Abstract: In this lecture I will describe in simple terms the basic ideas of gauge symmetry in phase space, its consequences in the form of a deeper redefinition of space and time, and some observable manifestations of an extra space and extra time dimensions.
Main Concepts & Themes

• Basic principle: gauge symmetry in phase space. (Evolution of Einstein: fundamental eqs. independent of observer)

• A consequence: higher space-time that redefines 1T-physics: 1 extra time + 1 extra space. Gauge degrees of freedom.

• Relation between 1T-physics and 2T-physics. 2T is unifying structure for 1T with extra gauge degrees of freedom. New insights and predictions that 1T-physics misses systematically. Observable manifestations of extra 1+1 dims in the language of 1T-physics.

• Outline of progress 1995-2012 and future paths.
Some aspects of any gauge symmetry

- Gauge symmetry dictates **existence** of certain degrees of freedom (photon, gluons, etc), requires **unique equations** and interactions.
- Gauge symmetry requires **redundant** (non-physical) gauge degrees of freedom. In their presence the theory is more beautiful and more transparent.
- Gauge generators must vanish (constraints)! Physical phenomena are associated with only the gauge invariant subspace of the degrees of freedom. **Physical subspace** satisfies the constraints.
- Many ways to **gauge fix**, and solve constraints, to identify the **physical subspace**. All gauge fixed forms of a gauge fixed theory are dual to each other. The **duality** transformation is a gauge transformation from one fixed gauge to another fixed gauge.
- Think of M-theory in this light!! (dualities & gauge symmetries)
**2T-physics**: Gauge symmetries that dictate extra spacetime gauge degrees of freedom

1 extra time + 1 extra space required (not by hand!)
No ghosts or causality problems in physical subspace.

With extra 1+1 dimensions the equations of physics in d dims get unified, are more beautiful, and more predictive in d+2; Verifiable predictions of 2T are systematically missed in 1T; Is a more powerful approach, provides new tools for computing.

Evident symmetries of d+2 dimensions in 2T-physics appear as hidden symmetries in 1T systems in d dimensions in 1T-physics

Gauge choices in 2T-physics lead to many dualities in 1T, (1T “shadows” in d dims of the same 2T system in d+2) 
⇒ unsuspected relations among 1T-physics systems
Review familiar example in 1T: relativistic particle

\[ L = \dot{x}^{\mu} p_{\mu} - e(\tau) \frac{p^2}{2} \]

1 non-compact parameter gauge symmetry, local on worldline

\[
\delta_{\Lambda} x^{\mu}(\tau) = \Lambda(\tau) \left\{ x^{\mu}, \frac{p^2}{2} \right\} = \Lambda(\tau) \frac{p^2}{2}
\]

\[
\delta_{\Lambda} p^{\mu}(\tau) = \Lambda(\tau) \left\{ p^{\mu}, \frac{p^2}{2} \right\} = 0
\]

\[
\delta_{\Lambda} e(\tau) = \partial_{\tau} \Lambda(\tau)
\]

\[
\delta_{\Lambda} L(\tau) = \left( \partial_{\tau} (\Lambda p^{\mu}) p_{\mu} + \dot{x} \cdot 0 - \partial_{\tau} \frac{p^2}{2} \right) = \partial_{\tau} \left( \Lambda \frac{p^2}{2} \right)
\]
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\[ \delta_\Lambda x^\mu (\tau) = \Lambda (\tau) \left\{ x^\mu, \frac{p^2}{2} \right\} = \Lambda (\tau) p^\mu (\tau) \quad \delta_\Lambda p^\mu (\tau) = \Lambda (\tau) \left\{ p^\mu, \frac{p^2}{2} \right\} = 0 \quad \delta_\Lambda e (\tau) = \partial_\tau \Lambda (\tau) \]

\[ \delta_\Lambda L (\tau) = \left( \partial_\tau (\Lambda p^\mu) p_\mu + \dot{x} \cdot 0 - \partial_\tau (\Lambda \frac{p^2}{2}) \right) = \partial_\tau \left( \Lambda \frac{p^2}{2} \right) \]

Equation of motion for the gauge field \( e \): \( p.p = 0 \)

Trivial solution if metric is Euclidean (no content).

