Title: (Jean-Sebastien Caux) Exact solutions for quenches in 1d Bose gases and quantum spin chains

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Abstract:
Exact solutions for quenches in 1d Bose gases and quantum spin chains

Quantum Many-Body Dynamics Workshop, Perimeter Institute, 15 May 2014

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Work done in collaboration with (among others):
Bethe Ansatz (1931)

Integrable Hamiltonian:

\[ H = \int_0^L dx \hat{H}(x) \]

"Reference state": vacuum, FM state,

"Particles": atoms, down spins,
Bethe Ansatz (1931)

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Exact many-body wavefunctions (in N-particle sector):

$$\Psi_N(\{x\}| \{\lambda\}) = \sum_{F} (-1)^{|F|} A_F(\{\lambda\}) e^{ix_{\lambda_j}^{(\lambda)}(\lambda_j \cdot \rho_j)}$$
Bethe Ansatz (1931)

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Exact many-body wavefunctions (in N-particle sector):

$$\Psi_N(\{x\},\{\lambda\}) = \sum_P (-1)^{|P|} A_P(\{\lambda\}) e^{i\sum_{j<k}(\lambda_j - \lambda_k)}$$

... and obeying some form of Pauli principle.
Bethe Ansatz (1931)

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Exact many-body wavefunctions (in N-particle sector):
\[ \Psi_N(\{x\}|\{\lambda\}) = \sum_P (-1)^{[P]} A_P(\{\lambda\}) e^{ix_jk(\lambda_{P_j})} \]

... and obeying some form of Pauli principle
The general idea, simply stated:

Start with your favourite quantum state
(expressed in terms of Bethe states)

$\mathcal{O} \rightarrow |\{\lambda\}\rangle$

Apply some operator on it

Reexpress the result in the basis of Bethe states:

$\mathcal{O} |\{\lambda\}\rangle = \sum_{\{\mu\}} F^\mathcal{O}_{\{\mu\},\{\lambda\}} |\{\mu\}\rangle$

using ‘matrix elements’ $F^\mathcal{O}_{\{\mu\},\{\lambda\}} = \langle \{\mu\}|\mathcal{O}|\{\lambda\}\rangle$
The simple pendulum on its head
The Kapitza pendulum
Out-of-equilibrium using integrability
It's possible to treat some situations using
ABA-based reasonings
BEC to repulsive Lieb-Liniger quench
Interacting Bose gas (Lieb-Liniger)

\[ \mathcal{H}_N = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1 \leq j < l \leq N} \delta(x_j - x_l) \]

Exact eigenstates from Bethe Ansatz:

\[ \Psi(x|\lambda) = F_{\lambda} \sum_{P \in S_N} A_P(x|\lambda) \prod_{j=1}^{N} e^{i\lambda x_j} \]

\[ F_{\lambda} = \frac{\prod_{j=1}^{N} (\lambda_j - \lambda_k)}{\sqrt{N!} \prod_{j<k} ((\lambda_j - \lambda_k)^2 + c^2)} \]

\[ A_P(x|\lambda) = \prod_{j<k} \left( 1 - \frac{ic \text{ sgn}(x_j - x_k)}{\lambda_{P_j} - \lambda_{P_k}} \right) \]
BEC vs Tonks-Girardeau

Intuitive picture

BEC
\(c = 0\)
Quench from BEC to repulsive gas

Start from GS of noninteracting theory,

\[ |0_N \rangle \equiv \frac{1}{\sqrt{L^N N!}} \left( \psi^\dagger_{k=0} \right)^N |0\rangle \]

Turn repulsive interactions on from \( t=0 \) onwards:
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Turn repulsive interactions on from t=0 onwards:

particles 'repel away' from each other, system heats up, momentum distribution broadens, ...
Adilet Imambekov
1982-2012
GGE approach to BEC-LL quench (take I)

Kormos, Shashi, Chou and Imambekov, arXiv:1204.3889

Conserved charges:

\[ \hat{Q}_n : \hat{Q}_n \{\lambda\}_N = Q_n \{\lambda\}_N \]

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Field operator representation:

\[ Q_{2n} = \int dx \left( \{ \text{derivative terms} \} + A_n \ c^n \ [\psi(x)]^n \psi^n(x) \right) \]

Idea: evaluate these (local) operators on BEC state!
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GGE approach to BEC-LL quench (take 1)

GGE conditions become condition on moments

\[ \int d\lambda \rho(\lambda) \lambda^m = 2^m \frac{(2m - 1)!!}{(m+1)!} n^{2m+1} \gamma^m \]
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$$\int d\lambda \rho(\lambda) \lambda^{2m} = 2^m (2m - 1)!! \frac{(m + 1)!}{n^{2m+1} \gamma^m}$$

