Title: Far from equilibrium energy flow in quantum critical systems
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Abstract: We investigate far from equilibrium energy transport in strongly coupled quantum critical systems. Combining results from gauge-gravity duality, relativistic hydrodynamics, and quantum field theory, we argue that long-time energy transport after a local thermal quench occurs via a universal steady-state for any spatial dimensionality. This is described by a boosted thermal state. We determine the transport properties of this emergent steady state, including the average energy flow and its long-time fluctuations.
Far-from-equilibrium dynamics in CFTs and holography

Koenraad Schalm

_Institute Lorentz for Theoretical Physics, Leiden University_

M.J. Bhaseen, B. Doyon, A. Lucas

Perimeter Institute
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The AdS/CFT correspondence

\[ Z_{\text{CFT}}(J; g, N) = \exp i S_{\text{AdS}}^{\text{on-shell}}(\phi(\phi_{\partial \text{AdS}} = J)) \]

Use AdS/CFT as a tool to generate strongly coupled critical theories
• Quantum Phase Transitions

Sachdev
- Quantum Phase Transitions

Quantum critical = Finite T CFT

2nd order phase transition: critical = conformal universal behavior
CFTs in experiment

- Quantum Phase Transitions

Sachdev

$Z_{\text{CFT}}(J; g, N) = \exp i S_{\text{on-shell}}^{\text{on-shell}}(\phi(\phi_{\partial \text{AdS}} = J))$

<table>
<thead>
<tr>
<th>CFT</th>
<th>AdS</th>
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<tbody>
<tr>
<td>$O_\phi$</td>
<td>$\phi$</td>
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<tr>
<td>$J^\mu$</td>
<td>$A^\mu_{\text{gauge}}$</td>
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<tr>
<td>$\Delta_\phi$</td>
<td>$m_\phi$</td>
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<td>...</td>
<td></td>
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<td>finite $T$</td>
<td>BH</td>
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Witten; Gubser, Klebanov, Polyakov
Real time responses

- A remarkable ability of AdS/CFT
  - Direct crossover to hydrodynamics

\[ \frac{1}{q^2} \text{Im} \langle J^0 J^0 \rangle_R \]

\[ q = \frac{3k}{4\pi T} \]

\[ w = \frac{3\omega}{4\pi T} \]
Far-from-equilibrium dynamics

• Driven Steady State?
  - Non-thermal distributions

• Universality in Non-equilibrium dynamics?
  - Kibble-Zurek scaling
  - Kolmogorov scaling

Chesler, Yaffe; de Boer, Kesko-Vakkuri +9; Bhaveen, Gauntlett, Simons, Sonner, Wiseman; Basu, Das, Nishioka Takanayagi; Albash, Johnson; Abajo-Arrastia, Aparicio, Lopez; Ebrahim, Headrick; Bhattacharyya, Minwalla; .... Buchel, Lehner, Myers, van Niekerk; Das, Galante, Myers, ....
Motivation: unique ability of holography

Actual: Combination of holography, hydrodynamics and QFT
Motivation: unique ability of holography

Actual: Combination of holography, hydrodynamics and QFT
• Thermal Quench in 1+1 CFTs
• Thermal Quench in 1+1 CFTs

\[ T_L \quad T_R \]

\[ t = 0 \]

- Intuitive expectation
• Thermal Quench in 1+1 CFTs

Bernard, Doyon

finite size effects at very late times...
- Thermal Quench in 1+1 CFTs

\[ x = -ct \quad \text{steady state with } J_{\text{heat}} \neq 0 \quad x = ct \]

\[ \langle J \rangle = \frac{c\pi}{12} (T_L^2 - T_R^2) \]

- Call \( \langle J \rangle = J(\beta_L, \beta_R) \) with \( \beta_L = 1/T_L, \ \beta_R = 1/T_R \)

\[ \langle J^{n+1} \rangle = \frac{d^n}{d\mu^n} J(\beta_L - \mu, \beta_R + \mu) \bigg|_{\mu=0} \]
- **Thermal Quench in 1+1 CFTs**

\[
\begin{align*}
T_L & & \text{steady state with } J_{\text{heat}} \neq 0 & & T_R \\
\end{align*}
\]

\[
x = -ct & & x = ct
\]

\[
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- Call \( \langle J \rangle = J(\beta_L, \beta_R) \) with \( \beta_L = 1/T_L, \beta_R = 1/T_R \)