MUST have 1T degree of freedom in target space \( x^\mu (\tau), p^\mu (\tau) \)

Time exists because the gauge symmetry demands it.
Gauge invariant **physical subspace** of phase space is reached by many possible **gauge choices**

**Examples:**
- Time-like gauge, $x^0(\tau) = \tau$
- Lightcone gauge, $x^0(\tau) + x^1(\tau) = \tau \sqrt{2}$
- Moving observer gauge, $[x^0(\tau) + v x^1(\tau)] = \tau (1 + v^2)^{1/2}$
- Other complicated gauges, $F(x^\mu(\tau), p^\mu(\tau)) = \tau$

Observers see the same system all differently (time, space, events, etc.).

Gauge fixed Lagrangians look different, but are all gauge equivalent = **duals**. Corresponding Hamiltonians related by canonical transformations (duality). Lorentz symmetry not manifest in fixed gauges; a hidden symmetry!

All observers in all gauges see the same gauge invariants.

All unified by the same covariant equations, $p.p = 0$, ...

**Different perspectives of same thing**

need some training to recognize they are the same thing
Motivation for 2T in M-theory

- The dualities in M-theory are discrete gauge transformations. They are canonical transformations (electric-magnetic charges). There must be a gauge invariant form of M-theory. What is the gauge group of M-theory?
- Even without knowing the full M-theory, it is known that a global symmetry of M-theory is the extended supersymmetry in 11-dimensions. I pointed out in 1995 that this is actually a symmetry in 12D with SO(10,2) symmetry in 10 space and 2 time dimensions. (Later, F-theory also 12D).
- This started me on the path of 2T-physics, and to the construction of a gauge symmetry that can solve the fundamental problem of ghosts in a theory with 2 timelike coordinates.
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- This started me on the path of 2T-physics, and to the construction of a gauge symmetry that can solve the fundamental problem of ghosts in a theory with 2 timelike coordinates.
- Starting with a single particle, it turned out that this gauge symmetry is uniquely $Sp(2,R)$ in phase space. This is the fundamental ingredient of 2T-physics, and must be one of the ingredients (a subgroup) of the gauge symmetry of M-theory.
Kaluza-Klein is the wrong idea for extra time dimensions:

e.g. $A_M(X^M)$ has wrong sign kinetic terms,
also causality problems with closed timelike curves

Need enough gauge symm. to remove extra field components from vectors, tensors & spinors,

*and the extra time dimensions from $X^M$.*

e.g. $A_M(X^M) \rightarrow A_\mu(x^\mu)$, etc

1T shadows! with many kinds of 1T spaces
The Fundamental Principle (1998)
Phase space \((X,P)\) gauge symmetry in the formulation of fundamental physics

- General coordinate invariance removes bias of observers in \(X\)-space (Einstein’s GR, etc).
- Next level: remove bias in phase space \((X^M,P_M)\), not just in \(X\)-space.
  - Hints: symmetries of Poisson brackets, quantum commutators, .... Canonical transformations!
  - dualities in M-theory are also phase space symmetries (electric-magnetic), ...
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- **Canonical transformations turned into gauge symmetry.** Start with worldline, then field theory, then ...
  (continuing project, ultimately field theory in phase space)
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• **Canonical transformations turned into gauge symmetry.** Start with worldline, then field theory, then ...
  (continuing project, ultimately field theory in phase space)
• **2T is not an input**, it is a consequence of gauge symmetry.
Phase space symmetry in particle classical mechanics

\[ L = \partial_\tau X^M P_M - \cdots \]

A huge symmetry of the first term in \( L \) under GLOBAL canonical transformations.

\[ \delta X^M = \frac{\partial \varepsilon (X, P)}{\partial P_M} = \{X^M, \varepsilon\}, \quad \delta \varepsilon \{X^M, P_N\} = 0 \quad \delta \varepsilon [X^M, P_N] = 0. \]
\[ \delta P_M = -\frac{\partial \varepsilon (X, P)}{\partial X^M} = \{P_M, \varepsilon\}, \quad \delta \left( \partial_\tau X^M P_M \right) = \partial_\tau \left( P_M \frac{\partial \varepsilon (X, P)}{\partial P_M} - \varepsilon (X, P) \right) \]

These \( \varepsilon (X, P) \) contain all gauge transformations Maxwell, Einstein, Yang-Mills, & MUCH MORE …

\[ L = \partial_\tau X^M P_M - H (X, P) \quad \delta \varepsilon H = \{H, \varepsilon\} \]

\[ \varepsilon (X, P) = \Lambda (X) + \varepsilon^M (X) P_M + \varepsilon^{MN} (X) P_M P_N + \cdots \]
\[ H (X, P) = G^{MN} (X) (P_M + A_M (X)) (P_N + A_N (X)) + \cdots \][Higher spin fields]

