Abstract: Charged colloidal particles present a controllable system for study a host of condensed matter/many body problems such as crystallization. 2D crystals are invariably hexagonal. Hexagons perfectly tile a flat plane but a soccer ball requires exactly 12 pentagons dispersed among the hexagons on its curved surface. Pentagons and hexagons are positive and negative topological charges, disclinations, sources for positive and negative curvature. But we have discovered that "Pleats", grain boundaries which vanish on the surface (and play a similar role to fabric pleats) can provide a finer control of curvature. We experimentally investigate the generation of topological charge as flat surfaces are curved. For positive curvature, domes and barrels, there is one pentagon added for every 1/12 of a sphere. Negative curvature is different! For capillary bridges forming catenoids, pleats relieve the stress before heptagons appear on the surface. Pleats are important for controlling curvature from crystals on surfaces, to the shape of the spiked crown of the Chrysler building. Adding a particle to a flat surface produces an interstitial - usually an innocuous point defect. On a curved surface interstitials are remarkable, forming pairs or triplets of dislocations which can fission dividing the added particles into fractions which migrate to disclinations.

Classical Wigner Crystals on Flat and Curved Surfaces
topological defects, “Pleats” and classical particle fractionalization

Creating dislocation pairs

Dislocations and Disclinations on a Capillary Bridge
Classical Wigner Crystals on Flat and Curved Surfaces

topological defects, “Pleats” and classical particle fractionalization

Creating dislocation pairs

Dislocations and Disclinations on a Capillary Bridge
Packing Densities for Spheres

$\phi_{FCC} = 0.74$

$\phi_{RCP} = 0.64$

MRJ
maximally random jammed

Ball Bearings

Figure 14. Face-centered cubic "crystal" surrounded by "liquid" caused by shearing ball-bearing mass. H1 face is shown at the top surface.
van der Waals and excluded volume

\[
S = N k_B \ln(V - Nb) = N k_B \ln(V(1 - \phi/\phi_c))
\]

\[
P = \frac{N k_B T}{V - Nb} = \frac{N k_B T}{V(1 - \phi/\phi_c)}
\]

Exact in 1D

Exact asymptotic form in any dimension

\[
P \propto \frac{dnk_B T}{(1 - \phi/\phi_c)}
\]

\[\Rightarrow \text{Entropy drives liquid to crystal}\]

\[S_{\text{liquid}} \rightarrow 0 \text{ as } \phi \rightarrow 0.64 \quad S_{\text{crystal}} \rightarrow 0 \text{ as } \phi \rightarrow 0.74\]

Highest Packing Fraction determines Stable High Density Phase
Polymer Hard Spheres - Colloids in Oil

PMMA-PHSA

Steric stabilization in decalin - tetralin

~1 μ

methyl methacrylate (●); methacrylic acid (○); glycidyl methacrylate (●); 12-hydroxystearic acid (≡)

Originally Ron Ottewill - Bristol
ours were from Andy Schofield - Edinburgh
now home grown by Andy Hollingsworth
“Hard Sphere” Colloidal Sample

60% volume fraction

Crystallized in microgravity in space

Remelted in gravity forms “glass phase”

remains glass after ~1 year

7
"Hard Sphere" Colloidal Sample

60% volume fraction

Crystallized in microgravity in space

Remelted in gravity forms "glass phase"

remains glass after ~ 1 year
Try to density match in decalin – CHB (cyclohexylbromide)

Coulomb crystals

Get charge stabilized colloid in oil, and screening length is enormous, $\lambda > 30\mu$

Van Blaaderen - Utrecht
Only get colossal crystals in some samples.

What’s different? ———— Water?

- oil $\varepsilon \approx 3$
- water $\varepsilon \approx 80$

- water has much higher $\varepsilon$
- should suck ions out of oil

$$E = -\frac{\varepsilon E^2}{2}$$
Try to density match in decalin – CHB (cyclohexylbromide)

Coulomb crystals

Get charge stabilized colloid in oil, and screening length is enormous, \( \lambda > 30 \mu \)

Van Blaaderen - Utrecht
Try to density match in decalin – CHB (cyclohexylbromide)

Coulomb crystals

Get charge stabilized colloid in oil, and screening length is enormous, $\lambda > 30\mu$

Van Blaaderen - Utrecht
Only get colossal crystals in some samples.

