Outline

- Fermi Telescope

- Dispersion relationship

\[ E^2 - p^2 + \alpha \frac{E^3}{m_{\text{QG}}} = m^2 \]

http://fermi.gsfc.nasa.gov

- L. Freidel and L. Smolin, *Gamma ray bursts probe the geometry of momentum space*
Doubly Special Relativity

**DSR**
- Observer independence is preserved
- Two constants are invariant

**Problems with DSR**
- Soccer Ball Problem
- Observer independence leads to non-locality
Principle of Relative Locality

Physics takes place in phase space and there is no invariant global projection that gives a description of processes in space-time. From their measurements local observers can construct descriptions of particles moving and interacting in a spacetime, but different observers construct different spacetimes, which are observer-dependent slices of phase space.(2)

G. Amelino-Camelia, L. Freidel, J. Kowalski-Glikman and L. Smolin
Curved Momentum Space

- Linear momentum space = absolute space-times

- Non-linear momentum space = relative locality
  - Non-associative
  - Non-commutative
  - Non-metric:
    \[ N^{abc} = \nabla^a g^{bc} \]
Non-linear Composition in Momentum Space:

\[ \oplus : \mathcal{P} \times \mathcal{P} \rightarrow \mathcal{P} \]
\[ (p, q) \rightarrow (p \oplus q) \]

1. Unit 0 such that
   \[ (0 \oplus p) = p = (p \oplus 0) \]

2. Inversion \( \ominus : \mathcal{P} \rightarrow \mathcal{P} \) such that
   \[ (\ominus p \oplus p) = 0 \]
   \[ p \oplus (\ominus p \oplus q) = q = \ominus p \oplus (p \oplus q) \].
Action of Relative Locality

The action of a particle in the relative locality theory:

\[ S_{\text{total}} = \sum_j S^j_{\text{free}} + S^{\text{int}}. \]

The free action is given by

\[ S^j_{\text{free}} = \int ds \left( x^a_j k^a_j + L_j C^j(k) \right), \quad C^j(k) \equiv D^2(k) - m^2_j, \]

The interaction term is given by

\[ S^{\text{int}} = \mathcal{K}(k)a^{-a}, \]

\( \mathcal{K} \) is the composition rule for the particle interactions.

\[ \mathcal{K} = (p \ominus q) \ominus r \quad \text{or} \quad \mathcal{K} = p \ominus (q \ominus r) \]
Equations of Motion

**Velocity**

\[ \dot{x}^a_j = \mathcal{L}_j \frac{\delta C^j}{\delta k_a} = \mathcal{L}_j \frac{\delta D^2(k)}{\delta k_a}, \quad k^i_a = 0 \]

**Constraints**

- Conservation of Energy and Momentum
  \[ \mathcal{K}(k)_a = 0 \]

- Definition of mass
  \[ C^i(k) = 0 \quad \rightarrow \quad D^2(p) = m^2 \]
Types of Coordinates

- Conjugate Space-time Coordinates
  \[ \{ x^a, k_b \} = \delta^a_b \]

- Interaction Space-time Coordinates

- Relating the two coordinate
  \[ x_j^a(0) = z^b \frac{\delta K_{jb}}{\delta k^i_a} \]
Experimental Set Up

Figure: The set up for the gamma ray burst experiment.
Calculating $\Delta S$

Starting Point:

$$ (z_4 - z_2) + (z_2 - z_1) = (z_4 - z_3) + (z_3 - z_1) $$

Relating $z_i$ and $z_j$:

$$ z_2 - z_1 = \dot{z}(k_1) S_1 $$
What is $\dot{z}$?