Proposal (1998): To be able to remove ghost degrees of freedom from \( X, P \), promote canonical transformations to a gauge symmetry by localizing on the worldline (every instant of motion)

\[ \varepsilon (X (\tau), P (\tau), \tau) \]
Gauge Symmetry in Phase Space

\[ \epsilon (X (\tau), P (\tau), \tau) = \sum_a \epsilon^a (\tau) Q_a (X, P) \quad \{Q_a, Q_b\} = f_{ab}^{\ c} Q_c \]

\[
L = \partial_\tau X^M P_M - A^a (\tau) Q_a (X, P) - H (X, P)
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\[
\delta_\epsilon X^M = \epsilon^a (\tau) \frac{\partial Q_a (X, P)}{\partial P_M} = \{X^M, \epsilon\}
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\[
\delta_\epsilon A^a = D_\tau \epsilon^a (\tau) = \partial_\tau \epsilon^a (\tau) + f_{bc}^a A^b \epsilon^c (\tau)
\]

\[
\{H, Q_a\} = 0 \quad \delta L = \partial_\tau \left( P_M \frac{\partial \epsilon (X, P, \tau)}{\partial P_M} - \epsilon (X, P, \tau) \right)
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Equation of motion

\[ Q_a(X, P) = 0 \]

means only gauge invariant subspace of phase space is physical.

Analogous to Virasoro constraints in string theory.
Gauge Symmetry in Phase Space

\[ \varepsilon (X(\tau), P(\tau), \tau) = \sum_a \varepsilon^a(\tau) Q_a(X, P) \]
\[ \{Q_a, Q_b\} = f_{ab}^c Q_c \]

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\[ \{H, Q_a\} = 0 \]
\[ \delta L = \partial_\tau \left( P_M \frac{\partial \varepsilon(X, P, \tau)}{\partial P_M} - \varepsilon(X, P, \tau) \right) \]

Familiar example: relativistic particle
1 local parameter

More general, any \(Q(X, P)\):
1-parameter non-compact Abelian gauge symmetry. Requires 1T and removes ghosts from 1T phase space
Three generators $Q_{ij}(X,P)$: $\{Q_{11}, Q_{22}, Q_{12}=Q_{21}\}$ form $\text{Sp}(2,R)$ under Poisson brackets.

$A_{ij}(t)$ is the $\text{Sp}(2,R)$ gauge potential, $i=1,2$ is label for $\text{Sp}(2,R)$ doublet.

$$\mathcal{L}_{2T} = \partial_{\tau} X^M P_M - \frac{1}{2} A_{ij}^i Q_{ij} (X, P) - H(X, P)$$

Example, flat spacetime:

$Q_{11}=X.X$, $Q_{22}=P.P$, $Q_{12}=X.P$, any signature metric $\rightarrow$ doublet $(X, P)$.

General $Q_{ij}(X,P)$ — expand in powers of $P_M$, coefficients= fields
c.t. generate general coordinate symmetry, gauge symmetry, and much more symmetry (hep-th/0103042).
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C.T generate general coordinate symmetry, gauge symmetry, and much more symmetry (hep-th/0103042).

Gauge invariant sector $Q_{ij}=0$. It exists non-trivially only if spacetime has two times, no less and no more.
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c.t. generate general coordinate symmetry, gauge symmetry, and much more symmetry (hep-th/0103042).

Gauge invariant sector $Q_{ij} = 0$. It exists non-trivially only if spacetime has **two times**, no less and no more

In flat case

Global symmetry $SO(d,2)$

$L^{MN} = X^M P^N - X^N P^M$, \{$Q_{ij}$, $L^{MN}$\} = 0,

So $L^{MN}$ is gauge invariant.

Could add $H(L^{MN})$, could break $SO(d,2)$, is OK, still gauge inv.
Shadows from 2T-physics $\Rightarrow$ hidden info in 1T-physics

2T-physics
Sp(2,R) gauge symm.
generators $Q_{ij}(X,P)$ vanish

simplest example
$X^2=P^2=X\cdot P=0 \Rightarrow$ gauge inv.
space: flat 4+2 dims
SO(4,2) symmetry
Shadows from 2T-physics $\Rightarrow$ hidden info in 1T-physics

Massless relativistic particle
$(p_\mu)^2=0$
conformal sym Dirac

Harmonic oscillator
2 space dims
mass = 3\textsuperscript{rd} dim
SO(2,2)$\times$SO(2)

