Abstract: We propose an operationally motivated definition of the physical equivalence of states in General Probabilistic Theories and consider the principle of the physical equivalence of pure states, which turns out to be equivalent to the symmetric structure of the state space. We further consider a principle of the decomposability with distinguishable pure states and give classification theorems of the state spaces for each principle, and derive the Bloch ball in 2 and 3 dimensional systems.
ON BASIC PRINCIPLES
OF GENERAL PROBABILISTIC THEORIES

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OUTLINE

1: General Probabilistic Theories
2: Motivations and Goal
3: Symmetric GPT
   * Physical Equivalence of Pure States
4: Decomposability w.r.t. distinguishable Pure States
5: Obtain classical and quantum (Bloch Ball) in 3 dim.
6: What I don’t know...
GENERAL PROBABILISTIC THEORIES (GPT)

Mackey (1960), Araki (1961); Ludwig (1964-); Mielnik (1968), Devies and Lewis (1970), Gudder (1973), etc

* Operationally Most General Framework for Probability

State + Measurement $\Rightarrow$ Probability

* Probabilistic Mixture of states (Convex Structure)
* Separation postulates for states and measurements
* Physical Topology measured by probabilities
GENERAL PROBABILISTIC THEORIES (GPT)

* Operationally Meaningful States

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* Physical Topology measured by probabilities

\[ \forall \text{ states } \rho_1, \rho_2 \text{ and } p \in [0, 1], \]
\[ \exists \text{ state } \rho = \langle p; \rho_1, \rho_2 \rangle \]
as a preparation of \( \rho_1 \) with \( p \) and \( \rho_2 \) with \( 1 - p \):
GENERAL PROBABILISTIC THEORIES (GPT)

* Operationally Meaningful Operations

∀ states $\rho_1, \rho_2$ and $p \in [0, 1],
\exists$ state $\rho = \langle p; \rho_1, \rho_2 \rangle$
as a preparation of $\rho_1$ with $p$ and $\rho_2$ with $1 - p$:

$$\Pr\{M = m \mid \langle p; \rho_1, \rho_2 \rangle\} =$$

$$p\Pr\{M = m \mid \rho_1\} + (1 - p)\Pr\{M = m \mid \rho_2\}$$

for any measurement $M$

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* Operationally Most General Framework for Probability

1 Separation Postulate for states:
∀ measurement $M$ and outcome $m$,
$\Pr\{m|M, \rho_1\} = \Pr\{m|M, \rho_2\}$, then $\rho_1 = \rho_2$

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2. **Separation Postulate for measurements:**
   \[ \forall \text{ states } \rho, \]
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**State + Measurement ⇒ Probability**

★ State space $S$ is embedded into a convex subset in a real vector sp.
  s.t. $\langle p; \rho_1, \rho_2 \rangle = p \rho_1 + (1 - p) \rho_2$

★ Measurement is represented by effects:
  $E := (e_i \in \mathcal{E})_{i=1}^n$, s.t. $\sum_i e_i = u$, $e_i(\rho)$: prob. for $i$th output under $\rho$

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★ Physical Topology measured by probability

\[ e \in \mathcal{E}: \text{effect on } S: \]
\[ e : S \rightarrow [0, 1] \text{ affine:} \]
\[ e(p\rho_1 + (1 - p)\rho_2) = pe(\rho_1) + (1 - p)e(\rho_2) \]
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Weakest topology s.t.
all effects are continuous on $S.$
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State + Measurement $\Rightarrow$ Probability

★ State space $S$ is embedded into a (pre)compact convex subset in a locally convex Hausdorff topological vector space $V$.
★ Measurement is represented by effects:
\[ E := (e_i \in \mathcal{E})_{i=1}^{n}, \text{s.t. } \sum_{i} e_i = u, \ e_i(\rho) : \text{prob. for } i\text{th output under } \rho \]

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★ Physical Topology measured by probabilities

**Theory of Convex Set and Affine Function (PreCompact Convex Set in Locally Convex Hausdorff Topological Vector Space)**
If $S = d < \infty$

$S$ is a compact (i.e. closed bounded) convex of $E^d$
GENERAL PROBABILISTIC THEORIES (GPT)

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If \( S = d < \infty \)

\( S \) is a compact (i.e. closed bounded) convex of \( E^d \)

Any compact convex set you imagine,

you can consider its general probabilistic model.
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**Pure State**

\[ \text{def} \] State which cannot be prepared by probabilistic mixtures of different states.

\[ \leftrightarrow \] Extreme Point of State Space
★ Quantum System:  \( S_d^Q := \{ \rho \in \mathcal{L}(\mathcal{H}_d) : \rho \geq 0, \, \text{tr}\rho = 1 \} \)

\[ d = 2 \]

\[ d \geq 3 \]

