Title: Gravitational RG flows on foliated spacetimes

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Abstract: The role of time and a possible foliation structure of spacetime are longstanding questions which lately received a lot of renewed attention from the quantum gravity community. In this talk, I will review recent progress in formulating a Wetterich-type functional renormalization group equation on foliated spacetimes and outline its potential applications. In particular, I will discuss first results concerning the RG flow of Horava-Lifshitz gravity, highlighting a possible mechanism for a dynamical Lorentz-symmetry restoration at low energies.
Gravitational RG flows from foliated spacetimes

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A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017
G. D’Odorico, F.S., in preparation

Renormalization group approaches to quantum gravity
Perimeter Institute, April 24, 2014
Outline

- Hořava-Lifshitz gravity from a Wilsonian viewpoint

- Wetterich equation for projective Hořava-Lifshitz gravity

- Constructing RG flows:
  - finite-temperature type computations
  - anisotropic heat-kernels

- Conclusions
Quantum Gravity
from a Wilsonian perspective
Theory space underlying the Functional Renormalization Group

\[ \Gamma_0 = \Gamma \quad \text{effective action} \]

\[ \Gamma = \Gamma_* \quad \sim \text{bare action} \]

\[ \Gamma_\infty = \Gamma_* \]
Fixed points of the RG flow

Fixed points \{g^*_i\}: Central ingredient in Wilsons picture of renormalization

- \( \beta \)-functions vanish: \( (\beta_{g_i}(g_i)|_{g_i=g^*_i} = 0) \)
  - RG-trajectories may “end” at a UV-fixed point
- dimensionless couplings remain finite
  - absence of unphysical UV divergences

Perturbations of fixed point theory controlled by stability matrix

\[
B_{ij} \equiv \frac{\partial \beta_{g_j}}{\partial g_i} \bigg|_{g_i=g^*_i}
\]

- 2 classes of scaling directions:
  - relevant = attracted to FP in UV
  - irrelevant = repelled from FP in UV
- predictivity:
  - finite number of relevant directions
Proposals for UV fixed points (incomplete...)

- **isotropic Gaussian Fixed Point (GFP)**
  - fundamental theory: Einstein-Hilbert action
  - perturbation theory in $G_N$

- **isotropic Gaussian Fixed Point (GFP)**
  - fundamental theory: higher-derivative gravity
  - perturbation theory in higher-derivative coupling

- **non-Gaussian Fixed Point (NGFP)**
  - fundamental theory: interacting
  - Lorentz-invariant, non-perturbatively renormalizable

- **anisotropic Gaussian Fixed Point (aGFP)**
  - fundamental theory: Hořava-Lifshitz gravity
  - Lorentz-violating, perturbatively renormalizable

Gravity
Embedding of QEG in Hořava-Lifshitz gravity

Theory space: Horava-Lifshitz
Symmetry: foliation preserving

Subspace: Quantum Einstein Gravity
Symmetry: diffeomorphisms

NGFP → GFP

β

aGFP

A[N, N_{a\sigma_{ab}}]
Wetterich equation
for projective Hořava-Lifshitz gravity
projective Hořava-Lifshitz gravity in a nutshell


central idea: find a perturbatively renormalizable quantum theory of gravity

fundamental fields: \( \{N(\tau), N_i(\tau, x), \sigma_{ij}(\tau, x)\} \)

symmetry: \( \text{Diff}(\mathcal{M}, \Sigma) \subset \text{Diff}(\mathcal{M}) \)

- breaks Lorentz-invariance at high energies

Can construct the effective average action for projective HL-gravity


- scale-dependence governed by functional renormalization group equation

\[ k\partial_k \Gamma_k[\phi, \bar{\phi}] = \frac{1}{2} \text{STr} \left[ \left( \Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} k\partial_k \mathcal{R}_k \right] \]

- Complication: anisotropic models have two correlation lengths
RG flows for projective HL gravity
finite temperature type computations
Foliated functional renormalization group equation

Flow equation: formally the same as in covariant construction

\[ k \partial_k \Gamma_k[h, h_i, h_{ij}; \sigma_{ij}] = \frac{1}{2} \text{STr} \left[ \left( \Gamma_k^{(2)} + R_k \right)^{-1} k \partial_k R_k \right] \]

- covariant: \( M^4 \)
  \[ \text{STr} \approx \sum_{\text{fields}} \int d^4 p \]

- foliated: \( S^1 \times M^3 \)
  \[ \text{STr} \approx \sqrt{\epsilon} \sum_{\text{component fields}} \sum_{\text{KK-modes}} \int d^3 p \]
  - structure resembles: quantum field theory at finite temperature!