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H-atom
3 space dims
$H=p^2/2m - a/r$
SO(4)$\times$SO(2)
SO(3)$\times$SO(1,2)

Massive
relativistic
$(p_\mu)^2+m^2=0$
Non-relativistic
$H=p^2/2m$
Shadows from 2T-physics \(\Rightarrow\) hidden info in 1T-physics

- **Twistors**
  - \(su(2,2)\rightarrow so(4,2)\)
  - Twistors transform for all these

- **Massless relativistic particle**
  - \((p_\mu)^2=0\)
  - Conformal symmetry
  - Dirac

- **Particle in Robertson-Walker expanding Universe**

- **Harmonic oscillator**
  - 2 space dims
  - mass = 3rd dim
  - \(SO(2,2)\times SO(2)\)

- **Simplest example**
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  - space: flat 4+2 dims
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- **2T-physics**
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  - generators \(Q_{ij}(X,P)\) vanish

- **Massive**
  - \((p_\mu)^2+m^2=0\)
  - Non-relativistic
  - \(H=p^2/2m\)

- **Particle in any conformally flat space**
  - Some singularities
  - Black Holes

- **H-atom**
  - 3 space dims
  - \(H=p^2/2m - a/r\)
  - \(SO(4)\times SO(2)\)
  - \(SO(3)\times SO(1,2)\)

- **AdS\(_{4+n}\) x S\(_n\)**

- **AdS\(_4\) x S\(_2\)"
Shadows from 2T-physics $\rightarrow$ hidden info in 1T-physics

**Spin!** Anyonic atom etc

**SUSY, Backgrounds.**

Twistors
\( su(2,2) \approx so(4,2) \)

Transform for all these

Harmonic oscillator
2 space dims
Mass = 3rd dim
SO(2,2)xSO(2)

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SO(4,2) symmetry

Massive
Relativistic
\( (p_\mu)^2 + m^2 = 0 \)
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\( H = p^2 / 2m \)

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3 space dims
\( H = p^2 / 2m - a/r \)
SO(4)xSO(2)
SO(3)xSO(1,2)

Particle in any
Conformally flat space
Singular okay
Black holes

Particle in any
Maximally symmetric space, e.g.
AdS, \( n \times S^n \)

Shadows also for \( \infty \) choices of \( Q_{ij}(X,P), H(X,P) \) & in 2T-field theory
Shadows from 2T-physics $\rightarrow$ hidden info in 1T-physics

- Spin! Anyonic atom etc.
- Massless relativistic particle $(p_\mu)^2 = 0$
- Conformal symmetry
- Dirac

- Twistors
  - $\text{su}(2,2) = \text{SO}(4,2)$
  - Transform for all these

- Harmonic oscillator
  - 2 space dims
  - mass = 3rd dim
  - $\text{SO}(2,2) \times \text{SO}(2)$

- 2T-physics
  - $\text{Sp}(2,R)$ gauge symmetry
  - Generators $Q_{ij}(X,P)$ vanish
  - Simplest example
  - $X^2 = P^2 = X \cdot P = 0 \rightarrow$ gauge inv.
  - Space: flat 4+2 dims
  - $\text{SO}(4,2)$ symmetry

- Massive
  - Relativistic $(p_\mu)^2 + m^2 = 0$
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- Particle in any
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- H-atom
  - 3 space dims
  - $H = p^2 / 2m - a/r$
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- Particle in any
  - Maximal symmetric space, e.g.
  - $\text{AdS}_{4+n} \times S^n$

Shadows also for $\infty$ choices of $Q_{ij}(X,P)$, $H(X,P)$ & in 2T-field theory
Shadows from 2T-physics \(\Rightarrow\) hidden info in 1T-physics

- Spin! Anyonic atom etc
- Susy, Backgrounds.

Twistors
- \(\text{su}(2,2)\) 3-soliton
- Twist for all these

Massless relativistic particle
- \((p_\mu)^2=0\)
- Conformal sym
- Dirac

Particle in Roberston-Walker expanding Universe

Emergent spacetimes and emergent parameters:
- Mass, couplings, curvature, etc.

Harmonic oscillator
- 2 space dims
- Mass = 3rd dim
- \(\text{SO}(2,2)\times\text{SO}(2)\)

Simplest example
- \(X^2=P^2=X\cdot P=0\)
- Gauge inv.
- Space: flat 4+2 dims
- \(\text{SO}(4,2)\) symmetry

2T-physics
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Shadows also for \(\infty\) choices of \(Q_{ij}(X,P), H(X,P)\) & in 2T-field theory
Observers like us are stuck on the "walls" (3+1 dims.), no privilege to be in the room (4+2).