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\langle J^{n+1} \rangle = \left. \frac{d^n}{d\mu^n} J(\beta_L - \mu, \beta_R + \mu) \right|_{\mu=0}
\]
• What is this steady state?
  - Not an obvious driven state
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  - Not an obvious driven state
**Time dependent DMRG** (density matrix renormalization group)

- **XXZ Hamiltonian**

\[ h_n = J_n \left( S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta_n S_n^z S_{n+1}^z \right) + b_n (S_n^z - S_{n+1}^z) \]

\[ J_n = \begin{cases} 1 & n \text{ odd} \\ \lambda & n \text{ even} \end{cases} \quad \Delta_n = \Delta \quad b_n = \frac{(-1)^n b}{2} \]
• Time dependent DMRG
  - XXZ Hamiltonian

\[ h_n = J_n \left( S_n^x S_{n+1}^x + S_n^y S_{n+1}^y + \Delta_n S_n^z S_{n+1}^z \right) + b_n (S_n^z - S_{n+1}^z) \]

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1 & n \text{ odd} \\
\lambda & n \text{ even} 
\end{cases}, \quad \Delta_n = \Delta, \quad b_n = \frac{(-1)^n b}{2} \]

homogeneous constant heat flow
• Time dependent DMRG
  - XXZ Hamiltonian

\[ \langle J \rangle \sim f_L(T_L) + f(T_R) \]
• How to understand this state?
  - Constant Heat flow vs Temperature relaxation

  Intuitive expectation

  \[ T_{L} \quad T_{R} \]

  \[ T^{00} = -aT(x)^2 \]

  EM Conservation plus CFT equation of state.

  \[ \partial_0 T^{0x} = -\partial_x T^{xx} \]

  \[ T^{xx} = -T^{00} \]

  \[ \partial_0 T^{0x} = 0 \quad \Rightarrow \quad T(x) = T \]
- How to understand this state?
  - Constant Heat flow vs Temperature relaxation

Intuitive expectation

\[ T^00 = -\alpha T(x)^2 \]

EM Conservation plus CFT equation of state

\[ \partial_\alpha T^{\alpha x} = -\partial_x T^{xx} \]

\[ T^{xx} = T^{00} \]

\[ \partial_\alpha T^{\alpha x} = 0 \quad \Rightarrow \quad T(x) = T \]
- How to understand this state?
  - holomorphic factorization (integrability)
  - In a 1+1 dim CFT left and right movers do not interact

Left of the interface

$$J_{p>0} \sim T^2_L, \quad J_{p<0} \sim T^2_L$$

Right of the interface

$$J_{p>0} \sim T^2_R, \quad J_{p<0} \sim T^2_R$$

At the interface $t = 0$

$$J_{p>0} \sim T^2_L, \quad J_{p<0} \sim T^2_R$$

very special to 1+1 D
• How to understand this state?
  - holomorphic factorization (integrability)
  - In a 1+1 dim CFT left and right movers do not interact

Left of the interface
\[ J_{p>0} \sim T_L^2, \quad J_{p<0} \sim T_L^2 \]

Right of the interface
\[ J_{p>0} \sim T_R^2, \quad J_{p<0} \sim T_R^2 \]

At the interface \( t = 0 \)
\[ J_{p>0} \sim T_L^2, \quad J_{p<0} \sim T_R^2 \]

very special to 1+1 D
• Holography

- Only Heat, i.e. pure AdS-gravity

\[ S = \int \sqrt{-g} (R - 2\Lambda) \]

- aAdS Solution to Einstein with a constant unsourced Heat current

\[ ds^2 = \frac{L^2}{r^2} dr^2 + g^{(0)}_{ij}(r) dx^i dx^j \]

\[ g^{(0)}_{ij} = \frac{r^2}{L^2} + \ldots + \frac{1}{r^d} \langle T_{ij} \rangle + \ldots \]

\[ \langle T_{ij} \rangle = \begin{pmatrix} -\rho & J \\ J & \rho \end{pmatrix} \]
• Holography