Advantages of the foliated flow equation:

- captures RG-flow on theory space of Hořava-Lifshitz gravity
- same structure as CDT
- \( \epsilon \)-dependence: keep track of signature effects
ADM-decomposed Einstein-Hilbert truncation

fundamental fields: $\{N(\tau), N_i(\tau, x), \sigma_{ij}(\tau, x)\}$

ADM-decomposed Einstein-Hilbert action:

$$\Gamma_k^{ADM} = \frac{\sqrt{\epsilon}}{16\pi G_k} \int d\tau d^3 x N \sqrt{\sigma} \left\{ \epsilon^{-1} \sum_{ij} K_{ij} \left[ \sigma^{ik} \sigma^{jl} - \sigma^{ij} \sigma^{kl} \right] K_{kl} \right\} + \frac{R^{(3)}}{2} + 2\Lambda_k$$

- lives on foliation $S^1_T \times M^{(3)}$
- running couplings: $G_k, \Lambda_k$
- signature parameter $\epsilon$

$\beta$-functions depend parametrically on $m = \frac{2\pi T_k}{\epsilon}$:

$$k \partial_k g_k = \beta_g(g, \lambda; m), \quad k \partial_k \lambda_k = \beta_\lambda(g, \lambda; m)$$

- $m$: anisotropy between cutoff in spatial/time direction
result: signature dependence of NGFP

for $m$ finite NGFPs separate:

- $\epsilon = +1$: Euclidean signature (blue)
- $\epsilon = -1$: Lorentzian signature (magenta)
result: phase diagrams

covariant computation

Euclidean

Lorentzian
RG-flows of HL-gravity in the IR

A. Contillo, S. Rechenberger, F.S., JHEP 1312 (2013) 017

RG-flow of anisotropic Einstein-Hilbert truncation

\[ \Gamma_k^{\text{grav}}[N, N_i, \sigma_{ij}] = \frac{1}{16\pi G_k} \int d\tau d^3 x N \sqrt{g} \left[ K_{ij} K^{ij} - \lambda_k K^2 - \lambda^{(3)} R + 2\Lambda_k \right] \]

Fixed points of the beta functions:

- Wheeler-de Witt metric ⇒ line of GFPs
  \[ \tilde{G}_* = 0, \quad \tilde{\Lambda}_* = 0, \quad \lambda = \lambda_* \]
  - one IR attractive, one IR repulsive, one marginal direction

- NGFP:
  \[ \tilde{G}_* = 0.49, \quad \tilde{\Lambda}_* = 0.17, \quad \lambda = 0.44 \]
  - three UV-attractive eigen-directions
  - imprint of Asymptotic Safety

- aGFP providing UV-limit of HL-gravity not in truncation
Hořava-Lifshitz gravity: recovering general relativity in the IR
Scale-dependence of dimensionful couplings

GFP governs IR-behavior of HL-gravity
small value of cosmological constant makes $\lambda$ compatible with experiments
RG flows for projectable HL gravity
anisotropic heat-kernels
Zooming into the aGFP in $D = 3 + 1$

Compute matter-induced gravitational $\beta$-functions

$$\Gamma_k = \Gamma_k^{HL} + S^{LM}$$

where

$$\Gamma_k^{HL} = \frac{1}{16\pi G_k} \int dt d^3 x \sqrt{\sigma} \left[ (K_{ij} K^{ij} - \lambda_k K^2) - g_7 R \Delta_x R - g_8 R_{ij} \Delta_x R^{ij} + \ldots \right]$$

$$S^{LM} = \frac{1}{2} \int dt d^3 x \sqrt{\sigma} \left[ \phi (\Delta_t + (\Delta_x)^z) \phi \right]$$

- 8 running couplings including two wave-function renormalizations

key ingredient: anisotropic Laplace operator

$$D = \Delta_t + (\Delta_x)^z$$

$$\Delta_t = -\sqrt{\sigma}^{-1} \partial_t \sqrt{\sigma} \partial_t , \quad \Delta_x = -\sigma^{ij} (t, x) D_i D_j$$
Zooming into the aGFP in $d = 4$