The relation between 2T-physics and 1T-physics described by an analogy:
Consider object in the room ≈ (phase space, \(X^M, P_M\) in 4+2 dims.)

and its MANY shadows on walls ≈ (MANY phase spaces, \(x^m, p_m\) in 3+1.)
2T-Physics as a completion and unifying framework for 1T-physics

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holographic

ONE 2T system → MANY 1T systems

Predict many relations among the shadows (dualities, symmetries d+2).

Contains systematically missed information in 1T-physics approach.

This info related to higher spacetime:
Instead of interpreting the shadows as different dynamical systems (1T), we recognize they are perspectives in higher spacetime. Then, we can indirectly “see” the extra 1+1 dims.
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1) 2T-physics makes new testable predictions, e.g. \( \text{SO}(4,2) C_2=-3 \) for all shadows, etc. (duals) and new computational tools.
2) 1T-physics is incomplete !!!

This info related to higher spacetime:
Instead of interpreting the shadows as different dynamical systems (1T), we recognize they are perspectives in higher spacetime. Then, we can indirectly "see" the extra 1+1 dims.
Some manifestations of 4+2 dimensions in 1T-physics at various distance/energy scales

Some “windows” where 4+2 is easier to spot:

- Conformal symmetry SO(4,2) in massless systems, in many contexts, at various energy scales.
- Celestial mechanics: why planetary orbits always in same plane, and axis of ellipse does not precess year after year? Hidden SO(4), Runge-Lenz. (rotation symm including 4th dim)
- H-atom spectrum, degeneracies and patterns (next slide).
- Cosmology – (i) FRW has hidden SO(4,2), 
  (ii) scale invariant spectrum of perturbations etc. ,
  (iii) nature of singularities (recent, IB, Chen, Steinhardt, Turok)

A lot more “windows” harder to spot, but predicted by 2T-physics
“Seeing” 4+2 dimensions through the H-atom

Energy
(n)
0
1
2
3

Angular momentum (L)

D=16

D=9

D=4

E_n~(-1/n^2), L=0,1,…(n-1)

n=4
1
3
5

n=3
1
3
5

n=2
1
3

n=1
1

D=1
"Seeing" 4+2 dimensions through the H-atom

Seeing 4 dimensions

Why different L at same energy? Hidden SO(4) symm

$n=2$: 4 states at the same energy = vector of SO(4)

Higher levels: symmetric traceless tensors of SO(4) with $n-1$ indices.

Seeing 1 space (the 4th), and 2 times:

At each fixed value of $L$ (3-space), the energy towers are patterns of space-time symmetry group SO(1,2):

$SO(4,2) > SO(3) \times SO(1,2)$ or $SO(4) \times SO(2)$

$E_n \sim (-1/n^2)$, $L=0,1,\ldots(n-1)$
Subtle effects in 1T:

“Seeing” 4+2 dimensions through the H-atom

Seeing 4 dimensions
Why different L at same energy? Hidden SO(4) symm

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Seeing 1 space (the 4th), and 2 times:
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SO(4,2) > SO(3)xSO(1,2) or SO(4)xSO(2)

Quantum level L^{MN} Lie algebra. A single unitary representation of SO(4,2) the singleton, C_2 = -3. Same as the massless particle.
Why levels of the H-atom are degenerate?

Why in celestial mechanics planetary orbits never change?

Because of the rotation symmetry between the 4th dim (w) and the other 3 dims (x,y,z)

1) Looks like rotation in x,y plane L=1

2) Looks like radial motion L=0
The same principle for all physics: Sp(2,R) gauge symmetry in phase space, and appropriate extensions with spin and SUSY.
The same principle for all physics: $\text{Sp}(2,R)$ gauge symmetry in phase space, and appropriate extensions with spin and SUSY.

- Shadows and 1T surprises
- Background fields
- Spin
- SUSY
- Twistors
- Strings & Branes (partially)

**Local Field Theory**

- BRST field theory approach
- 2T Standard Model in 4+2
- 2T Gravity in d+2
- 2T SUGRA in 4+2
- N=1,2,4 SUSY field theory in 4+2
- SYM in 10+2 ($\rightarrow$ Matrix theory)
- M-theory limit as 11+2 sugra (project)
- Cosmology – Big Bang + Inflation
- Phenomenology: LHC, other
- New computational methods in d+2 using the shadows (Weinberg)

**Phase space field theory**

- Non-commutative field theory
- Derived phase space $\text{Sp}(2,R)$ eqs from variation principle in NC field theory
- Expect it to be most fundamental form of 2T physics
Progress in 2T-physics

• **Local Sp(2,R): A general principle in Class. & Quant. Mech.**
  A principle for a higher unification and deeper insight into physics & space-time
  Reveals more physics phenomena that are systematically missed in 1T-physics.