Solution found as 'semicircle law'

$$\rho(\lambda) = \frac{1}{\pi \sqrt{\gamma}} \sqrt{1 - \frac{\lambda^2}{\lambda_*^2}}, \quad \lambda_* = 2n \sqrt{\gamma}$$
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Solution found as ‘semicircle law’

\[ \rho(\lambda) = \frac{1}{\pi \sqrt{\gamma}} \sqrt{1 - \frac{\lambda^2}{\lambda^2_*}} \quad \lambda_* = 2n \sqrt{\gamma} \]

This is all very nice and beautiful.
Evaluated carefully, the higher charges have infinite expectation values!

J-SC and J. Mossel, unpublished
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Explicit calculations with few particles:
Overlaps fall off at large rapidity as $c_\lambda \sim 1/\lambda^2$

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Therefore, we have $\langle Q_2 \rangle \sim \sum_\lambda |c_\lambda|^2 \lambda^2 \sim \text{finite}$
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Therefore, we have \(\langle Q_2 \rangle \sim \sum_\lambda |c_\lambda|^2 \lambda^2 \sim \text{finite}\)

but \(\langle Q_4 \rangle \sim \sum_\lambda |c_\lambda|^2 \lambda^4 \sim \sum_\lambda 1 \sim \delta(x = 0) \rightarrow \infty\)

and even worse divergences for higher charges
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These divergences are ‘Fourier’ infinities, and are in no way resolved by infinite size/thermodynamic limit
GGE approach to BEC-LL quench (take 2)

J-SC, unpublished

Idea: if the charges are the problem, use other charges!

Elementary symmetric polynomials of rapidities:

$$\hat{J}_n : \hat{J}_n(\{\lambda\}_N) = J_n(\{\lambda\}_N)$$

$$J_n(\{\lambda\}_N) = \left\{ \begin{array}{ll}
\sum_{1 \leq j_1 < j_2 < \ldots < j_n \leq N} \lambda_{j_1} \lambda_{j_2} \ldots \lambda_{j_n}, & 1 \leq n \leq N \\
0, & n > N.
\end{array} \right.$$
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0, & n > N.
\end{cases}
\]

Nice property: bounded on any finite-energy eigenstate

\[
|J_n(\{\lambda\}_N)| \leq \sum_{1 \leq j_1 < j_2 < \ldots < j_n \leq N} |\lambda_{j_1} \lambda_{j_2} \ldots \lambda_{j_n}| \leq \sum_{1 \leq j_1 < j_2 < \ldots < j_{n-1} \leq N} |\lambda_{j_1} \lambda_{j_2} \ldots \lambda_{j_{n-1}}| \times \sum_{j_n=1}^{N} |\lambda_{j_n}| \leq \left( \sum_{j=1}^{N} |\lambda_j| \right)^n \equiv \hat{J}_1^n
\]
GGE approach to BEC-LL quench (take 2)

Seem to work just fine: just evaluate 2nd quantized form

\[ \hat{J}_{2n} = (\text{deriv}) + \frac{(-1)^n c^n}{2^{n} n!} \int dy_1...dy_n (\psi^\dagger(y_1))^2... (\psi^\dagger(y_n))^2 \psi^2(y_n)...\psi^2(y_1) \]
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on BEC state \[ |0_N\rangle \equiv \frac{1}{\sqrt{L^N N!}} \left( \psi_{k=0}^\dagger \right)^N |0\rangle \]

yielding the conditions

\[ \langle 0_N | J_{2n} | 0_N \rangle = \frac{(-1)^n c^n}{2^n n! L^n} \frac{N!}{(N - 2n)!} \]
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Careful calculation (need all finite-size effects):
GGE approach to BEC-LL quench (take 2)

Seem to work just fine: just evaluate 2nd quantized form

\[ \hat{J}_{2n} = (\text{deriv}) + \frac{(-1)^n e^n}{2^n n!} \int dy_1 \ldots dy_n (\psi(\psi^+)(y_1))^2 \ldots (\psi^+(y_n))^2 \psi^2(y_n) \ldots \psi^2(y_1) \]

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yielding the conditions \[ \langle 0_N | \hat{J}_{2n} | 0_N \rangle = \frac{(-1)^n e^n}{2^n n!} \frac{N!}{L^n (N - 2n)!} \]

Careful calculation (need all finite-size effects):

Gives back the Q-GGE semicircle law?!?!?!

