Abstract: Dark matter models with an annihilation cross section enhanced by a Sommerfeld mechanism have been proposed in the past years to explain a number of observed anomalies, such as the excess of high energy positrons in cosmic rays reported by PAMELA. However, this enhancement can not be arbitrarily large without violating a number of astrophysical measurements. In this talk, I will discuss the degree to which these measurements can constrain Sommerfeld-enhanced models. In particular, I will talk about constraints coming from the observed abundance of dark matter and the extragalactic background light measured at multiple wavelengths.
Astrophysical constraints on dark matter annihilation with Sommerfeld enhancement

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Papers: PRD, 81, 083502 (0910.5221), PRD, 83, 123513 (1103.0776)

Unravelling Dark Matter, September 2011
Outline

- Sommerfeld enhancement (brief motivation and description)
- Relic density constraints
- CMB constraints
- DM annihilation in halos and the Extragalactic Background Light (X- and Gamma-rays)
- EBL constraints
Why boosting DM annihilation?

The case for DM annihilation

- WIMP annihilation can explain cosmic ray anomalies but: large cross section BF > O(100) over thermal relic value: $3 \times 10^{-26} \text{cm}^3\text{s}^{-1}$ (e.g. Bergström et al. 2009) and annihilation mainly to leptons, proton/antiproton channel suppressed (PAMELA data).
- A new force carrier ($m_\phi \sim \text{GeV}$) acting between the annihilating WIMPs enhances the cross section via a Sommerfeld mechanism (Hisano et al. 2004, Arkani-Hamed et al. 2009, ...). If $m_\phi < 2m_p$, then decay into antiprotons is kinematically forbidden.
Sommerfeld enhancement

Simplified case, a scalar boson as a force carrier, Yukawa potential

\[
\frac{1}{m_\chi} \frac{d^2 \Psi(r)}{dr^2} + V(r) \Psi(r) = -m_\chi \beta^2 \Psi(r) \quad V(r) = -\frac{\alpha_c}{r} e^{-m_\phi r}
\]

Lattanzi and Silk 2009

\[
\alpha_c = \frac{1}{30} \quad m_\phi = 90 \text{ GeV}
\]

Coulomb approximation (\(m_\phi \to 0\)):

\[
\sigma = \sigma_0 S_k \quad S_k = \frac{|\psi_k(0)|^2}{|\psi_k(0)|^2}
\]

\[
S = \frac{\pi \alpha_c}{\beta} \left(1 - e^{-\pi \alpha_c / \beta}\right)^{-1}
\]

\[
S(\beta) \propto 1/\beta \quad \text{if} \quad \beta \ll \pi \alpha_c
\]

General behaviour:

1) if \(\beta^2 \gg m_\phi \alpha_c/m_\chi\) \(\to\) Coulomb case

2) if \(\beta^2 \ll m_\phi \alpha_c/m_\chi\) \(\to\) bound states if \(m_\chi = 4m_\phi n^2/\alpha\)

3) Close to "resonances" \(\to\) \(S(\beta) \propto 1/\beta^2\)

4) Saturation at very low velocities, finite life time of the bound states
Relic density constraints

Boltzmann equation: \[ \frac{dn_x}{dt} + 3Hn_x = -\langle \sigma v \rangle \left( n_x^2 - (n_x^{EQ})^2 \right) \]

Thermal average: \[ x_x = m_x / T_x \]

\[ \langle \sigma v \rangle = \langle \sigma v \rangle_S \left( \frac{x^{3/2}}{2\pi^{1/2}} \int_0^1 S(\beta) \beta^2 e^{-x\beta^2/4} d\beta \right) = \langle \sigma v \rangle_S S(x_x) \]

Note that:

\[ S(\beta) \propto 1/\beta \rightarrow S(x_x) \propto x_x^{1/2} \propto 1/\sigma_{vel} \]

\[ S(\beta) \propto 1/\beta^2 \rightarrow S(x_x) \propto x_x \propto 1/\sigma_{vel}^2 \]

Dark matter abundance: \[ \Omega_x h^2 = \Omega_{DM} h^2 \sim 0.1143 \]

Kinetic decoupling: after freeze-out, scattering with SM particles keep $T_x = T$, after kinetic decoupling $T_x$ drops as $1/a^2$ ("colder" than radiation):

\[ x_x = x \quad \text{for} \quad t < t_{KD} \]

\[ x_x = x^2 / x_{KD} \quad \text{for} \quad t > t_{KD} \]
Relic density constraints

Dent et al. 2010

\[ \langle \sigma v \rangle_0 = 3 \times 10^{-26} \text{cm}^3 \text{s}^{-1} \]

\[ m_x = 500 \text{GeV} \]

- If \( S \sim 1/\sigma \) then \( Y \sim 1/\ln x \) for \( x > x_{kd} \)
- If \( S \sim 1/\sigma^2 \) then \( Y \sim 1/x \) for \( x > x_{kd} \)
- \( \langle \sigma v \rangle_0 \) needs to be lower than the case without enhancement (a factor of a few) to give the correct relic density
- Kinetic decoupling temperature is a relevant parameter, the larger it is, the stronger the suppression on the relic density