What's different? ------- Water?

oil $\varepsilon \sim 3$ \quad $\downarrow$ \quad water $\varepsilon \sim 80$

\[ E = -\frac{\varepsilon E^2}{2} \]

Water has much higher $\varepsilon$.

Should suck ions out of oil.
Add water to half of cell

Confocal view of a layer in the cell

H₂O

Oil

Depleted zone

Colossal crystal

Interface is probably charged – surface chemistry?

Monolayer held by image charge
If water Droplets differentially pump ions from oil then we should see charging effects without colloids present.

Two samples of clean cc CHB/decalin, with a drop of water added and sonicated:
- Wigner Crystal of 2-3 micron water droplets
- Emulsion stabilized by charge alone – no surfactants
- Shake instead of sonicate, big, polydisperse, charge stabilized emulsion

New form of crystallized water
- Ice 10?

H2O - CHB

H2O – CHB/decalin
If water Droplets differentially pump ions from oil then we should see charging effects without colloids present.

Two samples of clean cc CHB/decalin, with a drop of water added and sonicated:
• Wigner Crystal of 2-3 micron water droplets
• Emulsion stabilized by charge alone – no surfactants
• Shake instead of sonicate, big, polydisperse, charge stabilized emulsion

New form of crystallized water
- Ice 10?

H2O - CHB

H2O – CHB/decalin
If water droplets differentially pump ions from oil then we should see charging effects without colloids present.

Two samples of clean cc CHB/decalin, with a drop of water added and sonicated:
- Wigner Crystal of 2-3 micron water droplets
- Emulsion stabilized by charge alone – no surfactants
- Shake instead of sonicate, big, polydisperse, charge stabilized emulsion

New form of crystallized water
- Ice 10?

H2O - CHB

H2O – CHB/decalin
Add water to half of cell

Confocal view of a layer in the cell

Depleted zone

Colossal crystal

Interface is probably charged — surface chemistry?

Monolayer held by image charge
Principle Of Confocal Microscopy

Sees only an illuminated fluorescent particle
Scan xyz to get image
Everything I'm going to show you is data
An experiment based course on topological defects

\[ \psi = \frac{1}{6} \sum_{n=1}^{6} e^{i\theta_n} \]
An experiment based course on topological defects

\[ \psi = \frac{1}{6} \sum_{n=1}^{6} e^{i6\theta_n} \]
An experiment based course on topological defects
Disclinations

`elementary charges`

dipole

Very expensive in the plane
An experiment based course on topological defects

React

[Diagram showing reactions and microns]
Stretching: From commensurate to incommensurate

Stretch in steps - t=76.032 sec
Organization under dynamical potentials
The Euler Characteristic

\[ V - E + F = 2 \]

<table>
<thead>
<tr>
<th>Polyhedron</th>
<th>V</th>
<th>E</th>
<th>F</th>
<th>V-E+F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tetrahedron</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cube</td>
<td>12</td>
<td>30</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>Icosahedron</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ 1 - \frac{6}{2} + \frac{6}{3} = 0 \]
The Euler Characteristic

\[ V - E + F = \chi \]

Vertices \quad Faces

Edges

Sphere

\[ \chi \]

1

2
Curvature and non-euclidean geometry

Gaussian curvature

\[ K = \frac{1}{R_1} \frac{1}{R_2} \]

Positive Gaussian curvature

\[ \int K \, dA = \frac{\pi}{3} \]

Negative Gaussian curvature

\[ \int K \, dA = -\frac{\pi}{3} \]
Pleats
- Add width as one traverses their length
- As do aligned strings of dislocations
  - Grain boundaries which vanish on the surface
- Add an angular wedge from 0 to 30°
  (disclinations add 60°)
- Pleats produce “coneyness”
- Pleat gradients produce curvature
Disclinations 60°  Pleats ~20°
When do you pleat instead of making disclinations?

Area relieved by disclination

\[ \int \kappa dA = \frac{\pi}{3} \]

\[ \frac{\pi r^2}{R_1 R_2} = \frac{\pi}{3} \]

\[ r = \sqrt{\frac{R_1 R_2}{3}} \]