What is going on???
GGE approach to BEC-LL quench (take 2)

Fundamental problem:
this is not a 'complete' set of charges

Relation between $Q$'s and $J$'s: Girard-Newton formula

$$J_n = \frac{1}{n} \sum_{j=1}^{n} (-1)^{j-1} J_{n-j} Q_j$$

or

$$J_n = \frac{1}{n!}$$

<table>
<thead>
<tr>
<th>$Q_1$</th>
<th>$Q_2$</th>
<th>$Q_3$</th>
<th>$Q_4$</th>
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Relation between Q's and J's: Girard-Newton formula

\[ J_n = \frac{1}{n} \sum_{j=1}^{n} (-1)^{j-1} J_{n-j} Q_j \]  \hspace{1cm} \text{or} \hspace{1cm} J_n = \frac{1}{n!} \left[ \begin{array}{cccccc}
Q_1 & 1 & 0 & 0 & \cdots & 0 \\
Q_2 & Q_1 & 2 & 0 & \cdots & 0 \\
Q_3 & Q_2 & Q_1 & 3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
Q_{n-1} & Q_{n-2} & \cdots & Q_1 & n-1 & 0 \\
Q_n & Q_{n-1} & \cdots & Q_2 & 1 & 0 \\
\end{array} \right] 

For example,  \[ J_4 = \frac{1}{8} Q_2^2 - \frac{1}{4} Q_4 \]
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& & & & & & \\
& & & & Q_{n-1} & Q_{n-2} & \cdots & Q_1 & n-1
\end{array} \right]
\]

For example,

\[
J_4 = \frac{1}{8} Q_2^2 - \frac{1}{4} Q_4
\]

Complete set: free algebra of J’s

Problem: moments of J’s have infinite initial evaulves!

M. Rigol: GGE OK only if widths vanish!
GGE approach to BEC-LL quench (take 3)


Idea: if the model poses problems, change the model

Lattice version:

\[ B_j B_j^\dagger - q^{-2} B_j^\dagger B_j = 1, \quad q > 1 \]

\[ [N_j, B_j] = -B_j \quad [N_j, B_j^\dagger] = B_j^\dagger \]

\[ H_q = -\frac{1}{\delta^2} \sum_{j=1}^{M} (B_j^\dagger B_{j+1} + B_{j+1}^\dagger B_j - 2N_j) \]
GGE approach to BEC-LL quench (take 3)


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q-bosons

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Continuum limit takes one back to Lieb-Liniger

\( \delta \to 0, \quad M \to \infty, \quad q \to 1 \quad L = M \delta, \quad c = 2\kappa \delta^{-1} \)
Conserved (local) charges built from trace identities:

\[ I_m = \delta \sum_{j=1}^{M} J_j^{(m)} \]

\[ J^{(1)}(n) = \frac{1}{\delta} \chi^2 B_j^\dagger B_{j+1} \]

\[ \chi = \sqrt{1 - q^{-2}} \]
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\[ J^{(2)}(n) = \frac{1}{\delta} \chi^2 \left( 1 - \frac{\chi^2}{2} \right) \left( B_j B_{j+2} - \frac{\chi^2}{2} B_j^1 B_{j+1} B_{j+1} - \chi^2 B_j^1 B_{j+1} B_{j+2} \right) \]

Truncated GGE with a few charges: very complicated calculation (Kormos, Shashi, Chou) giving approximation

\[ \rho_{LL}^{(1)}(\lambda) = \frac{1}{2\pi} \frac{\chi^2}{(\lambda/n)^4 + \gamma(\gamma/4 - 2)(\lambda/n)^2 + \gamma^2} \]
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At large rapidity, \( 2\pi \rho(\lambda) = \frac{n^4 \gamma^2}{\lambda^4} - \frac{n^6 \gamma^3 (\gamma - 24)}{4 \lambda^6} + \ldots \)
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Recovers the expected tails (giving divergences)
The ‘quench action’ approach

in pictures...

Initial state:
The ‘quench action’ approach

in pictures...

\[ \mathcal{H}_0 \quad \text{initial state:} \quad \mathcal{H} \]

in pre-quench Hilbert space basis

in post-quench Hilbert space basis

J-SC & F.H.L. Essler, PRL 2013
The ‘quench action’ approach

J-SC & F.H.L. Essler, PRL 2013

$S_Q[\rho]$
The ‘quench action’ approach

Now in equations...

