Abstract: A number of recent proposals for a quantum theory of gravity are based on the idea that spacetime geometry and gravity are derivative concepts and only apply at an approximate level. Two fundamental challenges to any such approach are, at the conceptual level, the role of time in the emergent context and, technically, the fact that the lack of a fundamental spacetime makes difficult the straightforward application of well-known methods of statistical physics and quantum field theory to the problem. We initiate a study of such problems using spin systems as toy models for emergent geometry and gravity. These are models of quantum networks with no a priori geometric notions. In this talk we present two models. The first is a model of emergent (flat) space and matter and we show how to use methods from quantum information theory to derive features such as speed of light from a non-geometric quantum system. The second model exhibits interacting matter and geometry, with the geometry defined by the behavior of matter. This is essentially a Hubbard model on a dynamical lattice. We will see that regions of high connectivity behave like analogue black holes. Particles in their vicinity behave as if they are in a Schwarzschild geometry. Time permitting, I will show our study of the entanglement entropy of the system, which suggests particle localization near these traps.
Outline

Intro
- Quantum Gravity
- Emergent gravity & geometry
- Background independent spin systems as models for emergent gravity & geometry

Model 1
- Emergent (flat) space & matter
- Speed of light from a local Hamiltonian
- Spin systems on a dynamical lattice

Model 2
- Emergent gravity: matter/geometry interactions
- A toy black hole
- What does the matter see? effective curved geometry
- Thermalization from matter/geometry entanglement

Time, Gravity, Emergence, etc.

Summary
Emergent gravity

Gravity may be emergent:

- Thermodynamical aspects of gravity
  Hawking, Unruh, Jacobson, Padmanabhan, Horava, Verlinde, ...

- AdS/CFT, matrix models

- Emergent gravity in the condensed matter sense
  Volovik, Hu, Gu&Wen, Xu, ...

- Analogue models for gravity
  Unruh, Cirac, Visser, Weinfurtner, Liberati,...
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Emergence:

Behavior of whole system has no explanation in terms of the constituting particles, but instead comes from their collective behavior and interactions.

Distinguish between:  
- complex patterns emerge from simple rules (e.g. game of life)

- simple structures emerge from messy and complicated building blocks (e.g. emergence of order)
Quantum gravity = GR" + "QFT

Emergent gravity: spacetime geometry and gravity are derivative concepts, they apply only at an approximate level.

Quantum gravity as a problem in statistical physics: we know the macrophysics (GR and QFT), we are looking for the microphysics.

Our question: How does gravity/geometry emerge from a more fundamental quantum microtheory?

- Emergence is studied in cond mat/ statistical physics. Paradigm: Ising model

What is the Ising model for gravity? A universality class for gravity?
The problem of time in quantum gravity

- Matter tells spacetime how to curve and spacetime tells matter where to go
- \( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = T_{\mu\nu} \)
- \( g_{\mu\nu} \) dynamical
- Physical quantity: \( [g_{\mu\nu}]_{\text{Diff} M} \)
- Diffeomorphism invariance: only events and their relations are physical

- In pure gravity (\( T_{\mu\nu} = 0 \)), time evolution is a diffeomorphism (timelessness).
  - If we quantize GR (LQG) we find that the Hamiltonian is a constraint: \( \hat{H} |\Psi_U\rangle = 0 \)
    - Wheeler-deWitt equation instead of a Schroedinger equation.
    - What does the RHS mean?
  - Diffeos present serious problems when we try to construct local observables (already a problem in classical GR)
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What is the Ising model for gravity? A universality class for gravity?

- Method: A relativist’s viewpoint (the microscopic analogue of spacetime should be dynamical) with a cond matter toolbox (quantum many body physics, quantum information theory).

- The scale of quantum gravity is not beyond observable physics.
Quantum gravity = GR + QFT

What is the Ising model for gravity? A universality class for gravity?

