,t)$ is the number of systems with the beable $b$ at time $t$.
- Auxiliary hypothesis: the dynamics evolves to states where $\varphi_l(t) = \varphi(b_l(t))$.
- Hence the ensemble has a probability density $\rho(b)$ and a phase $\varphi(b)$.

$$\rho(b) = \frac{n(b)}{N}$$
How could the members of the ensemble of systems with the same quantum states interact, so as to produce the Schrödinger equation?

The dependence of the individual trajectory on $\rho(b)$ means that the dynamics by which individual beables change must depend on how many copies of the different beables there are in the ensemble in the universe.

Copy hypothesis: the beables evolve by systems copying the beables of other members of the ensemble.
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$$P(I \text{ copy } J) = F(n_I, \phi_I, n_J, \phi_J, b_I, b_J)$$

When this happens:

$$b_I \rightarrow b_J, \quad \phi_I \rightarrow \phi_J$$

Notation:

- $n_I$ is the number of systems in the ensemble with the same value of $b$.
- $n_b$ is the number of systems in the ensemble with the beable value $b$.
- $b_I$ or $a_I$ is the value of the beable in the system $I$.
- $n_I = n_{b_I}$
How do the phases \( \varphi_I(t) \) evolve?

- There is a continuous and deterministic evolution rule for the \( \varphi_I(t) \)

\[
\dot{\phi}_I = \sum_J G(n_I, \phi_I, n_J, \phi_J, b_I, b_J)
\]
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When this happens

$$b_I \rightarrow b_J, \quad \phi_I \rightarrow \phi_J$$

• There is otherwise a continuous and deterministic evolution rule for the $\varphi_I(t)$:

$$\dot{\varphi}_I = \sum_J G(n_I, \varphi_I, n_J, \varphi_J, b_I, b_J)$$
Summary of dynamics

Copy hypothesis: the beables evolve by systems copying the beables of other members of the ensemble. This is given by a stochastic rule:
The rate $P(I \text{ copy } J)$ that system $I$ copies the beables of system $J$:

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- There is otherwise a continuous and deterministic evolution rule for the $\varphi_I(t)$

$$\dot{\phi}_I = \sum_J G(n_I, \phi_I, n_J, \phi_J, b_I, b_J)$$

Can we choose these evolution rules to reproduce QM?
The evolution rules consistent with quantum dynamics are:

\[ \mathcal{P}(I \copy J) = \frac{1}{\sqrt{n_in_J}} R_{a_I a_J} \sin^+(\phi_I - \phi_J + \delta_{a_I a_J}) \]

\[ \dot{\phi}_I = \omega_{a_I} + \sum_{J \neq I} \frac{1}{\sqrt{n_in_J}} R_{a_I a_J} \cos(\phi_I - \phi_J + \delta_{ab}) \]

- The dynamics is specified by \( R_{ab}, \delta_{ab} \) which go into the Hamiltonian.

- Note that the rates for two systems to interact depend on the number of copies of each in the universe. This is highly non-local.

- \( n_I \) cannot vanish.

- \( E_{ij} \) and \( D_{ij} \) are terms that are subdominant when the \( n_I \gg 1 \).

- These make sense for general \( \phi_I \) but they preserve the condition \( \phi_I = \phi_{a_I} \).

  In particular if the preparation gives a unique initial state concentrated on one \( a \), then the phases are aligned initially and stay aligned.
Recovery of quantum dynamics: details
Simplifying assumptions:

The beable states are discrete: \( b \in \{b_a\}, \ a = 1, \ldots P \)

\( b \) is given by \( b_a \), \( a \) labels the possible beable states

The phases of systems with the equal \( b \)'s are equal \( \phi^I = \phi_{b^I} \)