Consider a generic integrable model, with
eigenstates labeled by quantum numbers \( \{I\} \)
The ‘quench action’ approach

*now in equations...*

Consider a generic integrable model, with eigenstates labeled by quantum numbers \( \{I\} \)

Resolution of identity: \[ 1 = \sum_{\{I\}} \langle \{I\} | \{I\} \rangle \]

Arbitrary initial state can be decomposed in this basis:

\[ |\Psi(t = 0)\rangle = \sum_{\{I\}} e^{-S^\Psi_{\{I\}}} |\{I\}\rangle \]

using overlap coefficients \[ S^\Psi_{\{I\}} = -\ln \langle \{I\} | \Psi(t = 0) \rangle \in \mathbb{C} \]
Time dependence: trivially written as

$$|\Psi(t)\rangle = \sum_{\{I\}} e^{-S^\varphi_{\{I\}t} - i\omega_{\{I\}}t} |\{I\}\rangle$$

The expectation values we're interested in then become

$$\mathcal{O}(t) = \sum_{\{I^\prime\}} \sum_{\{I^\prime\}} e^{-(S^\varphi_{\{I^\prime\}t})^* - S^\varphi_{\{I^\prime\}t} + i(\omega_{\{I^\prime\}} - \omega_{\{I\}})t} \langle\{I^\prime\}|O|\{I\}\rangle \overline{|\{I\}|} e^{-2\Im S^\varphi_{\{I\}}}$$
Start by looking at wavefunction normalization:

\[ \langle \Psi(t) | \Psi(t) \rangle = \sum_{\{I\}} e^{-2\Re S_{\{I\}}^\psi} \]

In Th.Lim., would like to use the usual functional integral

\[ \lim_{Th} \sum_{\{I\}} (...) = \int D\rho \; e^{S_{\{Y\}\psi}[\rho]} (...) \]
Start by looking at wavefunction normalization:

$$\langle \Psi(t) | \Psi(t) \rangle = \sum_{\{I\}} e^{-2\mathcal{R}_i^\Psi}$$

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$$\lim_{Th} \sum_{\{I\}} (\ldots) = \int D\rho \ e^{S_{\Psi \Psi}[\rho]} (\ldots)$$

Including the effective overlaps yields

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Start by looking at wavefunction normalization:

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Including the effective overlaps yields

$$\lim_{Th} \langle \Psi(t)|\Psi(t) \rangle = \int D\rho \ e^{-S^Q[\rho]}$$

with ‘quench action’

$$S^Q[\rho] = S^o[\rho] - S^{Y Y}[\rho]$$

Saddle-point evaluation: $\rho_{sp}$ such that

$$\left. \frac{\delta S^Q[\rho]}{\delta \rho} \right|_{\rho_{sp}} = 0$$
For operator expectation values, we had

\[ \hat{O}(t) = \sum_{\{I\}} \sum_{\{I'\}} e^{-(S_{\{I\}}^\phi)^* - S_{\{I'\}}^\phi + i(\omega_{\{I\}} - \omega_{\{I'\}})t} \langle \{I\} | O | \{I'\} \rangle \sum_{\{I\}} e^{-2RS_{\{I\}}^\phi} \]

Considering operators which are ‘weak’ (creating a non-entropically large nr of excitations when acting on a given state): thermodyln limit is
For operator expectation values, we had

\[ \tilde{O}(t) = \sum_{\{I^t\}} \sum_{\{I^r\}} e^{-(S_y^{(t)} - S_y^{(r)}) + i(\omega^{(t)} - \omega^{(r)}) t} \langle \{I^t\} | O | \{I^r\} \rangle \]

\[ \sum_{\{I\}} e^{-2RS_y^{(t)}} \]

Considering operators which are 'weak' (creating a non-entropically large nr of excitations when acting on a given state): thermodynamic limit is

\[ \lim_{T \to \infty} \tilde{O}(t) = \frac{\int D\rho e^{-S_Q[\rho]} \lim_{T \to \infty} \sum_{\{\rho\}} e^{-\delta S_{(\rho)}[\rho] - i\omega_{(\rho)}[\rho]} t \langle \rho | O | \rho \rangle \{\rho\} \{\} \rangle}{\int D\rho e^{-S_Q[\rho]}} \]
For operator expectation values, we had

\[ \hat{O}(t) = \frac{\sum_{\{I^l\}} \sum_{\{I^r\}} e^{-\left(S_{(I^l)}^0 - S_{(I^r)}^0 + i(\omega_{(I^l)} - \omega_{(I^r)})t\right)} \langle \{I^l\} | O | \{I^r\} \rangle}{\sum_{\{I\}} e^{-2RS_{(I)}^0}} \]