Can we get gravity as an emergent phenomenon from a quantum many body system?

- Can emergence coexist with the timelessness of General Relativity?
- Is there a unified picture underlying the different emergence hints?
- Emergent gravity should be testable (e.g. approximate symmetries).

Look for robust signatures of emergent gravity.

We propose to study the questions raised by emergence of gravity in the explicit context of a spin system.
Background independent spin models

What is the Ising model for gravity? A universality class for gravity?

Model 1: A spin system on a dynamical lattice. \textbf{The links are the spins.} A role for quantum information theory: Information before spacetime geometry.

Model 2: “Geometry tells matter where to go and matter tells geometry how to curve”, in a quantum Hamiltonian.
The dynamical lattice: lattice links as spins
T. Konopka, FM & L. Smolin, hep-th/0611197

Basic idea:
Lattice = adjacency = locality (geometry) = interactions in a Hamiltonian

\[ H = \sum_{ij} J_{ij} s_i s_j \begin{cases} J_{ij} \neq 0 \\ J_{ij} = 0 \end{cases} \] if \( s_i, s_j \) are neighbors
if \( s_i, s_j \) are not adjacent

Promote link to a quantum degree of freedom \( \{ |1\rangle, |0\rangle \} \)
qubits of adjacency

For \( s_i = 1, \ldots, N \), there are \( \frac{N(N-1)}{2} \) possible links.

Quantum geometry:
Superposition of adjacent/not adjacent

State space of models: \( \mathcal{H} = \bigotimes_{e \in K_N} \mathcal{H}_e \)
Spin models on a dynamical lattice

**Task:** Find a Hamiltonian on a spin system that:
- shows how a regular geometry emerges from "disordered" or no geometry
- exhibits primitive notions of gravity: attraction, horizons, c<0, gravitons
- is quantum: matter/geometry entanglement, interference of quantum geometries, ...
- Investigate emergent gravity and develop new tools in an explicit context

**So far:** Questions we have looked at:
- emergence of (flat) space
- emergence of matter (Wen)
- speed of light from first principles
- matter/geometry interactions & entanglement
- combinatorial analogue of attraction
- quantum cosmology: quantum → statistical?
- Lorentz & diffeomorphism invariance vs The Lattice
Model 1
Model 1: Emergent space and matter

We currently assume an FRW geometry all the way to the Big Bang.

Alternative scenario: What if geometry is not fundamental?
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Alternative scenario: What if geometry is not fundamental?

- Translations, etc
- Locality
- 3+1 dim.
- No locality
- No geometric symmetries

Emergence of geometry as emergence of order in network of adjacencies.
Model 1: Emergent space and matter

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**geometric phase**
- translations, etc
- locality
- 3+1 dim.

**geometrogenesis**

**non-geometric phase**
- no locality
- no geometric symmetries

Emergence of geometry as emergence of order in network of adjacencies
Model 1: Emergent space and matter

T. Konopka, FM & S. Severini, PRD 08

permutation symmetry
no subsystems
- no locality
- no space

translation symmetry (FRW)
- meaningful distances
- space
Model 1: Emergent space and matter

\[ |\Psi_{K_N}\rangle \rightarrow \text{translation symmetry} \]

\[ \text{permutation symmetry} \]
\[ \text{no subsystems} \]
\[ \Rightarrow \text{no locality} \]
\[ \Rightarrow \text{no space} \]

\[ + \text{matter} \rightarrow |\Psi_0\rangle \]

\[ K_N : \text{complete graph on } N \text{ vertices.} \]
\[ N(N - 1)/2 \text{ edges.} \]

- \[ \mathcal{H}_{\text{edge} \, ab} = |j_{ab}, m_{ab}\rangle \]

\[ \mathcal{H} = \bigotimes_\text{edge} \mathcal{H}_{\text{edge}} \]
Model 1: Emergent space and matter

\[ H_V = g_V \sum_a e^P \left( v_0 - \sum_b N_{ab} \right)^2 \]