* ∞ Extreme Points
**Classical System:** \( \mathcal{P}_d := \{ \mathbf{p} = (p_1, \ldots, p_d) \in \mathbb{R}^d \mid p_i \geq 0, \sum_i p_i = 1 \} \)

Simplex of Affine Dimension \( d-1 \) with \( d \) Pure States
EXAMPLES OF GPT II

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* \( \infty \) Extreme Points
EXAMPLES OF GPT III

★ Hyper Cuboid System: $C_d = \{ x \in \mathbb{R}^d \mid 0 \leq x_i \leq 1 \}$

$d = 2$

$d = 3$

* $2^d$ Extreme Points
* Affine Dimension $d$
EXAMPLES OF GPT III

★ Hyper Cuboid System: \( C_d = \{x \in \mathbb{R}^d \mid 0 \leq x_i \leq 1\} \)

\( d = 2 \quad d = 3 \)

* 2\(^d\) Extreme Points
* Affine Dimension \( d \)

\[ \begin{array}{cccccccccc}
0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
\end{array} \]
**EXAMPLES OF GPT III**

Classical

\[ X_d = \{ \mathbf{x} \in \mathbb{R}^d \mid 0 \leq x_i \leq 1 \} \]

* 2^{d} Extreme Points
* Affine Dimension \( d \)

...
$S_d = \{ x \in \mathbb{R}^d \mid 0 \leq x_i \leq 1 \}$

* 2^{d} Extreme Points
* Affine Dimension $d$
$C_d = \{ x \in \mathbb{R}^d \mid 0 \leq x_i \leq 1 \}$

Prop: Hyper Cuboid system $C_d$ can be realized by a $d$ bit classical system by restricting to 1 bit measurement.

* 2^d Extreme Points
* Affine Dimension $d$
EXAMPLES OF GPT III

Classical

$\mathcal{C}_d = \{ \mathbf{x} \in \mathbb{R}^d \mid 0 \leq x_i \leq 1 \}$

HC

* $2^d$ Extreme Points
* Affine Dimension $d$

Prop: Hyper Cuboid system $\mathcal{C}_d$ can be realized by a $d$ bit classical system by restricting to $1$ bit measurement

Thm: [Holevo 1982] Any GPT can be realized by a Classical System with an appropriate restriction on measurements

...
INFORMATION THEORY BASED ON GPT

* Classical Prob. Theory  $\Rightarrow$ Classical Information Theory
* Quantum Theory  $\Rightarrow$ Quantum Information Theory

GPT  $\Rightarrow$ GPT Information Theory

Motivations

- Seek for Physical Principles of Quantum Theory in terms of information languages
- Understand logical connections between physical principles and information processing.
- Preparation for “post” quantum
- Classical/Quantum Information theory under Measurement restrictions
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INFORMATION THEORY BASED ON GPT


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☞ Seek for Physical Principles of Quantum Theory in terms of information languages
☞ Understand logical connections between physical principles and information processings.
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☞ (Classical/Quantum) Information theory under Measurement restrictions
“Natural” Physical Principles:

*1 Physical Equivalence of Pure States
   ⇔ Symmetricity of State Space (Davies)
   ⇔ Reversible Connections of Pure States (Hardy etc.)

*2 Decomposability w.r.t. Distinguishable Pure States

⇒ Classification of GPTs for each Principle

* Kimura, Nuida, Imai (2010); arXiv:1012.5361v2
* Kimura, Nuida, (2010); arXiv:1012.5350v2
THIS TALK...

What We also Assume:

* Existence of Objective (Classical) World
* Causality
* All Mathematically Well-defined Measurements:
  $$(e_i \in \mathcal{E})_i \text{ s.t. } \sum_i e_i = u$$
  are physically realizable.
SYMMETRICITY OF STATE SPACE

[P1] Group of Affine Bijection on $S$ acts transitively on $\partial S$

(Davies 1974)

$\Leftrightarrow$ [P1′] Any pure states are connected by Reversible Transformation

(Hardy, ...)
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meaning that there are no “special” Pure States
SYMMETRICITY OF STATE SPACE

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(Davies 1974)

\[ \Leftrightarrow \quad \text{[P1']} \quad \text{Any pure states are connected by Reversible Transformation} \]

(Hardy, ...)

\[ \Leftrightarrow \quad \text{[P1'']} \quad \text{Any pure states are Physically Equivalent} \]

meaning that there are no “special” Pure States
PHYSICAL EQUIVALENCE OF STATES

State s1 and s2 have physically the same properties if

* For any measurement E1,
  there uniquely exists measurement E2 s.t.
  prob. dist of E1 under s1 equals prob. dist. of E2 under s2
* The correspondence should preserves
  probabilistic mixture of measurements
PHYSICAL EQUIVALENCE OF STATES