Considering operators which are ‘weak’ (creating a non-entropically large nr of excitations when acting on a given state); thermodynamic limit is

\[ \lim_{T \to \infty} \hat{O}(t) = \frac{\int D\rho e^{-S^Q[\rho]} \lim_{T \to \infty} \sum_{\{e\}} e^{-\delta S(e)[\rho] - i\omega(e)[\rho]t} \langle \rho | O | \rho; \{e\} \rangle}{\int D\rho e^{-S^Q[\rho]}} \]

denumerable set of excitations
For operator expectation values, we had

\[ \tilde{\mathcal{O}}(t) = \frac{\sum_{\{I\}} \sum_{\{I'\}} e^{-(S^\Phi_{\{I\}})^* - S^\Phi_{\{I'\}}} + i(\omega_{\{I\}} - \omega_{\{I'\}})t} {\sum_{\{I\}} e^{-2RS^\Phi_{\{I\}}}} \langle \{I\}\{I'\}\rangle \langle O | O \rangle \]

Considering operators which are ‘weak’ (creating a non-entropically large nr of excitations when acting on a given state): thermodynamic limit is

\[
\lim_{T \to 0} \tilde{\mathcal{O}}(t) = \frac{\int \mathcal{D}p e^{-S^\Phi[p]} \lim_{T \to 0} \sum_\{\{e\}\} e^{-\delta S_{\{e\}}[p] - i\omega_{\{e\}}[p]t} \langle O | O(p; \{e\}) \rangle} {\int \mathcal{D}p e^{-S^\Phi[p]}}
\]

relative overlaps  
denumerable set of excitations
For operators with non-entropically large matrix elements, can perform a saddle-point evaluation (same saddle-point in numerator and denominator)

\[
\lim_{T \to 0} \bar{O}(t) = \lim_{T \to 0} \frac{1}{2} \sum_{\{e\}} \left[ e^{\frac{i}{\hbar} \delta S_{\{e\}}[\rho_{sp}]} \left( \frac{\rho_{sp}}{\rho_{sp}} \right)^t \left( \rho_{sp} \right) \left( \rho_{sp} \right)^t \right] 
\]

\[
+ e^{\frac{i}{\hbar} \delta S_{\{e\}}[\rho_{sp}]} \left( \frac{\rho_{sp}}{\rho_{sp}} \right)^t \left( \rho_{sp} \right) \left( \rho_{sp} \right)^t \]
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\[
\lim_{T \to 0} \bar{O}(t) = \lim_{T \to 0} \frac{1}{2} \sum_{\{e\}} \left[ e^{-\delta S_{\{e\}}[\rho_{sp}]} \left( \rho_{sp}|O|\rho_{sp}; \{e\} \right) + e^{-\delta S_{\{e\}}[\rho_{sp}]} \left( \rho_{sp}|O|\rho_{sp} \right) \right]
\]
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\]

Main message: the *full* time dependence is recoverable using a minimal amount of data
For operators with non-entropically large matrix elements, can perform a saddle-point evaluation (same saddle-point in numerator and denominator)

\[
\lim_{T \to 0} \bar{O}(t) = \lim_{T \to 0} \frac{1}{2} \sum_{\{\sigma\}} \left[ e^{-\delta S_{\{\sigma\}}[\rho_{sp}]}e^{-i\omega_{\{\sigma\}}[\rho_{sp}]t} \langle \rho_{sp} | \mathcal{O} | \rho_{sp} \rangle \{ \sigma \} \right]
\]

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- saddle-point distribution (from GTBA)
- excitations in vicinity of sp state (easy)
For operators with non-entropically large matrix elements, can perform a saddle-point evaluation (same saddle-point in numerator and denominator)

\[
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\]

Main message: the *full* time dependence is recoverable using a minimal amount of data

- saddle-point distribution (from GTBA)
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- differential overlaps
For operators with non-entropically large matrix elements, can perform a saddle-point evaluation (same saddle-point in numerator and denominator)

\[
\lim_{T \to 0} \tilde{O}(t) = \lim_{T \to 0} \frac{1}{2} \sum_{\{e\}} \left[ e^{-\delta S_{\{e\}}}[\rho_{s_p} - i \omega_{\{e\}}][\rho_{s_p}] t \langle \rho_{s_p} | O | \rho_{s_p}; \{e\} \rangle \\
+ e^{-\delta S_{\{e\}}}[\rho_{s_p} + i \omega_{\{e\}}][\rho_{s_p}] t \langle \rho_{s_p}; \{e\} | O | \rho_{s_p} \rangle \right]
\]

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For operators with non-entropically large matrix elements, can perform a saddle-point evaluation (same saddle-point in numerator and denominator)

\[
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\]

Main message: the *full* time dependence is recoverable using a minimal amount of data