This implies:

\[
\phi^I = \sum_J G(n_{a_I}, \phi_{a_I}, n_{a_J}, \phi_{a_J}, a_I, a_J)
\]

\[
= \sum_b n_b G(n_{a_I}, \phi_{a_I}, n_{a_J}, \phi_{a_J}, a_I, a_J)
\]

\[
= \sum_b G'(n_{a_I}, \phi_{a_I}, n_{a_J}, \phi_{a_J}, a_I, a_J)
\]
Recovery of quantum dynamics:
details
so we have:

\[ \dot{n}_a = \sum_{b \neq a} n_b T_{b \rightarrow a} - n_a T_{a \rightarrow b} \]

\[ = \sum_{b \neq a} n_b n_a [F_{ab} - F_{ba}] \]

\[ T_{b \rightarrow a} = P \left( I_{copy} J \right) |_{a_I = b, a_J = a} n_a = F(n_{a_I}, \phi_{a_I}, n_{a_J}, \phi_{a_J})_{ab} n_a \]

We can define a probability density:

\[ \rho_a = \frac{a_a}{N} \]

\[ \dot{\rho}_a = \sum_{b \neq a} (\rho_b T_{b \rightarrow a} - \rho_a T_{a \rightarrow b}) \]
We can check our reasoning by explicit calculation:

\[
\dot{n}_a = \sum_{I} \sum_{J \neq I} \delta_{aa_J} (1 - \delta_{aa_I}) [P(I \text{copy} J) - P(J \text{copy} I)] \\
= \sum_{I} \sum_{J \neq I} \sum_{b \neq a} \delta_{aa_J} \delta_{ba_I} [P(I \text{copy} J) - P(J \text{copy} I)] \\
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\]
Summary so far:

\[ \dot{\rho}_a = \sum_b \sqrt{\rho_a \rho_b} \left( \mathcal{R}(e^{\nu(\phi_a - \phi_b)})_{ab} - \mathcal{R}(e^{\nu(\phi_b - \phi_a)})_{ba} \right) \]

\[ \dot{\phi}_a = \omega_a + \sum_{b \neq a} \left( \frac{n_a}{n_b} \right)^r \mathcal{U}(e^{\nu(\phi_a - \phi_b)})_{ab} \quad \omega_a = \mathcal{U}_{aa} \]

The first term suffices:

\[ \mathcal{R}(e^{\nu(\phi_a - \phi_b)})_{ab} = R_{ab} \sin^+(\phi_a - \phi_b + \delta_{ab}) \quad R_{ab} = R_{ba} \]

\[ \mathcal{U}(e^{\nu(\phi_a - \phi_b)})_{ab} = R_{ab} \cos(\phi_a - \phi_b + \delta_{ab}) \]

This gives equations of motion:

\[ \dot{\rho} = \sum_{b \neq a} \sqrt{\rho_a \rho_b} R_{ab} \sin(\phi_a - \phi_b + \delta_{ab}) \]

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Correspondence to quantum mechanics:

\[ i\hbar \frac{d\Psi}{dt} = \hat{H}\Psi \]

\[
|\Psi> = \begin{pmatrix}
\sqrt{\rho_1}e^{-i\phi_1} \\
\sqrt{\rho_2}e^{-i\phi_2} \\
\vdots \\
\sqrt{\rho_M}e^{-i\phi_M}
\end{pmatrix}
\]

\[
\hat{H} = \begin{pmatrix}
\hbar\omega_1 & R_{12}e^{i\delta_{12}} & \ldots \\
R_{12}e^{-i\delta_{12}} & \hbar\omega_2 & \ldots \\
\vdots & \vdots & \ddots
\end{pmatrix}
\]