- on \( \mathcal{H} \| 1,0 \) \n
\[ H_{\text{loops}} = \sum_a \left( - \sum_b g_B \delta_{ab} \sum_{L=0}^{\infty} \frac{r^L}{L!} N_{ab}^{(L)} \right) \]

- on full \( \mathcal{H} \)

\[ H_{\text{loops}} = - \sum_a \sum_{L=0}^{\infty} \frac{r^L}{L!} g_B \prod_{i=1}^{L} M_i^{\pm} \]

\[ H_{\text{string}} = g_C \sum_a \left( \sum_b M_{ab} \right)^2 + g_D \sum_{ab} M_{ab}^2 \]

Flat space is stable local minimum

Emergent photons and fermions for \( g_B \gg g_C, g_D \)

X.G. Wen, Quantum Field Theory of Many Body Systems

\[ \rightarrow \text{Emergent matter when space emerges.} \]
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X.G. Wen, Quantum Field Theory of Many Body Systems

Emergent matter when space emerges.

- How do we do, e.g. mean field theory analysis, when there is no fixed lattice?
- Scenario assumes an external heat bath.
New tool: from a varying to a fixed lattice

$K_N \rightarrow \text{Line graph of } K_N$

\[ i = 1, \ldots, N \rightarrow I = 1, \ldots, \frac{N(N-1)}{2} \]

$\hat{H}$ on dynamical lattice $\rightarrow$ Ising on fixed lattice

Vertex degree is a good order parameter.

Mean field theory analysis: We do obtain a regular lattice but at $T = 0$ (when $N \rightarrow \infty$).

Is a zero temperature transition good enough?
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Can we use \( K_N \) in other approaches where geometry is dynamical?

\[ \hat{H} \text{ on dynamical lattice} \rightarrow \text{Ising on fixed lattice} \]

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New tool: speed of light from a Hamiltonian

A. Hamma, FM. I. Premont-Schwarz, S. Severini, PRL 2009

Given local Hamiltonian $H = \sum_{\langle i,j \rangle} h_{ij}$,

Lieb-Robinson speed of information propagation:

$$\|[O_P(t), O_Q(0)]\| \leq 2\|O_P\|\|O_Q\| \sum \frac{(2|t|h_{\text{max}})^n}{n!} N_{PQ}(n)$$

$$\|[O_P(t), O_Q(0)]\| \leq 2\|O_P\|\|O_Q\|C \exp[-a(d_{PQ} - vt)]$$

Find: $v_{LR} = \sqrt{2g_Bg_C}$

- $v_{LR} = c$ in the emergent Maxwell equations
- Effective finite light cones consistent with non-relativistic quantum mechanics
- $v_{LR} \sim d$

• Approximating the model with a hypercube whose dimension varies with time, $v_{LR}(t) \sim D(t)$
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Evolving speed of light and the horizon problem:

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Mismatched locality

Evolution of speed of light and the horizon problem:

Model illustrates the effect of a transition in the local structure.

A. Hamma, F.M., I. Premont-Schwarz, S. Severini, PRL 2009

Model 2
Motivation for Model 2:

- The regular geometry appears at low energy. How does the system lower its energy? What we did is equivalent to an external heat bath.
  
  We want a unitary model of the universe.

