This talk analyzes the limits that quantum mechanics imposes on the accuracy to which spacetime geometry can be measured. By applying the fundamental physical bounds to measurement accuracy ensembles of clocks and signals, as in the global positioning system, I present a covariant version of the quantum geometric limit, which states that the total number of ticks of clocks and clicks of detectors that can be contained in a four volume of spacetime of radius $R$ and temporal extent is less than or equal to $RT$ divided by the Planck length times the Planck time.

The quantum geometric bound limits the number of events or `ops' that can take place in a four-volume of spacetime and is consistent with and complementary to the holographic bound which limits the number of bits that can exist within a three-volume of spacetime.
Science 306, 1330 (2004); quant-ph/0412078
+ Gr-QC/0502170
Science 306, 1330 (2004); quant-ph/0402078
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Science 306, 1330 (2004); quant-ph/0412078

Gr-QC/0502970

Quantum limits to measurement
Science 306, 1330 (2004); quant-ph/0412078

+ Gr-QC/0502970

Quantum limits to measurement

Standard quantum limit
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Quantum limits to measurement
Standard quantum limit
δp
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Quantum limits to measurement

Standard quantum limit

$\Delta P \sim \frac{1}{\sqrt{n}}$
Quantum limits to measurement

Standard quantum limit

$\Delta x \Delta p \geq \frac{\hbar}{2}$

Wigner's clock
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+ Gr-QC/0502970

Quantum limits to measurement

Standard quantum limit

\[ \Delta Q \geq \hbar \]

Wigner's clock
Quantum limits to measuring space + time

Standard quantum limit

Wigner's clock
Science 306, 1330 (2004); quant-ph/0412078

+ Gr-Qc, tomorrow

Quantum limits to measuring space + time

Standard quantum limit

\( \Delta x \propto \frac{1}{\Delta t} \)

Wigner's clock
Quantize GPS
Quantize GPS

\[ \Delta E \Delta t \geq \frac{\hbar}{2} \]

Heisenberg relation:
Quantize GPS

Heisenberg relation:

\[ \Delta E \Delta t \geq \frac{\hbar}{2} \]

Mandelstam-Levine limit.
Quantize GPS

Heisenberg relation:

$\Delta E \Delta t \geq \frac{\hbar}{2}$

Marples-Leviton theorem

$E \Delta t \geq \frac{\hbar}{2}$

$E = \text{tr} (H - E_0)$
Quantize GPS

Heisenberg relation:
\[ \Delta E \Delta t \geq \frac{\hbar}{2} \]

Merryfield-Levitin theorem:
\[ E \Delta t \geq \frac{\hbar}{2} \]

\[ E = \text{trace} \left( \mathbf{H} - E_0 \right) \]
State $\rightarrow$ orthogonal state

$|+\rangle = \frac{1}{\sqrt{2}} \left( |0\rangle + e^{-i\omega t} |1\rangle \right)$
State $\rightarrow$ orthogonal state

$$1+\gamma = \frac{1}{\sqrt{2}} \left( 10 + e^{i\gamma} 11 \right)$$

when $\gamma t = \frac{\pi}{\omega}$

$$\Rightarrow 1-\gamma = \frac{1}{\sqrt{2}} \left( 10 - e^{i\gamma} 11 \right)$$

$$E_0 = 0$$

$$E_1 = \hbar \omega$$
State $\rightarrow$ orthogonal state

$|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$

when $\delta t = \frac{\pi}{\omega}$

$\Rightarrow |\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$

$E = h\omega$

$E_0 = 0$

$E_1 = h\omega$

$\Delta E = h\omega$

$\text{take} - E_0 = \text{take} - E_1$
State $\rightarrow$ orthogonal state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{i\theta} |1\rangle)$$

when $\Delta t = \frac{\pi}{\omega}$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle - e^{i\theta} |1\rangle)$$

$E_0 = 0$

$E_1 = \hbar \omega$

$\Delta E = \hbar \omega$

$\Delta E - E_0 = \hbar \omega/2$
Quantize GPS

Heisenberg relation:

\[ \delta E \delta t \geq \frac{\hbar}{4} \]

Margolus-Levitin thm

\[ E \delta t \geq \frac{\hbar}{2\pi} \]

\[ E = \text{tr} \rho H - E_0 \]
# tricks clocks/clucks of detectors/fflips

\[ \leq \]
# ticks / clocks / clicks of detector / bit flips

\[ \leq \frac{2 E}{\hbar} \]

Quantum tunneling 

but no black hole \( r \geq R_s = \frac{2 G m}{c^2} \)
$\text{# ticks} \leq \frac{2E \epsilon}{\hbar^2}$

Quantum: $r \geq Rs = \frac{26m}{c^2}$
# ticks clocks / clicks of detector / bit flips

\[ \leq \frac{2E}{c^4} \]

Quantum limits

\[ \text{but no black hole } \Rightarrow r \geq R_s = \frac{2GM}{c^2} \]

\[ \Rightarrow \# \leq \frac{4\pi r t}{c^4} = \frac{4\pi t}{c^4} \]
# ticks clocks / clicks of detector / bit flips

\[ \leq \frac{2 E T}{\pi h} \]

Quantum information:

\[ \text{bit} = \text{bit} \text{- flip} \]

but

\[ \text{black hole } \Rightarrow r \geq R_S = \frac{2 M}{C^2} \]

\[ \Rightarrow \frac{C^4}{r} t = \frac{R T}{\Pi h} \]

\[ = \frac{2 E}{C^4} \]
Quantum geometric limit →
'space-time holography'
encourages us to imagine each 'op'
Quantum geometric limit $\rightarrow$