With \( r = -1/2 \), this gives the same equations of motion:

\[
\dot{\rho} = \sum_{b \neq a} \sqrt{\rho_a\rho_b}R_{ab}\sin(\phi_a - \phi_b + \delta_{ab})
\]

\[
\dot{\phi}_a = \omega_a + \sum_{b \neq a} \sqrt{\frac{\rho_b}{\rho_a}}R_{ab}\cos(\phi_a - \phi_b + \delta_{ab})
\]
An ansatz:

\[
F\left(\frac{n_a}{n_b}, e^{i(\phi_a - \phi_b)}\right)_{ab} = \frac{1}{n_a} \left(\frac{n_a}{n_b}\right)^q \mathcal{R}\left(e^{i(\phi_a - \phi_b)}\right)_{ab}
\]

\[
G'\left(\frac{n_a}{n_b}, e^{i(\phi_a - \phi_b)}\right)_{ab} = \left(\frac{n_a}{n_b}\right)^r \mathcal{U}\left(e^{i(\phi_a - \phi_b)}\right)_{ab}
\]

Note $F > 0$ and $R > 0$

Some reversal invariance imposes:

\[
t \rightarrow -t
\]
\[
\rho_a \rightarrow \rho_a
\]
\[
\phi_a \rightarrow -\phi_a
\]
\[
q = \frac{1}{2}
\]
\[
\mathcal{U}(z)_{ab} = \mathcal{U}(\bar{z})_{ab}
\]
\[
\mathcal{R}\left(e^{i(\phi_a - \phi_b)}\right)_{ba} = \mathcal{R}\left(e^{i(\phi_a - \phi_b)}\right)_{ab}
\]

\[
\dot{\rho}_a = \sum_{b \neq a} (\rho_b F_{ab} - \rho_a F_{ba})
\]
\[
\dot{\phi}_a = \sum_b G' \left( n_a, \phi_a, n_b, \phi_b \right)_{ab}
\]
We can check our reasoning by explicit calculation:

\[
\dot{n}_a = \sum_{I} \sum_{J \neq I} \delta_{aa_J} (1 - \delta_{aa_I}) \left[ P(I\text{copy} J) - P(J\text{copy} I) \right]
\]

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= \sum_{I} \sum_{J \neq I} \sum_{b \neq a} \delta_{aa_J} \delta_{ba_I} \left[ P(I\text{copy} J) - P(J\text{copy} I) \right]
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= \sum_{b \neq a} n_b n_a \left[ F_{ab} - F_{ba} \right]
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\[
T_{b \rightarrow a} = P(I\text{copy} J)_{a_I = b, a_J = a} n_a = F(n_{a_I}, \phi_{a_I}, n_{a_J}, \phi_{a_J})_{ab} n_a
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In ansatz:

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\]

\[
G'(\frac{n_a}{n_b}, e^{i(\phi_a-\phi_b)})_{ab} = \left(\frac{n_a}{n_b}\right)^r \mathcal{U}(e^{i(\phi_a-\phi_b)})_{ab}
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\[ \hat{H} = \begin{pmatrix} \hbar\omega_1 & R_{12}e^{i\delta_{12}} & \cdots \\ R_{12}e^{-i\delta_{12}} & \hbar\omega_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \]

With r = -1/2, this gives the same equations of motion:

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P(I \text{copy } J) = \frac{1}{\sqrt{n_In_J}} R_{a_la_J} \sin^+(\phi_I - \phi_J + \delta_{a_la_J}) + \mathcal{E}_{IJ}
\]

\[
\dot{\phi}_I = \omega_a + \sum_{J \neq I} \frac{1}{\sqrt{n_In_J}} R_{a_la_J} \cos(\phi_I - \phi_J + \delta_{ab}) + D_{IJ}
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• The dynamics is specified by \( R_{ab}, \delta_{ab} \) which go into the Hamiltonian.

• Note that the rates for two systems to interact depend on the number of copies of each in the universe. This is highly non-local.

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• Note that the rates for two systems to interact depend on the number of copies of each in the universe. This is highly non-local.

• \( n_I \) cannot vanish.

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• These make sense for general \( \varphi_I \) but they preserve the condition \( \phi_I = \phi_a \).