- Matter and geometry come from the same microscopic degrees of freedom. This is a very interesting feature of this model but we also want to study a model with state space $\mathcal{H} = \mathcal{H}_{\text{geometry}} \otimes \mathcal{H}_{\text{matter}}$

- Can we write an Ising model-type system that realizes “geometry tells matter where to go and matter tells geometry how to curve”? If so, investigate:
  
  Gravity $\leftrightarrow$ geometry tells matter where to go and matter tells geometry how to curve.
Model 2: Interacting matter-geometry

Gravity:
Geometry tells matter where to go and matter tells geometry how to curve.

\[ \mathcal{H} = \bigotimes_e \mathcal{H}_e \bigotimes_v \mathcal{H}_v \]

\[ H_{\text{ex}} = \sum_{ij} P^L_{ij} (|0\rangle\langle1|_{ij} b_i^\dagger b_j^\dagger + |1\rangle\langle0|_{ij} b_i b_j ) \]

\[ H_{\text{hop}} = \sum_{ij} P_{ij} (b_i^\dagger b_j + b_i b_j^\dagger) \]

\[ P_{ij} = |1\rangle\langle1|_{ij} \]

The matter gives the graph the meaning of space:
Adjacency is defined by interactions. The graph determines where the matter is allowed to go.

A Bose-Hubbard model on a dynamical lattice.
Model 2: A toy black hole

\[ c^A \propto 4 \]

\[ c^B \propto N^B \]

\[ \theta_c = \sin^{-1} \left( \frac{c^B}{c^A} \right) \sim \sin^{-1} N^B \]

Probability of light escaping \[ \sim \frac{1}{N^B} \to 0 \]
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Probability of light escaping \( \sim \frac{1}{N^B} \rightarrow 0 \)

- The dense geometry creates more bosons and the bosons make more links (cf gravitational collapse)
- Eventually the “black hole” evaporates (faster as it gets smaller)
- Unitary evolution, mixed radiation (no singularity): matter entangled with remnant geometry
Model 2: curved geometry and gravity

\[ |r\rangle = \frac{1}{\sqrt{N}} \sum_{\theta=1}^{N} |r, \theta\rangle \]

Ignoring the backreaction of the matter on the lattice, and on states with certain symmetries (foliated graphs) the transition coefficients of the Hamiltonian correspond to the wave equation for particles propagating in a curved spacetime.

Can explicitly study the evolution of matter and show that the highly connected regions trap matter.

Can also show trapping as localization of particles at the “black hole” by studying the entanglement entropy.

Non-local” physics in deep quantum gravity regime?
Matter as the heat bath for geometry:

Thermalization in quantum cosmology: While the whole system evolves unitarily, locally it can look thermal (Page 13). How to prove?

New results in thermalization in closed quantum systems:
- Consider part of a closed system:
  \[ H = H_{\text{graph}} + H_{\text{matter}} + H^I \]
  \[ \rho_{\text{matter}}(t) = \text{Tr}_{\text{graph}} \rho(t) \]
- Insulator/superfluid regions: thermalization of initial state of no particles on \( K_N \)
- Typical values of observables correspond to those of a canonical ensemble:
  \[ \langle \Gamma \rangle(t) = \text{Tr} \left\{ \rho(t) \Gamma \otimes 1_{\text{matter}} \right\} \]
  \[ \lim_{t \to \infty} \langle \Gamma \rangle(t) \simeq \frac{\text{Tr} \left\{ \Gamma e^{-\beta(E)H_{\text{graph}}} \right\}}{\text{Tr} \left\{ e^{-\beta(E)H_{\text{graph}}} \right\}} \]

The system is not in equilibrium, but every time you look at
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Timescale of thermalization:
- Hayden and Preskill, 2007, Black holes as mirrors: BHs are nearly optimal thermalizers.
- Sekino & Susskind, 2008, Fast scramblers (≡thermalizers):
  Conjecture: BHs are the fastest scramblers in nature.

Our toy BHs are an explicit example of the above (infinite-dim spin systems).

Non-local physics in deep quantum gravity regime?
Analogue models for gravity, or more?

Hindsight-aided motivation: analogue gravity

World of observer inside fluid (water, BEC, ...):
- is Lorentzian
- has horizons
- has Hawking radiation

⇒ Gravity analogues in the lab.