- saddle-point distribution (from GTBA)
- excitations in vicinity of sp state (easy)
- differential overlaps
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For operators with non-entropically large matrix elements, can perform a saddle-point evaluation (same saddle-point in numerator and denominator)

\[
\lim_{T_h} \tilde{O}(t) = \lim_{T_h} \frac{1}{2} \sum_{\{e\}} \left[ e^{-\delta S(e)} |\rho_{sp}\rangle - i\omega(e) |\rho_{sp}\rangle t \langle \rho_{sp}| \mathcal{O} |\rho_{sp}\rangle \{e\} \right] \\
+ e^{-\delta S(e)} |\rho_{sp}\rangle + i\omega(e) |\rho_{sp}\rangle t \langle \rho_{sp}| \{e\} \mathcal{O} |\rho_{sp}\rangle
\]

Main message: the *full* time dependence is recoverable using a minimal amount of data

Large time limit: \[
\lim_{t\to\infty} \lim_{T_h} \tilde{O}(t) = \lim_{T_h} \langle \rho_{sp}| \mathcal{O} |\rho_{sp}\rangle
\]
Back to BEC-LL quench

Problem: need to calculate overlaps.

Remark: only parity-invariant states contribute since

\[ 0 = \langle 0 | \hat{Q}_{2m+1} | I \rangle = \langle 0 | I \rangle \sum_{j=1}^{N} \lambda_j^{2m+1} \]
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Known large \( c \) limit:  

Gritsev, Rostunov & Demler, JSTAT 2010

\[ \langle \{ \lambda_j \}_{j=1}^{N/2}, \{-\lambda_j\}_{j=1}^{N/2} | 0 \rangle \propto \prod_{\lambda_j > 0} \frac{1}{\lambda_j} \]
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Back to BEC-LL quench

M. Brockmann JPA 2014

\[
\langle \{\lambda_j\}_{j=1}^{N/2}, \{-\lambda_j\}_{j=1}^{N/2}\rangle = \sqrt{\frac{(cL)^{-N} N!}{\det_{j,k=1}^N G_{jk}}} \frac{\det_{j,k=1}^{N/2} G_{jk}^Q}{\prod_{j=1}^{N/2} \frac{\lambda_j}{c} \sqrt{\frac{\lambda_j^2}{c^2} + \frac{1}{4}}}
\]

(reminiscent of Gaudin formula)

with matrix 
\[
G_{jk}^Q = \delta_{jk} \left( L + \sum_{l=1}^{N/2} K^Q(l_j, l_l) \right) - K^Q(l_j, l_k)
\]

\[
K^Q(\lambda, \mu) = K(\lambda - \mu) + K(\lambda + \mu)
\]

\[
K(\lambda) = \frac{2c}{\lambda^2 + c^2}
\]
Quench action solution to BEC-LL quench


It is in fact possible to give a closed form solution of the GTBA for the saddle-point state, for any value of the interaction:
Quench action solution to BEC-LL quench


It is in fact possible to give a closed form solution of the GTBA for the saddle-point state, for any value of the interaction:

\[
\rho(\lambda) = -\frac{\gamma}{2\pi} \frac{\partial a(\lambda)}{\partial \gamma} (1 + a(\lambda))^{-1}
\]

\[
a(\lambda) = \frac{2\pi/\gamma}{\Lambda_c \sinh \left( \frac{2\pi \Lambda_c}{c} \right)} I_{1-2i\Lambda_c/c} \left( \frac{4}{\sqrt{\gamma}} \right) I_{1+2i\Lambda_c/c} \left( \frac{4}{\sqrt{\gamma}} \right)
\]
Quench action solution to BEC-LL quench

Quench action solution to BEC-LL quench


Subplot: scaled fn

\[ \rho_s(x) = \sqrt{\gamma} \rho(c\sqrt{\gamma}/2) \]

Large c:

\[ \rho(\lambda) = \frac{1}{2\pi} \frac{4n^2}{\lambda^2 + 4n^2} \]

Small c: semicircle

\[ \rho(\lambda) \sim \frac{1}{\pi \sqrt{\gamma}} \sqrt{1 - \frac{\lambda^2}{4\gamma n^2}} \]

Asymptotics as from q-bosons:

\[ 2\pi \rho(\lambda) \sim \frac{n^4 \gamma^2}{\lambda^4} + \frac{n^6 \gamma^3 (24 - \gamma)}{4\lambda^6} + \ldots \]
Quench action solution to BEC-LL quench


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Tail explains divergences of evals of conserved charges
Quench action solution to BEC-LL quench


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Tail explains divergences of evalus of conserved charges
Relaxation & equilibration

Time evolution driven by ‘excitations’ around saddle-point of quench action.