In particular if the preparation gives a unique initial state concentrated on one \( a \), then the phases are aligned initially and stay aligned.
A model for phase alignment
model for phase alignment:

To complete the proposal we need a dynamics that drive $\phi_I \rightarrow \phi_{a_I}$

Here is a possible model which has solutions which do this:

\[
\dot{\phi}_I = \int dt \sum_I \left[ \pi^I (\dot{\phi}_I - \Omega_I(\phi, n)) - \frac{1}{2} (\pi^I)^2 - \frac{f^2}{2} \sum_{J \in a_I} \sin^2(\phi_I - \phi_J) \right]
\]

Where:

$\Omega_I = \omega_{a_I} + \sum_{J \neq I} \frac{1}{\sqrt{n_In_J}} R_{a_Ia_J} \cos(\phi_I - \phi_J + \delta_{ab}) = \dot{\phi}_I$
model for phase alignment:

To complete the proposal we need a dynamics that drive \( \phi_I \to \phi_{a_I} \).

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\[
\begin{align*}
\dot{\pi}^I &= \sum_I \left[ \pi^I (\dot{\phi}_I - \Omega_I(\phi, n)) - \frac{1}{2}(\pi^I)^2 - \frac{f^2}{2} \sum_{J \in a_I} \sin^2(\phi_I - \phi_J) \right] \\
\Omega_I &= \omega_{a_I} + \sum_{J \neq I} \frac{1}{\sqrt{n_In_J}} R_{a_Ia_J} \cos(\phi_I - \phi_J + \delta_{ab}) = \dot{\phi}_I \\
\text{Canonical momenta:} \quad \pi^I &= \dot{\phi}_I - \Omega_I(\phi, n) \quad \{\phi_I, \pi^J\} = \delta_{IJ} \\
\text{Hamiltonian:} \quad H &= \sum_I \left[ \frac{1}{2}(\pi^I)^2 + \pi^I \Omega_I(\phi, n) + \frac{f^2}{2} \sum_{J \in a_I} \sin^2(\phi_I - \phi_J) \right]
\end{align*}
\]
model for phase alignment:

To complete the proposal we need a dynamics that drive $\phi I \rightarrow \phi_{aI}$

Here is a possible model which has solutions which do this:

\[
\Omega_I = \omega_{aI} + \sum_{J \neq I} \frac{1}{\sqrt{n_In_J}} R_{aIaJ} \cos(\phi_I - \phi_J + \delta_{ab}) = \dot{\phi}_I
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Canonical momenta:

$$\pi^I = \dot{\phi}_I - \Omega_I(\phi, n) \quad \{ \phi_I, \pi^J \} = \delta^J_I$$

Hamiltonian:

$$H = \sum_I \left[ \frac{1}{2} (\pi^I)^2 + \pi^I \Omega_I(\phi, n) + \frac{f^2}{2} \sum_{J \in a_I} \sin^2(\phi_I - \phi_J) \right]$$
Consider first the approximation: \( f \gg \omega_a, |R_{ab}| \)

the approximate Hamiltonian:

\[
H = \sum_i \left[ \frac{1}{2}(\pi I)^2 + \frac{f^2}{2} n_I (\phi_I - \bar{\phi}_{a_i})^2 \right]
\]

there is a degenerate ground state which drives the phase to its average:

\[
\phi_I \to \bar{\phi}_{a_i} \quad \bar{\phi}_{a_i} = \frac{1}{n_I} \sum_{J \in a_i} \phi_J
\]

the other eqn of motion tells us that when the phases align the momenta vanish:

\[
\dot{\pi}^I = -f^2 n_I (\phi_I - \bar{\phi}_{a_i}) + \ldots
\]

at which point the quantum evolution is recovered:

\[
\Omega_I = \omega_{a_i} + \sum_{J \neq I} \frac{1}{\sqrt{n_I n_J}} R_{a_i a_J} \cos(\phi_I - \phi_J + \delta_{ab}) = \dot{\phi}_I
\]
The approximate solutions:

\[ \phi_I = \bar{\phi}_{a_I}, \quad \pi^I = 0, \quad \dot{\pi}^I = 0 \]

remains solution to the full equations of motion:

\[ \dot{\pi}^I = -f^2 \sum_{J \in a_I} \sin(\phi_I - \phi_J) \cos(\phi_I - \phi_J) - \sum_K \pi^K \frac{\partial \Omega_K(\phi, n)}{\partial \phi_I} \]

\[ \pi^I = \dot{\phi}_I - \Omega_I(\phi, n) \]