• **Principles of 2T field theory in d+2 dimensions**
  The Standard Model, General Relativity, and GUTS in 2T-physics,
  IB+Y.C.Kuo 0605267, IB 0606045, 0610187, 0804.1588; IB+S.H.Chen 0811.2510
  Phenomenological applications: Cosmology 1004.0752, LHC 0606045, 0610187,
  Path integral quantization in d+2 field theory – in progress

• **SUSY in 4+2 field theory, and in Higher dimensions**
  IB+Y.C.Kuo, 0702089, 0703002, 0808.0537 (N=1,2,4 in 4+2 dims) Klein-Gordon, Dirac, Yang-Mills fields.
  IB+Y.C.Kuo , N=1 SYM in 10+2 – (parent of N=4 SYM in 3+1; parent of M(atrix) theory).
  IB: SUGRA 4+2, 10+2 and 11+2 (toward M-theory in 2T framework): path is clear, details in progress.

• **Strings, Branes, M-theory in 2T-physics (partial progress)**
  IB+Dellduman+Minic, 9906223, 9904063 (tensionless string); IB 0407239 (twistor string)
  M-theory: expect 11+2 dims → OSp(1|64) global SUSY, related to S-theory (IB 9607112)

• **Non-perturbative technical tools in 1T-field theory**
  Sp(2,R)-induced dualities among 1T field theories (many shadows of 2T-field theory), “AdS-CFT”.
  IB+Chen+Quelin 0705.2834; IB+Quelin 0802.1947 + in progress. Note: S. Weinberg 1006.3480.
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• **A more fundamental approach – field theory in phase space (full Q,P symm)**
  IB + Deliduman 0103042, IB + S.J.Rey 0104135, IB 0106013, + under development.

• **Cosmological Applications (conformal symmetry)** - IB + C.H. Chen + N. Turok + P. Steinhardt
How shadows might be used in "AdS-CFT" type correspondence

Shadows from Flat 5+2, all have SO(5,2) in the same repr.

Flat 4+1
Robertson-Walker 4+1
AdS$_5$
AdS$_4$ x S$^1$
AdS$_3$ x S$^2$
AdS$_2$ x S$^3$
RxS$^4$
deSitter$_5$
All conf. flat in 5D
H-atom$_{4+1}$
Massive particles$_{4+1}$
Etc.

Shadows from Flat 4+2, all have SO(4,2) in the same repr.

Flat 3+1
Robertson-Walker 3+1
AdS$_4$
AdS$_3$ x S$^1$
AdS$_2$ x S$^2$
RxS$^3$
deSitter$_4$
All conf. flat spacetimes 4D
H-atom$_{3+1}$
Massive particles$_{3+1}$
Etc.

The usual case
AdS$_5$
When there exist phenomena not explained by current theory, it is signal to evolve to better formulation.

"Of course the elements are earth, water, fire and air. But what about chromium? Surely you can’t ignore chromium."
When there exist phenomena not explained by current theory, it is signal to evolve to better formulation.

Examples and general principle make it amply evident that 1T physics is incomplete.

"Of course the elements are earth, water, fire and air. But what about chromium? Surely you can't ignore chromium."
Do you need 2T? YES!

When there exist phenomena not explained by current theory, it is signal to evolve to better formulation.

Examples and general principle make it amply evident that 1T physics is incomplete

"Of course the elements are earth, water, fire and air. But what about chromium? Surely you can't ignore chromium."

2T-physics is a new direction in higher dimensional unification.

Expect more predictions at every scale of distance or energy, and more powerful computational tools in future research...
Where to find more information on 2T-physics

For concepts and technical guidance on over 50 papers:
My recent talk at Gell-Mann’s birthday celebration
arXiv:1004.0688

A book at an elementary level for science enthusiasts (Springer 2009):

By
Itzhak Bars
and
John Terning

It can be downloaded at your university if your library has a contract with Springer
DOI: 10.1007/978-0-387-77638-5

Search for I. Bars and J. Terning at SPIRES
A quotation from Gell-Mann

(Once you have a consistent theory),

Anything which is not forbidden is compulsory!
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