'space-time holography'

encourages us to imagine each 'op'
(=elementary quantum event)
as projected onto a $1+1$ space-time

'world sheet'
Quantum geometric limit → 'space-time holography'

encourages us to imagine each 'op'
(elementary quantum event)
as projected onto a 1+1 space-time
'world sheet' at density of
no greater than $O(1/\ell_{\text{ptp}})$
Quantize GPS

Hermann relation:

\[ \delta E \delta t \geq \frac{\hbar}{2} \]

Maryplus-Levinthin

\[ E \delta t \geq \frac{\hbar}{2} \]

\[ E = \text{tr} e^{iH} E_0 \]
Quantize GPS

Hessenberg relation:

\[ \delta E \Delta t \geq \frac{\pi}{2} \]

Margolus-Levitin thin

\[ E \Delta t \geq \pi \]

\[ E = \frac{\pi}{\Delta t} \]
Make covariant cylinder.
Make covariant cylinder

$\mathbf{g} + 1$ cylinder of radius $r$. 

Pirsa: 12060061
Quantize GPS

Hessanal relation:

Margolis-Levitin thin

\[ \Delta x_m \geq \frac{\pi}{\hbar} \]

\[ E = \text{tr} \rho \mathcal{H} - E_0 \]
Shorm clocks \Rightarrow V^m \text{ tangent vectors}

T^m V^n V^n = \text{energy density according local clock}

V^m V^n = -1
Shawm clocks $\Rightarrow v^m$ tangent vectors

$T_{nmn} = \text{energy density according local clock}$

$\int T_{nmn} \, dv = 'E \cdot t'$
Shawm clocks \Rightarrow V^m \text{ tangent vectors}

\[ T_{\mu \nu} V^\mu V^\nu = \text{energy density} \]

according local clock

\[ \frac{2}{\pi k} \left( \int T_{\mu \nu} V^\mu V^\nu ds - \frac{\gamma E \cdot t}{\pi k} \right) \leq f \]
Scharm clocks $\Rightarrow V^m$ tangent vectors

$T^m V^n V^r V^s = \text{energy density according local clock}$

$$\frac{1}{\mathcal{H}} \int T^m V^n V^r \, dv \geq \frac{\mathcal{E}}{\mathcal{H}} t$$
is an orthonormal tetrad or vierbein

\[ \eta_{ab} = \text{diag}(1,1,1,1) \]

\[ \eta_{ab} e_a e_b = g_{mn} \]
\{ e^m \}_{\alpha \beta} \text{ is an orthonormal tetrad or vierbein}

\eta_{ab} = \text{diag}(-, +, +, +)

\eta_{ab} e^a e^b = g_{\mu \nu}
The tetrad or vierbein is an orthonormal basis for the tangent space at each point on a manifold. The metric tensor $g_{\mu\nu}$ encodes the metric information and relates the tetrad components $e^\mu$ to the coordinate basis vectors $\partial_\mu$.

The equation $v = g_{\mu\nu} e^\nu$ expresses the projection of a vector $v$ onto the tangent space at a point, where $v$ is a vector in the tangent space, and $e^\nu$ are the tetrad components.
\{e^m_{\nu}\}

is an orthonormal
tetrad or
vierbein

\eta_{\alpha\beta} e^\alpha_m e^\beta_r = g_{mr}
\[ \eta_{ab} \epsilon^a_{\mu} \epsilon^b_{\nu} = g_{\mu\nu} \]
Rule: for each op remove an area $\alpha l_p^2$ from the six sections defined be $e_{m} \times e_{v}$ etc. ($e_{m} \times e_{v}$)
Rule: for each op remove an area $aLP^2$

from the six sections defined be $e_m \times e_v$ etc. $(e^n_m \times e^n_v)$
is an orthonormal tetrad or vierbein.
\[ e^m_5 \]

is an orthonormal tetrad or vierbein

\[ \eta_{ab} e^a e^b = g_{\mu \nu} \]
Rule: for each op, remove an area $\alpha^2$ from the six sections defined
be $e_m \times e_n \ldots$ etc.

$\Rightarrow$ Transform des Einsteins equations.
Area removed = \frac{2 \times 2}{\pi h}
Area removed = \( \alpha \cdot \# \)

\[ \frac{2 \pi}{3} \Theta \]

\[ \Delta \sum_{\alpha \neq 6} K_{\alpha 6} \]

Gaussian curvature
Area removed = \( \alpha \# = \frac{2 \pi}{11} \times \text{energies} \)

\[
\Delta \frac{E}{2} \sum_{a \neq b} K_{ab}
\]

Gaussian curvature

\[
\sum_{a \neq b} \text{Ricci tensor}
\]

\[
\sum_{a \neq b} K_{ab}
\]
\[ T^\mu_{\mu} e_\mu e^\nu = \frac{2}{\pi^2} K_a c \]

\[ = \frac{1}{2} \left( R_{\mu \nu} e^\mu e^\nu + R_{\rho \sigma} e^\rho e^\sigma + R_{\mu \rho} e^\mu e^\rho + R_{\mu \nu} e^\mu e^\nu \right) \]

\[ = \left( R_{\mu \nu} - g_{\mu \nu} R \right) e^\mu e^\nu = \frac{2 \xi}{T^\mu_{\mu}} T^{\mu \nu} e_\mu e_\nu \]

\[ \Rightarrow \text{Einstein's equations} \]

\[ x = \frac{\pi^2}{3} \]