Thus, phase alignment and Schrödinger dynamics are achieved in a set of solutions to the model.

\[ \Omega_I = \omega_{a_I} + \sum_{J \neq I} \frac{1}{\sqrt{n_In_J}} R_{a_Ia_J} \cos(\phi_I - \phi_J + \delta_{ab}) = \dot{\phi}_I \]

Here remain open issues about the stability of these solutions.
Hamiltonian:

\[
H = \sum_I \left[ \frac{1}{2}(\pi^I)^2 + \pi^I \Omega_I(\phi, n) + \frac{f^2}{2} \sum_{J \in a_I} \sin^2(\phi_I - \phi_J) \right]
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Equations of motion:

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There remain open issues about the stability of these solutions.
When the phases are aligned the system reduces to a lagrangian system with:

\[ s = \int dt \sum_a \left( \rho_a (\dot{\phi}_a - \omega_a) - \sum_{b \neq a} \sqrt{\rho_a \rho_b} R_{ab} \cos(\phi_a - \phi_b + \delta_{ab}) \right) \]
model for phase alignment:

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$$\mathcal{L} = \int dt \sum_I \left[ \pi_I (\dot{\phi}_I - \Omega_I(\phi, n)) - \frac{1}{2} (\pi^I)^2 - \frac{f^2}{2} \sum_{J \in a_I} \sin^2(\phi_I - \phi_J) \right]$$

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Thus, phase alignment and Schrodinger dynamics are achieved in a set of solutions to the model.

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An action for quantum dynamics

When the phases are aligned the system reduces to a lagrangian system with:

\[
\mathcal{L} = \int dt \sum_a \left( \rho_a \left( \dot{\phi}_a - \omega_a \right) - \sum_{b \neq a} \sqrt{\rho_a \rho_b} R_{ab} \cos(\phi_a - \phi_b + \delta_{ab}) \right)
\]
The evolution rules consistent with quantum dynamics are:

\[
\mathcal{P}(I \text{ copy } J) = \frac{1}{\sqrt{n_Im_J}} R_{a_Ia_J} \sin^+(\phi_I - \phi_J + \delta_{a_Ia_J}) + \mathcal{E}_{IJ}
\]

\[
\dot{\phi}_I = \omega_{a_I} + \sum_{J \neq I} \frac{1}{\sqrt{n_Im_J}} R_{a_Ia_J} \cos(\phi_I - \phi_J + \delta_{ab}) + \mathcal{D}_{IJ}
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- The dynamics is specified by \( R_{ab}, \ \delta_{ab} \) which go into the Hamiltonian.
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- These make sense for general \( \varphi_I \) but they preserve the condition \( \phi_I = \phi_{a_I} \).

In particular if the preparation gives a unique initial state concentrated on one \( a \), then the phases are aligned initially and stay aligned.
Correspondence to quantum mechanics:

\[ \frac{i\hbar}{dt} \Psi = \hat{H} \Psi \]

\[ |\Psi\rangle = \begin{pmatrix} \sqrt{\rho_1} e^{-i\phi_1} \\ \sqrt{\rho_2} e^{-i\phi_2} \\ \vdots \\ \sqrt{\rho_M} e^{-i\phi_M} \end{pmatrix} \]

\[ \hat{H} = \begin{pmatrix} \hbar\omega_1 & R_{12} e^{i\delta_{12}} & \cdots \\ R_{12} e^{-i\delta_{12}} & \hbar\omega_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \]