BF (relative to \( \langle \sigma v \rangle_0 = 3 \times 10^{-26} \text{cm}^3 \text{s}^{-1} \)) < 100

for \( \alpha < 10^{-2} \), \( m_\phi/m_x < 10^{-3} \), \( m_x \sim 100 \text{GeV} \), \( T_{kd} = 8 \text{MeV} \)

\[ BF = \frac{\langle \sigma v \rangle_0^{\Omega DM S(\sigma_{vel,h})}}{3 \times 10^{-26} \text{cm}^3 \text{s}^{-1}} \]

Feng et al. 2010, “maximal” BF up to \( m_x \sim 3 \text{TeV} \) is < 300

Zavala et al. 2010

\[ \Omega_{DM h^2} \sim 0.1143 \]
CMB constraints

CMB energy spectrum: energy injection at $10^4 < z < 10^6$ effectively produces a Bose-Einstein energy spectrum with chemical potential $\mu$ instead of a pure black body spectrum (Illarionov and Sunyaev 1975). Limit by COBE/FIRAS $|\mu| < 9 \times 10^{-5}$. "$f$" is the fraction that ionizes and heats the IGM.

$$\mu = 1.4 \frac{\delta \rho_\gamma}{\rho_\gamma} = 1.4 \int_{t_1}^{t_2} \frac{1}{\rho_\gamma} \frac{d}{dt} \left( \frac{i m_X}{\rho_\gamma} \frac{d n_X}{dt} \right) dt$$

Injection at $10^3 < z < 10^4$ produces a $\gamma$-type distortion to the CMB (Hannestad and Tram 2011). Both are weak constraints.

CMB power spectrum: e.g. Slatyer et al. 2009, limits based on WMAP5:

$$\frac{\lim_{v \to 0} \langle \sigma v \rangle}{3 \times 10^{-26} \text{cm}^3/\text{s}} \lesssim \frac{120}{f} \left( \frac{m_X}{1 \text{TeV}} \right)$$

$f \sim 0.25$ for annihilation into SM particles, except electrons ($f \sim 0.7$) and neutrinos ($f \sim 0$)
Cosmic background radiation from dark matter annihilation

- Energy of photons per unit area, time, solid angle and energy range received by an observer located at $z=0$.

$$I = \frac{1}{4\pi} \int \mathcal{E}(E_0(1+z), z) \frac{dr}{(1+z)^4} e^{-\tau(E_0, z)}$$

- Contribution from all dark matter structures along the line of sight of the observer (assumption: no contribution from unclustered DM).

- The volume emissivity of photons (energy of photons produced per unit volume, time and energy range) can be written as:

$$\mathcal{E} = \frac{f_{WIMP}}{2} E \rho_\chi (\vec{x})^2$$

- Properties of dark matter as a particle:

$$f_{WIMP} = \frac{dN}{dE} \frac{\langle \sigma v \rangle_c}{m^2_\chi}$$

- The density squared dependence is connected to the gravitational interactions of dark matter.
Photon yield

Zavala et al. 2011

Main annihilation channel into leptons (PAMELA fit)

Prompt emission (secondary decay of $\pi^0$)

Up-scattered CMB photons

$E^2 dN/dE(\text{GeV})$

SUSY example:

$m_\chi$ 200 GeV

$\chi \chi \rightarrow b \bar{b}$

$\sigma v \sim 6.2 \times 10^{-27} \text{cm}^3\text{s}^{-1}$

e+e- equilibrium spectrum (not normalized)

e+e- injection spectrum
Annihilation in DM halos

Virgo Consortium's Aquarius Project
MW-like halo
\( m_{DM} \sim 1500 \text{ Msun} \) (Springel et al. 2008)

Via-Lactea II simulation
MW-like halo
\( m_{DM} \sim 4100 \text{ Msun} \) (Kuhlen et al. 2009)

- Total luminosity of a smooth DM halo (formula roughly agrees with summation over particle densities):

\[
L_h' = \int \rho_{\text{NFW}}(r) \, dV = \frac{1.23 \, V_{\text{max}}^4}{G^2 r_{\text{max}}}
\]

Keep in mind: uncertainty on the DM density profile (+effects of baryons)
**Annihilation in DM halos (substructure)**

*Substructures within halos have a dominant role for external observers. Their contribution to the total luminosity is uncertain ~ 2 - 2000 times the contribution of the smooth component for a MW-like halo (once their minimum mass is extrapolated to ~Earth mass).*
Same cosmology as Millennium I
100 Mpc/h box and \( \varepsilon = 1 \text{kpc/h} \)
\( N_p = 2160^3, \ m_{\text{DM}} = 6.89 \times 10^9 \text{Msun/h} \)

Bound substructures found using SUBFIND (Springel et al. 2001):

\[ 11 \times 10^6 \text{ subs at } z=0 \]

\[ M_{\text{ab}} (\text{min}) \sim 1.4 \times 10^9 \text{Msun/h} \]
Same cosmology as Millennium I
100 Mpc/h box and \( \epsilon = 1 \) kpc/h
\( N_p = 2160^3, \ m_{dm} = 6.89 \times 10^8 \) Msun/h

