Title: The Unreasonable Effectiveness Of Quantum Physics in Modern Mathematics

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Abstract: Mathematics has proven to be "unreasonably effective" in understanding nature. The fundamental laws of physics can be captured in beautiful formulae. In this lecture I want to argue for the reverse effect: Nature is an important source of inspiration for mathematics, even of the purest kind. In recent years ideas from quantum field theory, elementary particles physics and string theory have completely transformed mathematics, leading to solutions of deep problems, suggesting new invariants in geometry and topology, and, perhaps most importantly, putting modern mathematical ideas in a `naturalâ€™ context.
The Unreasonable Effectiveness of Quantum Physics in Modern Mathematics

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Mathematics & Physics
“The Unreasonable Effectiveness of Mathematics in the Natural Sciences.”

— Eugene Wigner (1960)
Galileo’s `Book of Nature’
“Philosophy is written in this grand book — I mean the universe — which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language and interpret the characters in which it is written. **It is written in the language of mathematics**, and its characters are triangles, circles, and other geometrical figures, without which it is humanly impossible to understand a single word of it; without these, one is wandering around in a dark labyrinth.”
“To those who do not know mathematics it is difficult to get across a real feeling as to the beauty, the deepest beauty, of nature ...

If you want to learn about nature, to appreciate nature, it is necessary to understand the language that she speaks in.”

Anonymous Mathematician

Richard Feynman
18541^2 = 13500^2 + 12709^2
Platonic Solids
The Four Elements

Earth  Fire  Universe  Water  Air

Quintessence

Scotland, 2000 BC.
Old Hall of the House of Representatives, The Hague
Dodecahedral space topology as an explanation for weak wide-angle temperature correlations in the cosmic microwave background

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The current ‘standard model’ of cosmology posits an infinite flat expanding under the pressure of dark energy. In the Wilkinson Microwave Anisotropy Probe this model to spectacular precision on all but a few scales. Temperature correlations across the micro-variations on angular scales narrower than 60⁰ predictions, vanish on scales wider than 60⁰. Observations have been proposed⁵,⁶. One natural is the underlying geometry of space—namely, topology⁶. In an infinite flat space, waves from should fill the universe on all length scales. The temperature correlations on scales beyond 60⁰ broadest waves are missing, perhaps because big enough to support them. Here we present a
Johannes Kepler (1571-1630)
Mysterium Cosmographicum (1596)
Kepler’s First Cosmology
Kepler’s First Law

- Parabola
- Circle
- Ellipse
- Hyperbola
A = B
Clinton’s Principle

\[ A = B \]

It depends on what the meaning of `is’ is.
Image vs Text
Mathematical Brain

Algebra

Geometry
Geometry

General Relativity

Algebra

Quantum Mechanics
Reduction

**Macrophysics**

**Microphysics**

*Standard Model*
Quantization

Geometry

\[ K \]

geometric object

Algebra

\[ Z(K) \in C \]

quantum invariant
Emergence

**Macrophysics**
- hydrodynamics
- thermodynamics

**Microphysics**
- molecules
- statistical mechanics
Emergence

Geometry

effective geometry

Algebra

quantum system
“I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has recently ended in divorce.”
Anomalous magnetic moment $e^-$
$1.0011596521859 \pm 0.000000000000038$
“Time is the fourth dimension”
Who’s Afraid Of Extra Dimensions?
Four Dimensions
Why is every electron exactly the same?
There is only one electron in the universe!

John Wheeler
Each turn in + twice minus -

\[ \text{cancels} \]

\[ \text{cancels} \]

\[ \text{cancels} \]

\[ \text{cancels} \]

Rule:

- If a path from \( A \) to \( B \)
  - is followed in one direction, it
  - cancels the amp in \( X_{AB} \). If followed
  - in the other direction \( B \rightarrow A \), it
  - cancels \( X_{BA} = -X_{AB} \)

Because each turn in + is a turn for

\[ \text{must be in reverse (1 \\ X) except when it} \]

\[ \text{cancels a W or a M or a path.} \]

\[ \text{But the MAX + MIN to add, hence} \]

\[ \text{the sign changes.} \]

\[ \text{If we start + step going in same} \]

\[ \text{direction, then } X_{AB} = X_{BA} \text{ because the} \]

\[ \text{no turns in our} \]

\[ \text{cancels the} \]

\[ \text{cancels the} \]
Virtual Particles
Vacuum Fluctuations
Dark Energy

Quantum Space-Time
Knot Theory
The Book Of Knots
Chern-Simons Gauge Theory
Quantum Amplitude

quark

gluons
Counting

Enumerative Geometry
The Quintic

\[ x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 = 0 \]
Gromov-Witten theory

\[ N_d = \# \text{curves of degree } d \]

\[ x_1 = a_{1,d} z^d + a_{1,d-1} z^{d-1} + \ldots + a_{1,1} z + a_{1,0} \]

\[ \ldots \]

\[ x_5 = a_{5,d} z^d + a_{5,d-1} z^{d-1} + \ldots + a_{5,1} z + a_{5,0} \]
$d=1$ Lines

$N_1 = 2,875$
$d=2 \quad \text{Conics}$

$N_2 = 609,250$
\[ d=3 \quad \text{Cubics} \]
\[ N_3=317,206,375 \]
\[ N_1 = 2875 \]
\[ N_2 = 609250 \]
\[ N_3 = 317206375 \]
\[ N_4 = 242467530000 \]
\[ N_5 = 2293058888887625 \]
\[ N_6 = 248249742118022000 \]
\[ N_7 = 295091050570845659250 \]
\[ N_8 = 375632160937476603550000 \]
\[ N_9 = 503840510416985243645106250 \]
\[ N_{10} = 704288164978454686113488249750 \]
Strings
Worldsheet

Riemann surface
algebraic curve
Hidden Dimensions
String Theory

\[ F(t) = \sum_{d \geq 0} N_d e^{-dt} \]
Calabi-Yau Spaces
Mirror Symmetry

\[ F(t) = \sum_{d \geq 0} N_d e^{-td} \]
\[ \tilde{F}(t) = \int \Omega \]

quantum

classical
Symmetry
Strong Force (QCD)

3 colors of quarks
Global Symmetry
Local Gauge Symmetry
Gauge Fields
Intermediate Gauge Bosons
Gluons

$N \times N$ matrix
D-branes
multiplicity $N$

Internal space

space-time
U(N) Yang-Mills Theory

$N \times N$ matrix of open strings $A_{ij}$
U(N) Yang-Mills Theory

matrix multiplication $\sum_k A_{ik} A_{kj}$
Mathematics/Physics Complementarity

Mathematical rigor

Physical intuition
Plato’s Cave

Mathematical Dream

Physical Reality
Quantum Cave

Physical Dream

Mathematical Reality