Outline

● The speed-up of quantum computation
● Review of de Broglie-Bohm theory
● Quantum computing in de Broglie-Bohm
● Conclusion and afterthoughts
What aspect of quantum mechanics enables this speed-up?
Deutsch’s algorithm I

Scenario

We’re given a black box whose dynamics can be abstractly described by one of the four possible functions \( f : \{0, 1\} \rightarrow \{0, 1\} \).

\[
\begin{align*}
 f_0(0) &= 0 & f_0(1) &= 0 \\
 f_1(0) &= 0 & f_1(1) &= 1 \\
 f_2(0) &= 1 & f_2(1) &= 0 \\
 f_3(0) &= 1 & f_3(1) &= 1
\end{align*}
\]

We are not told \textit{which} of these describes the box.
Deutsch’s algorithm I

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We are not told which of these describes the box.
Deutsch's algorithm III

Determine $f$
Feed the black box with an initial state $|0\rangle$ and measure the output. Repeat with $|1\rangle$.
→ To determine $f$ (two bits of information) we had to make two 1-bit measurements.

Determine if $f \in \{f_0, f_1\}$
Feed the black box with an initial state $|0\rangle$ and measure the output. (Remember $f_0(0) = f_1(0) \neq f_2(0) = f_3(0)$)
→ To determine if $f \in \{f_0, f_1\}$ (one bit of information) we had to make one 1-bit measurement.
Deutsch's algorithm IV

Determine if \( f \in \{f_0, f_1\} \) (i.e. if \( f \) is odd or even)

Feeding either \( |0\rangle \) or \( |1\rangle \) into the box won't allow us to determine whether \( f \) is odd or even, but we need to do both. **Classically this is all we can do.**

\[ \Rightarrow \] To determine if \( f \) is odd or even (one bit of information) we had to use the oracle twice and make two one-bit measurements.

However, by exploiting quantum mechanics we only require **one** use of the black box and **one** measurement.
Deutsch's algorithm V

Data qubit $|0\rangle$

Auxiliary qubit $|1\rangle$

Hadamard Gate

$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$|1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

Black Box

$|x\rangle \rightarrow |x\rangle \circ f(x)$

Measurement in $\{|0\rangle, |1\rangle\}$ basis

State prior to measurement = \[
\begin{cases}
    \pm \frac{1}{\sqrt{2}} |0\rangle_D (|0\rangle_A - |1\rangle_A) & \text{if } f(0) = f(1) \\
    \pm \frac{1}{\sqrt{2}} |1\rangle_D (|0\rangle_A - |1\rangle_A) & \text{if } f(0) \neq f(1)
\end{cases}
\]
The speed-up I

So what explains the speed-up?

- Entanglement? (Josza)
- “Wavelike” dynamics? (Mermin)
- Many worlds? (e.g. Deutsch)
The speed-up I

So what explains the speed-up?

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- “Wavelike” dynamics? (Mermin)
- Many worlds? (e.g. Deutsch)
In order to be able to answer what explains the speed-up, we must look at quantum computation from the point of view of all contending formulations (or theories) of quantum mechanics.

Here we look at what de Broglie-Bohm pilot-wave theory has to say.
The **pilot wave**: a complex field $\psi(\vec{x}, t)$ in the configuration space of the chosen system $\mathcal{S}$, obeying the equation

$$i \frac{\partial}{\partial t} \psi(\vec{x}, t) = \hat{H}_\mathcal{S} \psi(\vec{x}, t).$$

(***)

The **actual configuration** as a function of time, tracing out a path $\vec{x}_c(t)$ in configuration space according to the "guidance equation"

$$\dot{\vec{x}}_c(t) = \frac{\vec{j}(\vec{x}_c, t)}{|\psi(\vec{x}_c, t)|^2},$$

where $\vec{j}(\vec{x}, t)$ is the current implicitly defined in the continuity equation $\frac{\partial}{\partial t} |\psi(\vec{x}, t)|^2 + \nabla \cdot \vec{j}(\vec{x}, t) = 0$ as derived from (**).
Trajectories in the double slit experiment.
Trajectories in the double slit experiment.
Consider a large ensemble of identical systems with normalised ensemble density \( \rho(\vec{x}, t) \) of obtaining configurations.

**Theorem: The existence of quantum equilibrium**

If \( \rho(\vec{x}, t_0) = |\psi(\vec{x}, t_0)|^2 \) for some time \( t_0 \), then \( \rho(\vec{x}, t) = |\psi(\vec{x}, t)|^2 \) for all times \( t \).