State $s_1$ and $s_2$ have physically the same properties if

* For any measurement $E_1$, there uniquely exists measurement $E_2$ s.t.
  prob. dist of $E_1$ under $s_1$ equals prob. dist. of $E_2$ under $s_2$
* The correspondence should preserves probabilistic mixture of measurements

[Def] State $s_1$ and $s_2$ are physically equivalent iff there exists unit preserving affine bijection $\Lambda$ on $\mathcal{E}$ such that

$$e(s_1) = \Lambda(e)(s_2) \forall e \in \mathcal{E}$$

* Physical Equivalence is an equivalence relation
* Physical Equivalence in QM is unitary equivalence:

$$\rho_1 \sim \rho_2 \iff \exists \text{unitary } U \text{ s.t. } \rho_1 = U\rho_2U^\dagger$$
[Lem] For any affine functional $\Phi : \mathcal{E} \rightarrow [0, 1]$ satisfying $\Phi(u) = 1$ and $\Phi(0) = 0$, there uniquely exists a state $s \in \mathcal{S}$ such that $\Phi(e) = e(s)$ for any $e \in \mathcal{E}$.

[Thm] State $s_1$ and $s_2$ are physically equivalent iff there exists an affine bijection $\Psi$ on $\mathcal{S}$ such that $s_1 = \Psi(s_2)$.

[P1′] Any pure states are connected by Reversible Transformation

$\iff$ [P1″] Any pure states are Physically Equivalent

We call Symmetric GPT if it satisfy [P1] ( [P1′] or [P1″])
[Thm] State Space of Symmetric GPT with finite numbers of pure states is isomorphic to Isogonal Figure
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CLASSIFICATION OF SYMMETRIC GPTS

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[Thm] State Space of 2-dimensional Symmetric GPT with infinite numbers of pure states is isomorphic to a disk.
CLASSIFICATION OF SYMMETRIC GPTS

[Thm] State Space of Symmetric GPT with finite numbers of pure states is isomorphic to Isogonal Figure

[Thm] State Space of 2-dimensional Symmetric GPT with infinite numbers of pure states is isomorphic to a disk.

[Thm] State Space of 3-dimensional Symmetric GPT with infinite numbers of pure states is either a ball or a circular cylinder.
**DIM S = 3**

Classical Sys.

or

or

Quantum Sys.
(Bloch Ball)
DIM $S = 3$

Classical Sys.

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Classical Sys.  or  Quantum Sys. (Bloch Ball)

[P2] Decomposability w.r.t. Distinguishable Pure States
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For any state $s$, there exists a set of distinguishable pure states $\{s_i\}_i$ such that

$$s = \sum_i p_i s_i$$

with some probability dist. $(p_i)_i$

Any state can be prepared as an ensemble of distinguishable pure states.

In QM, this corresponds to Eigenvalue Decomposition of Density Op.
ANOTHER PRINCIPLES

[P2] Decomposability w.r.t. Distinguishable Pure States:

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[P3] Dynamical Generation of State Space by Classical System:

$$S = \bigcup_{\Lambda \in G} \Lambda(Simplex)$$
[P2] Decomposability w.r.t.

For any state \( s \), there exists a decomposition

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[P3] Dynamical Generation of State Space by Classical System:

$$S = \bigcup_{\Lambda \in G} \Lambda(\text{Simplex})$$
SYMMETRIC GPT WITH [P2] (OR [P3])

[Prop] GPT with finite numbers of pure states satisfying [P2] is a simplex.
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[Prop] (Dim $S = 3$) Circular Cylinder does not satisfy [P2].
WHAT I DON’T KNOW...

* Complete Classification of Symmetric GPT in arbitrary Dimension
* Complete Classification of GPT with \([p2]\) in arbitrary Dimension
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[Thm] 3-dimensional Symmetric GPT with [P2] is either classical or quantum (Bloch Ball)
[Prop] GPT with finite numbers of pure states satisfying [P2] is a simplex.
[P2] Decomposability w.r.t. state $s$

For any state $s$, there exists a decomposition $s = \sum_i p_i s_i$ with some probability $p_i$.

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[P3] Dynamical Generation of State Space by Classical System:

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* To obtain QM with Infinite Dimension
  or Quantum Field Theory with Infinite Numbers of Degrees
* Operational Approach..
  -- Identification of states and the use of affinity seems not universal
  -- Is there any massage for interpretation of QM?
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