From the equations of motion

$$x^a_j(0) = z^b(W_{x_i})^a_b, \quad (W_{x_i})^a_b = \pm \frac{\delta K_b}{\delta k^i_a}$$

Invert the relationship

$$z_i(x) = x_i W^{-1}_{x_i}$$

Differentiate with respect to proper-time

$$\dot{z}_i(\dot{x}_i) = \dot{x}_i W^{-1}_{x_i}, \quad \dot{x}_j = L_j \frac{D^2(k)}{\delta k^a}$$
Which coordinate do we use?

\[ x_2 S_1 = x_2 - u_1. \]

Parallel transport to \( z_2 \)

\[ x_2 \mathcal{W}^{-1}_{x_2} S_1 = z_2 - z_1 \mathcal{W}_{u_1} \mathcal{W}^{-1}_{x_2}. \]

Final result:

\[ \dot{z}(k^2) S_2 - \dot{z}(k_1) S_1 + \dot{z}(p^1) T_1 - \dot{z}(p^2) T_2 = z_1 \left( \mathcal{W}_{u_1} \mathcal{W}^{-1}_{x_2} - \mathcal{W}_{u_3} \mathcal{W}^{-1}_{x_4} \right) \]
First Order Approximations

Expanding composition to first order

\[(p \oplus q)_a = p_a + q_a - \Gamma^b_c (0)p_b q_c\]

and

\[(U^k_r)_a^b = \delta^b_a - \Gamma^b_c (r - k)_c\quad (V^k_r)_a^b = \delta^b_a - \Gamma^b_c (r - k)_c\]

To first order

\[\dot{z}(k^2)S_2 - \dot{z}(k_1)S_1 + \dot{z}(p^1)T_1 - \dot{z}(p^2)T_2 = 0\]

Assuming \(\dot{z}(k_1) \approx \dot{z}(k_1)\)

\[(S_2 - S_1)|\dot{z}(k_1)|\dot{K} = \dot{z}(p^2)T_2 - \dot{z}(p^1)T_1\]
The Time Delay

\[ \Delta S \approx \frac{(\dot{z}(p^2))^2}{2\hat{K} \cdot \dot{z}(p^2)} T_2 - \frac{(\dot{z}(p^1))^2}{2\hat{K} \cdot \dot{z}(p^1)} T_1 \]
Calculation in Connection Normal Coordinates

We take $g^{ab}(0) = \eta^{ab}$ and set

$$\Gamma^{(ab)}_{c}(0) = \left\{^{ab}_{c}\right\} + \frac{1}{2}g_{ci} \left( T^{iab} + T^{iba} - N^{abi} - N^{bai} + N^{iab} \right) = 0$$

This fixes the first derivative of the metric at the origin:

$$g^{bi,a}(0) = -\frac{1}{2} \left( T^{iab} + T^{bai} - 2N^{abi} \right) = N^{bia}(0)$$

Distance function is:

$$D^2(p) = E^2 - \vec{p}^2 + N^{cab}(0)p_cp_ap_b = m^2$$
Time Delay in Connection Normal Coordinates

To first order in connection normal coordinates

$$\Delta S \approx E_{p_2} M^{cbd} \hat{p}_b \hat{p}_d \hat{p}_c^2 T_2 - E_{p_1} M^{cbd} \hat{p}_b \hat{p}_d \hat{p}_c \hat{p}_1 T_1$$

where

$$M^{abc} \equiv \frac{1}{2} \left( N^{abc} + N^{bac} + N^{bca} \right).$$

*Note the absence of Torsion
Non-linear Composition in Momentum Space:

\[ \oplus : \mathcal{P} \times \mathcal{P} \rightarrow \mathcal{P} \]
\[ (p, q) \rightarrow (p \oplus q) \]

1. Unit 0 such that
\[ (0 \oplus p) = p = (p \oplus 0) \]

2. Inversion \( \ominus : \mathcal{P} \rightarrow \mathcal{P} \) such that
\[ (\ominus p \ominus p) = 0 \]
\[ p \oplus (\ominus p \ominus q) = q = \ominus p \oplus (p \ominus q). \]
Gamma Ray Bursts and the Principle of Relative Locality

Anna E. McCoy

Perimeter Institute

Advisor: Lee Smolin
Outline

- Fermi Telescope

- Dispersion relationship

\[ E^2 - p^2 + \alpha \frac{E^3}{m_{QG}} = m^2 \]

http://fermi.gsfc.nasa.gov

- L. Freidel and L. Smolin, Gamma ray bursts probe the geometry of momentum space