**Theorem: Approach of equilibrium**

An ensemble density \( \rho(\vec{x}, t) \) not in quantum equilibrium monotonically approaches quantum equilibrium (given certain assumptions about the initial conditions).
A “measurement” is just a particular kind of interaction: one with an apparatus with a pointer or dial and whose effective interaction Hamiltonian can be written in the form

\[ \hat{H}_{\text{meas}} = a \hat{Q} \hat{p}_{\text{dial}}. \]

- \( a \): coupling parameter of the interaction
- \( \hat{Q} \): operator associated with the “observable” in the SQF
- \( \hat{p}_{\text{dial}} \): momentum operator of the pointer or dial

**Theorem: Recovering the Born Rule**

If an ensemble is in quantum equilibrium and some particular measurement is performed on each system in the ensemble, the relative frequencies of measurement outcomes will match those predicted by the Born rule.
Identify abstract state vectors with spin states in the obvious way, i.e. initial state is

$$\psi(y, t = 0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}_D \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}_A \otimes \phi_0(y)$$

Evolution in gates leaves pointer (and hence the ensemble distribution) unaffected but only changes spin states. The final state just before the time $t_{meas}$ of measurement is

$$\psi(y, t_{meas} - \epsilon) = \begin{cases} \pm \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix}_D \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}_A \otimes \phi_0(y) & \text{if } f(0) = f(1) \\ \pm \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}_D \otimes \begin{pmatrix} 1 \\ -1 \end{pmatrix}_A \otimes \phi_0(y) & \text{if } f(0) \neq f(1) \end{cases}$$
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Now look at a different implementation using particles with spatial degrees of freedom: A particle in an $n$-dimensional infinite well ($n = \text{number of qubits}$).

- Use two lowest energy eigenstates.
- Two qubits in algorithm $\Rightarrow$ particle in a two-dimensional well
- Focus on trajectories inside the Black Box.
Position basis and associated energies:

\[ \langle x|0 \rangle = \sqrt{2} \sin(\pi x), \quad E(0) = \frac{1}{2} \pi^2 m \]

\[ \langle x|1 \rangle = \sqrt{2} \sin(2\pi x), \quad E(1) = 2\pi^2 m \]

Hamiltonian is of the form \( H = \frac{p^2}{2m} + V(x, y) \), so the guidance equation for the particle is

\[ \frac{d\mathbf{x}}{dt} = \frac{1}{m} \nabla S(x, y, t) \quad \text{where} \quad \psi(x, y, t) = |\psi(x, y, t)|e^{iS(x,y,t)} \]
Example of dynamical implementation of gates:

- Free time evolution (Z-rotation):

\[ |\psi\rangle = a |0\rangle + b |1\rangle \rightarrow e^{-i(E_0 + E_1)t/2} (ae^{i\omega t} |0\rangle + be^{-i\omega t} |1\rangle) \]

with \( \omega = \frac{1}{2} (E_0 - E_1) \).

This corresponds to \( |\psi\rangle \rightarrow e^{iZ\omega t} |\psi\rangle \) plus an overall phase factor.

- To achieve an X-rotation, add a perturbation \( \delta V(x) \) inside the box:

\[ \delta V(x) = -\frac{9\pi^2}{16} \left( x - \frac{1}{2} \right) \Rightarrow \langle i | \delta V | j \rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{ij} = X_{ij} \]

- X and Z rotations are sufficient to achieve any overall rotation in the Bloch sphere.
One possibility to implement the two-qubit evolution inside the black box:

\[ U(x, y) = (A + B \cos(x) + Cx \cos(x)) \left[ -\frac{9\pi^2}{16} \left(y - \frac{1}{2}\right) - 1 \right] \]

with

\[ A = \frac{52}{27}, \quad B = -\frac{225}{432}\pi^2, \quad C = \frac{225}{216}\pi^2 \]
Numerically computed trajectories for the evolution inside the Black Box for the function $f = f_2$, i.e. $f(0) = 1$, $f(1) = 0$, with $T = \frac{\pi}{2}$. 
We could have ignored the trajectories inside the black box and only looked at the measurement – this would have been sufficient to account for the success of the algorithm in a dBb description.

But do the trajectories inside the black box (or the rest of the circuit) add any explanatory value?

— The answer will influence the debate over what causes the speed-up.

What about an ensemble in non-equilibrium?
- Analysis in terms of Bell's model for spin is the same as before: the configuration only moves in the process of measurement.
- The motion of the ensemble can be inferred from $\psi$ (in equilibrium).
- We can look at the trajectories themselves through numerical simulations, although it remains open what insights we can gain from that.
We have seen the details of examples of quantum computing in de Broglie-Bohm, both the wavefunction evolution and the trajectories.

As such, we have provided a technical description from which one may draw one's own conclusion concerning the debate over the speed-up of quantum computers. However . . .
Most of the “work” is done by the wavefunction (especially clear in case of Bell’s model), at least in quantum equilibrium.

The particle’s role is merely to make the configuration concrete and thereby define the outcome of a measurement.

In non-equilibrium, other kinds of measurements are possible (Valentini, 2002) and new “gates” are conceivable (e.g. gates that separate non-orthogonal states), leading to new kinds of algorithms. The set of possible equilibrium quantum algorithms would then be a subset of all possible “subquantum” algorithms. (Future investigation!)
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