Bound substructures found using SUBFIND (Springel et al. 2001):
11 \times 10^6 subs at \( z = 0 \)
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All-sky maps

- Simulation of the past light cone with resolved DM halos and subhalos as sources (assuming scaling law for luminosity, NFW profile)
- Spatial distribution and temporal evolution given by MS-II
- Extrapolation to unresolved sources down to earth masses (two orders of magnitude uncertainty)
- Photon yield given as input from a particle physics model
- Sommerfeld enhancement included as a \( S(\sigma_{vel}) \) function
All-sky maps
(resolved structures up to z~10, E=10GeV)

\[ N_{\text{pix}} = 12(512)^2 \sim 3 \times 10^6 \quad \text{ang. res.} \sim 0.115^\circ \]
Isotropic component (example)

\[ m_\chi \sim 200 \text{ GeV}, \chi\chi \rightarrow b\bar{b} \text{ and } \langle \sigma v \rangle \sim 6.2 \times 10^{-27} \text{ cm}^3 \text{s}^{-1} \]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{isotropic_component.png}
\caption{Example of isotropic component with different datasets and theoretical predictions.}
\end{figure}
Constraints on particle physics models

"factoring out" the astrophysical part of the signal

\[ N(0) = \frac{c}{8\pi} E_0 f_{\text{WIMP}}(E_0(1 + z^*)) \]

\[ \int \frac{\rho_x^2(x, z) e^{-\tau(E_0, z)}}{(1 + z)^3 H(z)} dz \]

$z^* < 4$ for X-rays

$z^* < 1$ for $E > 10 \text{GeV}$

Minimum extrapolation unresolved subhalos

Maximum extrapolation unresolved subhalos

same SUSY example
Sommerfeld-enhanced models fitting the cosmic ray excesses \textbf{(Finkbeiner et al. 2011)}

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- New force carrier in the "dark sector"
- Annihilation cross section enhanced by a Sommerfeld mechanism: $\langle \sigma v \rangle = \langle \sigma v \rangle_0 S(\sigma_{\text{vel}})$
- Correct relic density
- Fit to the cosmic ray excesses measured by PAMELA and Fermi
- Allowed by bounds on $S_{\text{max}}$ from the CMB
- IC contribution dominates the photon yield
Sommerfeld-enhanced models fitting the cosmic ray excesses

- Minimum contribution from subhalos
- SFG = 53% of EGB (E > 1GeV)
- Blazars = 16% of EGB (E > 1GeV)

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$n_\chi \sim 200 \text{ GeV}, \chi \chi \rightarrow b\bar{b}$ and $(\sigma v) \sim 6.2 \times 10^{-27} \text{ cm}^3 \text{s}^{-1}$
Isotropic component (example)

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\[ E_0 \text{ [GeVcm}^{-2}\text{sr}^{-1}\text{s}^{-1}] \]

\[ 10^{-11} \quad 10^{-10} \quad 10^{-9} \quad 10^{-8} \quad 10^{-7} \quad 10^{-6} \quad 10^{-5} \quad 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10^0 \quad 10^1 \quad 10^2 \]

Chandra, INTEGRAL, COMPTEL, EGRET, Fermi

Star forming galaxies (53\%) and blazars (16\%)

Max. subhalos, Min. subhalos

lava1a et al. 2011, arXiv:1103.0776
Constraints on particle physics models

"factoring out" the astrophysical part of the signal

\[(E_0) = \frac{c}{8\pi} E_0 f_{\text{WIMP}}(E_0(1 + z^*)) \]
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Blazars = 16%
Sommerfeld-enhanced models fitting the cosmic ray excesses

Caveats

- Minimum mass for bound halos was assumed to be $10^{-6} M_\odot$. It can be higher for these models $\sim 0.1 M_\odot$ (Feng et al. 2010, Bringmann 2009). Signal would be reduced by a factor of $\sim 2$.

- Self-scattering cross section could deplete the central density cusps and disrupt low-mass halos (Loeb and Weiner 2011).

- Fits to PAMELA positron excess taking into account local substructure weakens the constraints (Tracy's talk).
Summary and Conclusions

- Sommerfeld-enhanced models can explain the cosmic-ray anomalies, but they need to be consistent with independent astrophysical constraints: correct relic abundance and CMB already constrain the boost factor to be less ~ few hundred.

- We have obtained predictions from the simulated all-sky maps of the cosmic X- and gamma-ray background from DM annihilation including:
  - Photon yield given by a WIMP model (in situ photons and up-scattered photons of the CMB). Model-independent, can be used for Sommerfeld-enhanced models.
  - Dark matter spatial distribution using Millennium-II simulation, uncertainty of ~2 orders of magnitude in extrapolation to unresolved structures.
  - Isotropic component constrained by observations of the cosmic background, and contributions from blazars and star forming galaxies: although is not as clean as the CMB, it is a powerful tool to constrain the intrinsic properties of dark matter.