- Unique solution: boosted BTZ black hole

\[ ds^2 = -\frac{r^2}{L^2} (1 - \frac{M^2 \cosh^2(\eta)}{r^2}) dt^2 + \frac{r^2 + M^2 \sinh^2(\eta)}{L^2} dx^2 + \frac{M^2}{L^2} \sinh(2\eta) dx dt + \frac{L^2}{r^2} \frac{dr^2}{(1 - \frac{M^2}{r^2})} \]

- This is dual to a state with constant heat current

\[ \langle T^{0x} \rangle = \frac{c\pi}{6} T_{BH} \sinh(2\eta) \]

\\

\[ \ldots \]

\[ \ldots \]

eg Figueras, Wiseman, Fischetti, Marolf
• “Boosts” to understand real transport
  - Classical Hall effect

rest frame

\[ E = 0 \]
\[ J = 0 \]

boosted frame

\[ E = -v \times B \]
\[ J = \rho v \]

\[ J_i = \sigma_{ij} E_j \quad \Rightarrow \quad \sigma_{xy} = \frac{\rho}{B_z} \]
• Holography

- **Unique** solution: boosted BTZ black hole

\[ ds^2 = -\frac{r^2}{L^2}(1 - \frac{M^2 \cosh^2(\eta)}{r^2})dt^2 + \frac{r^2 + M^2 \sinh^2(\eta)}{L^2} dx^2 + \frac{M^2}{L^2} \sinh(2\eta) dx dt + \frac{L^2}{r^2} \frac{dr^2}{(1 - \frac{M^2}{r^2})} \]

- This is dual to a state with constant heat current

\[ \langle T^{0,x} \rangle = \frac{c\pi}{6} T_{BH} \sinh(2\eta) \]

*The novel steady state coincides with the boosted equilibrium state identifying*

\[ T_L = T e^\eta, \]
\[ T_R = T e^{-\eta} \]

Bhaseen, Doyon, Lucas, KS
1+1 CFT/AdS$_3$ is special

- Factorization: can solve the full quench exactly

\[ d\bar{S}_{FG}^2 = \frac{L^2}{r^2} \left[ dr^2 + \bar{g}_{\mu\nu}(r,t,x) dx^\mu dx^\nu \right]. \]

\[
\bar{a}_n = - \left( 1 - \frac{c}{L^2} (f_R(x-t) + f_L(x+t)) \right)^2 + \left( \frac{c}{L^2} (f_R(x-t) - f_L(x+t)) \right)^2.
\]

\[
\bar{a}_c = -2 \frac{c^2}{L^2} (f_R(x-t) - f_L(x+t)).
\]

\[
\bar{a}_{xx} = \left( 1 + \frac{c^2}{L^2} (f_R(x-t) + f_L(x+t)) \right)^2 - \left( \frac{c^2}{L^2} (f_R(x-t) - f_L(x+t)) \right)^2.
\]

\[
(T_{xx}) = \frac{c}{6\pi L^2} \frac{f_R(x-t) - f_L(x+t)}{2L}.
\]

\[
f_L(x) = f_R(x) = \frac{\pi^2}{2L^2} \left( T_{xx}^L + (T_{R}^L - T_{L}^L) \Theta(x) \right)
\]
• Boosted equilibrium suggests hydro applies
  - $d+1$ Conformal Hydro for a thermal quench
    Effective dimensional reduction to $1+1$ dimension
    $d+1$ Conformal $\neq$ Integrable = dissipation

$$J(x, t) = \theta(t + x) + \theta(t - x) - 1$$

$J(x, t)$
$\rho(x, t)$

*sound fronts*
• Boosted equilibrium suggests hydro applies
  - \( d+1 \) Conformal Hydro for a thermal quench

  Effective dimensional reduction to \( 1+1 \) dimension

  \( d+1 \) Conformal \( \neq \) Integrable = dissipation

\[ J(x, t) = \theta(t + x) + \theta(t - x) - 1 \]
• Matching across shocks

\[ T_{\mu \nu} = T_{\mu \nu}(x + u_L t) + T_{\mu \nu}(x - u_R t) \]

\[ \int_{\text{shock}} \partial_\mu T^{\mu \nu} = \int_{\text{shock}} \partial_x T^{x \mu} + u_{\text{shock}} \partial_x T^{0 \mu} = 0 \]