With \( r = -1/2 \), this gives the same equations of motion:

\[ \dot{\rho} = \sum_{b\neq a} \sqrt{\rho_a \rho_b} R_{ab} \sin(\phi_a - \phi_b + \delta_{ab}) \]

\[ \dot{\phi}_a = \omega_a + \sum_{b\neq a} \sqrt{\frac{\rho_b}{\rho_a}} R_{ab} \cos(\phi_a - \phi_b + \delta_{ab}) \]
Model for phase alignment:

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Action principle:

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\]

Where:

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\Omega_I = \omega_{a_I} + \sum_{J \neq I} \frac{1}{\sqrt{n_In_J}} R_{a_Ia_J} \cos(\phi_I - \phi_J + \delta_{ab}) = \dot{\phi}_I
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Issues
The subsystem problem

Consider a quark. It is a subsystem of a proton, which is a subsystem of a nucleus, which is a subsystem of an atom, a molecule, etc. What determines which ensemble is relevant for purposes of the copy dynamics?

At each level the subsystem is described by a density matrix. \( \rho_I = Tr \rho_{I+1} \)

There is only one level, \( I \) such that \( \rho_1 \) is pure and not a product state. That is the pure state the ensemble refers to.
The classical limit problem

There is an apparent paradox: It appears we can reach the classical limit in two steps:

1. Copy dynamics
   \[ n_a > 1 \]
2. Quantum dynamics
   \[ \hbar \to 0 \]
3. Classical dynamics

But macroscopic systems have no copies. Hence the first step fails. So is there any reason for macroscopic systems to obey classical dynamics?
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Possible answer: all that is required is for the centre of mass of a classical body to obey classical dynamics. We can show this is the case if the ensemble of positions of the atoms in the body obey quantum dynamics.
model of the classical limit problem:

An ensemble of atoms on a one dimensional periodic lattice. P sites, lattice spacing a. Beables, a, b, ... refer to site

Hopping dynamics:

\[ R_{ab} = \frac{\hbar}{2ma^2} (\delta_{ab+1} + \delta_{ab-1}) \]
Start with the quantum action

\[ S = \int dt \sum_a \left( \rho_a (\dot{\phi}_a - \omega_a) - \sum_{b \neq a} \sqrt{\rho_a \rho_b} R_{ab} \cos(\phi_a - \phi_b + \delta_{ab}) \right) \]

Hopping dynamics:

\[ R_{ab} = \frac{\hbar}{2m \bar{a}^2} (\delta_{ab+1} + \delta_{ab-1}) \]

Define the potential energy

\[ V(a) = \hbar \omega_a + \frac{\hbar^2}{ma^2} \]

and let $S$ and $\rho$ be slowly varying on the scale $\bar{a}$.

\[ \dot{S} = \hbar S = \int dt \sum_a \rho_a \left( \dot{S} + \frac{1}{2m} (\partial_x S)^2 - V(a) - V_Q + O(\bar{a}) \right) \]

$\hbar$ only appears in the quantum potential:

\[ V_Q = \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} \]
neglect the quantum potential and we get an ensemble of classical systems.

\[ S = \int dt \sum \rho_a \left( \dot{S} + \frac{1}{2m} (\partial_x S)^2 - V(a) - V_Q + O(\tilde{a}) \right) \]

The equations of motion give classical ensemble dynamics:

\[ \dot{\rho} = \frac{1}{m} \partial_x (\rho \partial_x S) \quad \dot{S} = -\frac{1}{2m} (\partial_x S)^2 + V \]

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With a current velocity:

\[ v = \frac{1}{m} \partial_x S \]

To make both steps from copy dynamics to classical dynamics work choose the ensemble to be the collection of atoms making up the macroscopic body. Then choose the lattice spacing much larger than the atomic spacing so that the occupation numbers are large and the phases are classical.

\[ n_a >> 1, \quad \hbar/S << 1 \]
Thus, with the lattice spacing large we can reach the classical limit in two steps:

Copy dynamics

$\eta \gg 1$,

Quantum dynamics

$\hbar/S << 1$

Classical dynamics
The getting stuck at nodes problem

Suppose in the initial state defined by the preparation $n_a(t=0) = 0$ for some $a=a_0$. Then it follows that $n_a(t) = 0$ for all time. There is nothing to copy.