In the impenetrable limit, reproduces known result

Kormos, Collura & Calabrese, PRA 89, 2014

\[
\langle \hat{\rho}(x)\hat{\rho}(0) \rangle_t - \langle \hat{\rho}(x)\hat{\rho}(0) \rangle_{sp} = \left[ \int_{-\infty}^{\infty} \frac{dk}{\pi} \frac{nk}{4n^2 + k^2} e^{-2itk^2 + ikx} \right]^2
\]

where

\[
\langle \rho_{sp} | \hat{\rho}(x)\hat{\rho}(0) | \rho_{sp} \rangle = n\delta(x) + n^2(1 - e^{-4n|x|})
\]

Relaxation at finite c: ongoing work...
Thinking back on the (Q or J)-GGE

The solution of the BEC-LL quench is very instructive:

For Lieb-Liniger, we had charges $Q_n(\{\lambda\}_N) = \sum_{j=1}^{N} \lambda_j^n$

The GGE ‘free energy’ is

$$\sum_n \beta_n \hat{Q}_n = \sum_n \beta_n \sum_j \lambda_j^n = \sum_n \beta_n \int d\lambda \rho_{Q\text{-GGE}}(\lambda) \lambda^n$$

From the exact overlaps, the quench action is however

$$S^Q[\rho] = \frac{L}{2} \int_0^\infty d\lambda \rho(\lambda) \log \left[ \frac{\lambda^2}{c^2} \left( \frac{\lambda^2}{c^2} + \frac{1}{4} \right) \right] - S^{YY}[\rho]$$
Néel to XXZ quench
Quench from Néel to XXZ

Start from Néel state:
Quench from Néel to XXZ

Start from Néel state:

From $t=0$ onwards, evolve with XXZ Hamiltonian
Quench from Néel to XXZ

Start from Néel state:

From \( t=0 \) onwards, evolve with XXZ Hamiltonian

Positions of downturned spins start fluctuating wildly

Can one treat this problem exactly?
“Particle content” of XXZ: nontrivial

Solution of Bethe equations: rapidities + strings

\[ \lambda^{j,a}_\alpha = \lambda^{j}_\alpha + \frac{\zeta}{2} (n_j + 1 - 2a) + i\delta^{j,a}_\alpha \quad O(e^{-(cst)N}) \]
Quench action approach to Néel-XXZ quench
First step: exact overlaps of Néel state with XXZ eigenstates

(Gaudin-like form again!) M. Brockmann, J. De Nardis, B. Wouters & J-SC JPA 2014

\[
\frac{(\Psi_0|\{\pm \lambda_j\}_{j=1}^{M/2})}{\|\{\pm \lambda_j\}_{j=1}^{M/2}\|} = \sqrt{2} \left[ \prod_{j=1}^{M/2} \frac{\sqrt{\tan(\lambda_j + i\eta/2) \tan(\lambda_j - i\eta/2)}}{2 \sin(2\lambda_j)} \right] \frac{\det_{M/2}(G_{jk}^+)}{\det_{M/2}(G_{jk}^-)}
\]

\[
G_{jk}^\pm = \delta_{jk} \left( NK_{\eta/2}(\lambda_j) - \sum_{l=1}^{M/2} K_{\eta}^+(\lambda_j, \lambda_l) \right) + K_{\eta}^\pm(\lambda_j, \lambda_k)
\]

\[
K_{\eta}^\pm(\lambda, \mu) = K_{\eta}(\lambda - \mu) \pm K_{\eta}(\lambda + \mu)
\]

\[
K_{\eta}(\lambda) = \frac{\sinh(2\eta)}{\sin(\lambda + i\eta) \sin(\lambda - i\eta)}
\]
Quench action approach to Néel-XXZ quench

Second step: generalized TBA

\[
\ln \eta_n(\lambda) = -2h_n - \ln W_n(\lambda) + \sum_{m=1}^{\infty} a_{nm} \ast \ln \left(1 + \eta_m^{-1}\right)(\lambda)
\]

where \(\eta_n(\lambda) \equiv \rho_{n,h}(\lambda)/\rho_n(\lambda)\)

\[a_n(\lambda) = \frac{\sin n\eta}{\pi \cosh n\eta - \cos 2\lambda}\]

and the effective driving terms (pseudo-energies) are

\[
W_n(\lambda) = \begin{cases} 
\prod_{j=1}^{n-1} \frac{\cosh (2j-1)\eta - \cos 2\lambda}{\cosh (2j-1)\eta + \cos 2\lambda} & \text{if } n \text{ odd,} \\
\prod_{j=1}^{n-1} \frac{\cosh 2j\eta + \cos 2\lambda}{\cosh 2j\eta - \cos 2\lambda} & \text{if } n \text{ even.}
\end{cases}
\]
Quench action approach to Néel-XXZ quench