Our spin systems can be viewed as a new kind of analogue gravity models: no background fluid, fully dynamical

ush analogy:

Quantum many-body system \(\rightarrow\) Gravity (FAPP) : quantum gravity!
(no geometric dofs)

Find fundamental obstructions to the analogy: learn why gravity is not emergent
Summary

Underlying spatial geometry is the notion of adjacency.

metric: neighbours or not?

E.g. a 3d euclidean geometry is a particular order of adjacencies that exhibits certain symmetries. Our geometric world is a phase (geometrogenesis), we froze to that phase.

By promoting adjacency to a qubit and considering any network as a subgraph of $K_N$ we have a convenient way to deal with superpositions of quantum geometries.

When the adjacency qubits are dynamical dofs, we have an important ingredient of GR dynamical geometry).

Local interactions mean a finite speed of information propagation. Given local dynamics on network of adjacencies, we can define a spacetime with finite lightcone structure. (Is it also universal?)

If we find microscopic dynamics that simulate GR, we will have reconciled quantum with gravity. With emergent gravity, we need to explain, not quantize the Einstein Equations!
Summary

Spin systems on a dynamical lattice as models for emergent gravity.

Questions we have looked at:
- emergence of (flat) space
- emergence of matter (Wen)
- speed of light from first principles
- matter/geometry interactions & entanglement
- combinatorial analogue of attraction
- quantum cosmology: quantum vs statistical?
- Lorentz & diffeomorphism invariance vs The Lattice

What we'd like to understand:
- Emergence, time and background independence
- Fundamental time in Hamiltonian vs geometric time: can emergence allow us to have our cake and eat it?
- Can symmetries such as Lorentz invariance, diffeomorphisms, be emergent?
- Quantization of GR: fundamental vs "phonons". Observational signature?
- Gravity from first principles: emergent gravity requires explaining gravity.

Some of the things to do next:

- Thermalization and equilibration to a regular lattice
- Timescale of thermalization (scrambling)
- Lieb-Robinson speed of light in the continuum limit and for infinite-dimensional systems
- Tensor network renormalization for the low energy physics (Gu&Wen, Vidal, Vedral, ...)
- Recast the partition function of the spin system in the stabilizer formalism (Brriegel et al)
- A new type of analogue models? Can a designer fluid (instead of BEC or water) show dynamical aspects of GR?
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Non-local physics in deep quantum gravity regime?
Why don't we see quantum spacetimes?

Hamma, Lloyd, FM, Caravelli, Severini, Markstrom, PRD2010, Hamma, FM NJP2011

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Caravelli, Hamma, FM, Riera

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Geometry tells matter where to go and matter tells geometry how to curve.

$$\mathcal{H} = \bigotimes_e \mathcal{H}_e \bigotimes_v \mathcal{H}_v$$

$$H_{ex} = \sum_{ij} P_{ij}^L ( |0\rangle\langle 1|_{ij} b_i^\dagger b_j^\dagger + |1\rangle\langle 0|_{ij} b_i b_j )$$

$$H_{hop} = \sum_{ij} P_{ij} ( b_i^\dagger b_j + b_i b_j^\dagger )$$

$$P_{ij} = |1\rangle\langle 1|_{ij}$$

The matter gives the graph the meaning of space:
Adjacency is defined by interactions. The graph determines where the matter is allowed to go.

A Bose-Hubbard model on a dynamical lattice.
Model 2: A toy black hole

Hamma, Lloyd, FM, Caravelli, Severini, Markstrom, PRD2010

\[ c^A \propto 4 \]
\[ c^B \propto N^B \]

\[ \theta_c = \sin^{-1} \frac{c^B}{c^A} \sim \sin^{-1} N^B \]

Probability of light escaping \( \sim \frac{1}{N^B} \to 0 \)

- The dense geometry creates more bosons and the bosons make more links (cf gravitational collapse)
- Eventually the "black hole" evaporates (faster as it gets smaller)
- Unitary evolution, mixed radiation (no singularity): matter entangled with remnant geometry
New tool: from a varying to a fixed lattice

\[ K_N \longrightarrow \text{Line graph of } K_N \]

\[ i = 1, \ldots, N \]

\[ I = 1, \ldots, \frac{N(N-1)}{2} \]

\[ \hat{H} \text{ on dynamical lattice} \longrightarrow \text{Ising on fixed lattice} \]

Vertex degree is a good order parameter.