Compute matter-induced gravitational $\beta$-functions

$$\Gamma_k = \Gamma_k^{\text{HL}} + S^{\text{LM}}$$

where

$$\Gamma_k^{\text{HL}} = \frac{1}{16\pi G_k} \int dt d^3 x \sqrt{\sigma} \left[ (K_{ij} K^{ij} - \lambda_k K^2) - g_7 R \Delta x R - g_8 R_{ij} \Delta x R^{ij} + \ldots \right]$$

$$S^{\text{LM}} = \frac{1}{2} \int dt d^3 x \sqrt{\sigma} \left[ \phi (\Delta t + (\Delta x)^2) \right] \phi$$

- expansion: $\sigma_{ij} = \delta_{ij} + \sqrt{16\pi G_k} h_{ij}$

Gravitational propagators (flat space):

$$[G_{s=2}(\omega, p)] \propto \omega^2 - g_8 p^6$$

$$[G_{s=0}(\omega, p)] \propto \left( \frac{1}{3} - \lambda_k \right) \left( \omega^2 - \frac{1}{3 - \lambda_k} \left( \frac{8}{9} g_7 + \frac{1}{3} g_8 \right) p^6 \right)$$
Heat kernel expansion of anisotropic operators

FRGE computations use heat-kernel expansion of Laplacian \( \Delta \equiv -g^{\mu\nu} D_\mu D_\nu \)

\[
\text{Tr} e^{-s\Delta} \simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \sum_{n \geq 0} s^n a_{2n}
\]

\[
\simeq \frac{1}{(4\pi s)^{d/2}} \int d^d x \sqrt{g} \left[ 1 + \frac{s}{6} R + \ldots \right]
\]

Heat kernel expansion of anisotropic operators

\( D \equiv \Delta_t + (\Delta_x)^z \)

- apply the “Universal Renormalization Group Machine”


\[
\text{Tr} e^{-sD} \simeq (4\pi)^{-(d+1)/2} s^{1/2 (1+d/z)} \int dt d^d x \sqrt{\sigma} \left[ \frac{s}{6} \frac{\Gamma(\frac{d}{2})}{\Gamma(\frac{d+2}{2})} \left( \frac{d-s+3}{d+2} K^2 - \frac{d+2z}{d+2} K_{ij} K^{ij} \right) + \sum_{n \geq 0} s^{n/z} b_n a_{2n} \right]
\]
Heat kernel coefficients for anisotropic operators

<table>
<thead>
<tr>
<th></th>
<th>$d = 2$</th>
<th></th>
<th>$d = 3$</th>
</tr>
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<tbody>
<tr>
<td>$z = 1$</td>
<td>$1$</td>
<td>$\frac{\sqrt{\pi}}{2}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$z = 2$</td>
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</tr>
<tr>
<td>$z = 3$</td>
<td>$1$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$b_3$</td>
<td>$1$</td>
<td>$-2$</td>
<td>$0$</td>
</tr>
<tr>
<td>$b_4$</td>
<td>$1$</td>
<td>$0$</td>
<td>$6$</td>
</tr>
</tbody>
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- $z = 1$: reproduces standard heat-kernel
- $z = 2, d = 2$: reproduces

[M. Baggio, J. de Boer and K. Holsheimer, arXiv:1112.6416]

- $d$ even: zero coefficients in heat kernel expansion
matter-induced RG flows in $d = 4$

UV attractive anisotropic GFP

$G^* = 0, \quad \lambda^* = 1/3, \quad g_{7}^* = \frac{5\pi}{84}, \quad g_{8}^* = \frac{\pi}{42}$
Embedding of QEG in Hořava-Lifshitz gravity

Theory space: Horava-Lifshitz
Symmetry: foliation preserving

Subspace: Quantum Einstein Gravity
Symmetry: diffeomorphisms

NGFP $\rightarrow$ GFP
$\beta \rightarrow aGFP$

$A[N, N_a \sigma_{ab}]$
Conclusions
Conclusions

Wetterich equation for projectable HL gravity

- powertool for constructing RG flows in anisotropic gravity
- two correlation lengths

theory space of projective HL gravity

- contains NGFP from asymptotic safety
- GFP capable of providing IR completion

matter-induced RG flow possesses anisotropic GFP:

\[ G^* = 0, \quad \lambda^* = 1/3, \quad g_7^* = \frac{5\pi}{34}, \quad g_8^* = \frac{\pi}{42} \]

- anisotropic GFP is UV attractive in \( G_k \)

projective HL gravity is asymptotically free at large \( N \)