Equivalent form of generalized TBA:

\[
\ln(\eta_n) = d_n + s \star \left[ \ln(1 + \eta_{n-1}) + \ln(1 + \eta_{n+1}) \right]
\]

with driving terms

\[
d_n(\lambda) = \sum_{k \in \mathbb{Z}} e^{-2ik\lambda} \frac{\tanh(\eta k)}{k} (-1)^n - (-1)^k
\]
Quench action approach to Néel-XXZ quench

Equivalent form of generalized TBA:

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with driving terms

\[ d_n(\lambda) = \sum_{k \in \mathbb{Z}} e^{-2ik\lambda} \frac{\tanh(\eta k)}{k} \left( (-1)^{n} - (-1)^{k} \right) \]

GGE with local charges: same form of coupled equations, but driving term only for n=1:

\[ d_1(\lambda) = -\frac{1}{\pi} \sum_{m=1}^{\infty} \beta_{2m} \sum_{k \in \mathbb{Z}} e^{-2ik\lambda} \frac{k^{2m-2}}{\cosh k\eta} \]
Néel-XXZ quench: conserved charges

Initial expectation value of local charges:

\[ \lim_{N \to \infty} \frac{1}{N} \langle \text{Néel} | Q_{n+1} | \text{Néel} \rangle = -\frac{\Delta}{2} \frac{\partial^{n-1}}{\partial x^{n-1}} \frac{1 - \Delta^2}{\cosh[\sqrt{1 - \Delta^2 x}] - \Delta^2} \bigg|_{x=0} \]

In 1-to-1 correspondence with 1-string hole density:

\[ \sum_{k \in \mathbb{Z}} k^{2m-2} \left( \frac{e^{-|k|\eta} - \tilde{\rho}^t_{1}(k)}{2 \cosh k\eta} \right) = \langle Q_{2m} \rangle \quad m \in \mathbb{N} \]
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In 1-to-1 correspondence with 1-string hole density:

\[ \sum_{k \in \mathbb{Z}} k^{2m-2} \left( \frac{e^{-|k|\eta} - \tilde{\rho}_1^h(k)}{2 \cosh k\eta} \right) = \langle Q_2^m \rangle \quad m \in \mathbb{N} \]

which fixes

\[ \tilde{\rho}_1^N = \frac{\pi^2 a_1^3(\lambda) \sin^2(2\lambda)}{\pi^2 a_1^5(\lambda) \sin^2(2\lambda) + \cosh^2(\eta)} \]

Quench action nontrivially reproduces this; GGE also of course, but only by definition
The steady state: Néel to XXZ

Solid lines: quench action
Dashed lines: GGE (local charge)
The steady state: Néel to XXZ

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The steady state: Néel to XXZ

Solid lines: quench action
Dashed lines: GGE (local charge)
QA and GGE have different saddle-point densities

Large Delta expansion:
\[ \rho_1^{GGE} - \rho_1^{sp} = \frac{1}{4\pi\Delta^2} + O(\Delta^{-3}), \]
\[ \rho_2^{GGE} - \rho_2^{sp} = \frac{1 - 3\sin^2(\Delta)}{3\pi\Delta^2} + O(\Delta^{-3}). \]
The steady state: Néel to XXX
Difference in distribution: impact on correlations
Difference in distribution:
impact on correlations

Large Delta expansions:

\[
\langle \sigma_1^x \sigma_2^x \rangle_{QA} = -1 + \frac{2}{\Delta^2} - \frac{7}{2 \Delta^4} + \frac{77}{16 \Delta^6} + \ldots
\]

\[
\langle \sigma_1^x \sigma_2^x \rangle_{GGE} = -1 + \frac{2}{\Delta^2} - \frac{7}{2 \Delta^4} + \frac{43}{8 \Delta^6} + \ldots
\]
Not convinced?

Look at other results by Budapest group

B. Pozsgay, M. Mestyán, M. A. Werner, M. Kormos, G. Zaránd, G. Takács, arxiv 1405.2843

- reobtain our Néel results
- also consider initial dimer state
- obtain numerical (iTEBD) evidence for correlations being different in dimer case
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There remains little doubt about the correctness of the quench action results
What's going on?

Néel to XXZ: current situation

- quench action solution gives correct expectation value for all conserved charges, directly from microscopics
- quench action and (local) GGE steady state distributions do not coincide
- these different distributions lead to different observable expectation values

Possible explanations of this mismatch:
What’s going on?

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Possible explanations of this mismatch:
- GGE converges to QA once all (nonlocal) charges are added
- exceptional states invalidate the QA calculation ruled out
- of which there are exponentially many more than local ones!
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