Mean field theory analysis: We do obtain a regular lattice but at \( T = 0 \) (when \( N \to \infty \)).

Is a zero temperature transition good enough?
New tool: from a varying to a fixed lattice

$K_N \rightarrow$ Line graph of $K_N$

$\hat{H}$ on dynamical lattice $\rightarrow$ Ising on fixed lattice

Vertex degree is a good order parameter.

Mean field theory analysis: We do obtain a regular lattice but at $T = 0$ (when $N \rightarrow \infty$).

Is a zero temperature transition good enough?

Can we use $K_N$ in other approaches where geometry is dynamical?
New tool: speed of light from a Hamiltonian

A. Hamma, FM, I. Premont-Schwarz, S. Severini, PRL 2009

Given local Hamiltonian \( H = \sum_{\langle i,j \rangle} h_{ij} \).

Lieb-Robinson speed of information propagation:

\[
\|[O_P(t), O_Q(0)]\| \leq 2\|O_P\|\|O_Q\| \sum_n \frac{(2|t|h_{\max})^n}{n!} N_{PQ}(n)
\]

\[
\|[O_P(t), O_Q(0)]\| \leq 2\|O_P\|\|O_Q\|C \exp[-a(d_{PQ} - vt)].
\]

Find: \( v_{LR} = \sqrt{2g_B g_C} \)

- \( v_{LR} = c \) in the emergent Maxwell equations
- Effective finite light cones consistent with non-relativistic quantum mechanics
- \( v_{LR} \sim d \)
- Approximating the model with a hypercube whose dimension varies with time, \( v_{LR}(t) \sim D(t) \)
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Mismatched locality
A. Hamma, FM, I. Premont-Schwarz, S. Severini, PRL 2009


Evolving speed of light and the horizon problem:

geometric phase

gemetrogenesis — —

non-geometric phase

inflation
Matter as the heat bath for geometry:

Thermalization in quantum cosmology: While the whole system evolves unitarily, locally it can look thermal (Page 13). How to prove?

New results in thermalization in closed quantum systems:
- Consider part of a closed system:
  \[ H = H_{\text{graph}} + H_{\text{matter}} + H^I \]
  \[ \rho_{\text{matter}}(t) = \text{Tr}_{\text{graph}} \rho(t) \]
- Insulator/superfluid regions: thermalization of initial state of no particles on \( K_N \)
- Typical values of observables correspond to those of a canonical ensemble:
  \[ \langle \Gamma \rangle(t) = \text{Tr} \left\{ \rho(t) \Gamma \otimes 1_{\text{matter}} \right\} \]
  \[ \lim_{t \to \infty} \langle \Gamma \rangle(t) \simeq \frac{\text{Tr} \left\{ \Gamma e^{-\beta(E)} H_{\text{graph}} \right\}}{\text{Tr} \left\{ e^{-\beta(E)} H_{\text{graph}} \right\}} \]

The system is not in equilibrium, but every time you look at
Analogue models for gravity, or more?

Hindsight-aided motivation: analogue gravity

World of observer inside fluid (water, BEC, ...):
• is Lorentzian
• has horizons
• has Hawking radiation
⇒ Gravity analogues in the lab.

Our spin systems can be viewed as a new kind of analogue gravity models: no background fluid, fully dynamical

ush analogy:

Quantum many-body system → Gravity (FAPP) : quantum gravity!
(no geometric dofs)

Find fundamental obstructions to the analogy: learn why gravity is not emergent