4 equations for 4 unknowns

\[ x = -u_L t \quad x = u_R t \]
**d+1 dim Thermal Quench**

- **Matching across shocks**

\[
T_{\mu\nu} = T_{\mu\nu}(x + u_L t) + T_{\mu\nu}(x - u_R t)
\]

\[
\int_{\text{shock}} \partial_{\mu} T^{\mu\nu} = \int_{\text{shock}} \partial_x T^{x\mu} + u_{\text{shock}} \partial_x T^{0\mu} = 0
\]

4 equations for 4 unknowns

\[
T_{ss} = \sqrt{T_L T_R}
\]

\[
u_L = \frac{1}{d} \sqrt{\frac{\chi + d^{-1}}{\chi + d}}
\]

\[
u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}}
\]

\[
\eta_{ss} = \frac{\chi - 1}{\sqrt{(\chi + d^{-1})(\chi + d)}}
\]

\[
\langle T^{tx} \rangle = a_d \left( \frac{T_{T_L}^{d+1} - T_{T_R}^{d+1}}{u_L + u_R} \right)
\]

\[
\chi = \left( \frac{T_L}{T_R} \right)^{\frac{d+1}{2}}
\]

\[
x = -u_L t
\]

\[
x = u_R t
\]
- Shocks are non-linear sound waves

\[ u_L = \frac{1}{d} \sqrt{\frac{\chi + d}{\chi + d^2}} \quad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}} \]

\[ u_L u_R = c_s^2 = \frac{1}{d} \]

\[ \chi = \left( \frac{T_L}{T_R} \right)^{\frac{d+1}{2}} \]
• Confirming with numerical (ideal) hydro

\[ u_L = \frac{1}{d} \sqrt{\frac{\chi + d}{\chi + d^{-1}}} \quad u_R = \sqrt{\frac{\chi + d^{-1}}{\chi + d}} \]

asymmetric

formation of steady state
• Dissipative corrections should not change this
  - Small shocks can be traced in linear response

\[ T(x, t) = T_L + \frac{T_R - T_L}{4} \left[ 2 + \text{erf} \frac{x - t/\sqrt{d}}{4D_{||}t} + \text{erf} \frac{x + t/\sqrt{d}}{4D_{||}t} \right] \]

*Width \sqrt{D_{||}t} is smaller than the distance \( t/\sqrt{d} \)*

Numerically confirmed by Chang, Karch, Yarom

• Turbulence?
  - Assumption: completely smooth \( T \) discontinuity
  - Allows reduction to eff 1+1 dim system
• Dissipative corrections should not change this
  - Small shocks can be traced in linear response

\[
T(x, t) = T_L + \frac{T_R - T_L}{4} \left[ 2 + \frac{\text{erf} \frac{x - t/\sqrt{d}}{4D_{||} t}}{4D_{||} t} + \frac{\text{erf} \frac{x + t/\sqrt{d}}{4D_{||} t}}{4D_{||} t} \right]
\]

\[
\text{width } \sqrt{D_{||} t} \text{ is smaller than the distance } t/\sqrt{d}
\]

Numerically confirmed by Chang, Karch, Yarom

• Turbulence?
  - Assumption: completely smooth \( T \) discontinuity
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Non-Thermal distributions

- The Fluctuation Spectrum
  - So far we have looked at xpv $\langle T_{\mu\nu} \rangle$
  - Cumulants of the current at the interface

$$c_n \equiv \langle J^n (x = 0) \rangle$$

Extended Fluctuation Relation

$$\langle J^{n+1} \rangle = \left. \frac{d^n}{d\mu^n} J(\beta_L - \mu, \beta_R + \mu) \right|_{\mu=0}$$
Non-Thermal distributions

- **The Fluctuation Spectrum**
  - So far we have looked at xpv \( \langle T_{\mu \nu} \rangle \) (=hydro)
  - Cumulants of the current at the interface
    \[ c_n \equiv \langle J^n(x = 0) \rangle \]
    Extended Fluctuation Relation
    \[
    F(z) = \sum \frac{1}{n!} z^n c_n
    \]
    \[
    \frac{dF(z)}{dz} = J(\beta_L - z, \beta_R + z)
    \]

  *holds in any d!*
Outlook

- Direct Momentum relaxation vs dissipation.
- Including charge discontinuity in the quench.
- Effects of turbulence.

- Direct connection to cold atom experimental set-ups.