Indeed: \[ \dot{n}_a = \sum_{b \neq a} (n_b n_a F_{ab} - n_a n_b F_{ba}) = \sum_b \sqrt{n_a n_b} R_{ab} \sin(\phi_a - \phi_b + \delta_{ab}) \]

finishes when $n_a = 0$.

At the Schrödinger equation does not preserve $\rho_a = 0$. 

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\]

finishes when \( n_a = 0 \).

But the Schrödinger equation does not preserve \( \rho_a = 0 \). To see the problem look at the second derivative

\[
\ddot{n}_a = \sum_b \sqrt{\frac{n_b}{n_a}} \dot{n}_a R_{ab} \sin(\phi_a - \phi_b + \delta_{ab}) + \ldots
\]

\[
= \sum_{b \neq a} \sum_{c \neq a} \sqrt{\frac{n_b}{n_a}} \sqrt{n_a n_c} R_{ab} R_{ac} \sin(\phi_a - \phi_b + \delta_{ab}) \sin(\phi_a - \phi_c + \delta_{ab}) + \ldots
\]

\[
= \sum_{b \neq a} \sum_{c \neq a} \sqrt{n_b n_c} R_{ab} R_{ac} \sin(\phi_a - \phi_b + \delta_{ab}) \sin(\phi_a - \phi_c + \delta_{ab})
\]

The last line is incorrect when \( n_a = 0 \).
responses:

We assumed all $n_I$ and $n_a$ were large in arguing for the form that led to quantum theory. If this viewpoint is correct that quantum dynamics fails for systems that are states that are unique in the universe. There could be other terms that come in.

\[
\mathcal{P}(I \text{copy} J) = \frac{1}{\sqrt{n_I n_J}} R_{a_I a_J} \sin^+ (\phi_I - \phi_J + \delta_{a_I a_J})
\]

\[
\phi_I = \omega a_I + \sum_{J \neq I} \frac{1}{\sqrt{n_I n_J}} R_{a_I a_J} \cos (\phi_I - \phi_J + \delta_{ab})
\]

Nonetheless the problem is easy to address also within the current rules. All that is required is either

1) require that the basis chosen for the beables is such that no $n_a=0$ or

2) add to the universe a small number of spectator states in each possible $a$ so that no $n_a=0$.

3) Insist on a tiny admixture to every state of a state with all $n_a$ non-vanishing such as the ground state.
urther points that need investigation:

What exactly defines the ensemble?
• Same constituents, hence same hamiltonian.
• Same preparation, i.e., same initial quantum state.

The notion of similarly prepared and constituted systems defines the equivalence of a quantum state. This use of macrosystems to initialize and define preparations of microsystems as a primitive notion has in common with Bohr’s viewpoint that quantum physics requires a distinction between micro and macro systems. This demands that there be some more fundamental theory that quantum theory approximates for small subsystems of a universe.
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Does the ensemble require a preferred simultaneity to define it?
• Is that necessarily bad?
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How is the connection between linear operators and observables non-diagonal in the beables established? Presumably as in dBB probabilities computed in a single basis suffice but it would be good to clarify this.
Possible Empirical consequences:

Quantum dynamics should fail both for systems that have no duplicates in the universe and for systems in states that are unique in the universe.

• Can quantum informationalists produce a device that can be put into unique, coherent quantum states, unlikely to exist anywhere else?

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