Title: Universal dynamics and topological order in many-body localized states

Date: May 15, 2014  10:30 AM

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Abstract: It has been argued recently that, through a phenomenon of many-body localization, closed quantum systems subject to sufficiently strong disorder would fail to thermalize. In this talk I will describe a real time renormalization group approach, which offers a controlled description of universal dynamics in the localized phase. In particular it explains the ultra-slow entanglement propagation in this state and identifies the emergent conserved quantities which prevent thermalization. The RG analysis also shows, that far from being a trivial dead state, the MBL state admits phase transitions between distinct dynamical phases. For example, I will discuss the universal aspects of a transition between a paramagnetic localized state to one which exhibits spin-glass order. Finally, I will present a development of the RG scheme, defined on an effective coarse grained model, which allows to capture the transition from a many-body localized to a thermalizing state.
What are dynamical phase transitions? Do they exist?

Random transverse field Ising model + generic interactions:

\[ H = \sum_i [J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots] \]

Distinct quantum phases and universal critical behavior in ground state
(Infinite randomness fixed point separating Paramagnet and Ferromagnet)

Contrast this to:

Unitary evolution from an arbitrary initial state:

\[ e^{-iHt \mid \Psi_0 \rangle} \]

\[ \downarrow \uparrow \downarrow \downarrow \uparrow \downarrow \downarrow \uparrow \downarrow \uparrow \downarrow \downarrow \uparrow \downarrow \downarrow \downarrow \uparrow \downarrow \downarrow \uparrow \]

Involves all energies!

- Is there universality associated with the long time behavior?
- Transitions between different dynamical states with singular effect on the behavior of observables?
What are dynamical phase transitions? do they exist?

Random transverse field Ising model + generic interactions:

\[ H = \sum_i \left[ J_i^x \sigma_i^x \sigma_{i+1}^x + h \sigma_i^z + J_i^z \sigma_i^z \sigma_{i+1}^z + \ldots \right] \]

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\[ \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \uparrow \downarrow \]

Involves all energies!
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- Transitions between different dynamical states with singular effect on the behavior of observables?
What are dynamical phase transitions? Do they exist?

\[ H = \sum_i \left[ J_i^x \sigma_i^x \sigma_{i+1}^x + h_i \sigma_i^z + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right] e^{-iHt} |\Psi_0\rangle \]

Naïve answer: NO!
- System thermalizes at long times.
- Any singularities are just thermal phase transitions.
- No singularities in 1d

I will argue: YES!
- Thermalization prevented by strong disorder (Many body localization)
- Allows for distinct quantum dynamical phases and phase transitions
- Universal singularities in dynamics possible in 1d at finite energy density
Eigenstate thermalization hypothesis (ETH)

Deutsch 91, Srednicki 94

In a high energy eigenstate:

$$\rho_A = \frac{1}{Z_A} e^{-\beta H_A}$$

Extensive Von-Neuman entropy:

$$S_A \propto L^d$$
Outline

- ETH and MBL – brief intro

- RG theory for MBL states
  Dynamical quantum phase transition between MBL states

- RG theory for the MBL transition
  Surprising insight on the delocalization transition and the delocalized state
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Extensive Von-Neuman entropy:

$S_A \propto L^d$

Example where ETH fails:
Anderson localization

“Area law” entropy as in ground state also holds in high energy eigenstates

$S_A \propto L^{d-1}$
Thermalization Following a Quench

\[ \rho(0) = \rho_A \otimes \rho_B \]

\[ \rho(t_1) \]

\[ \rho(t_2) \]

Growing entanglement between the two halves is measured by the Von-Neuman entropy:

\[ S_A(t) = -\text{Tr}[\rho_A(t)\ln \rho_A(t)] \]

Ergodic system

\[ S_A = S_{\text{equilibrium}} \sim L \]

Localized system (?)

\[ S_A \sim \xi_{\text{localization}} \]
Generic exception to ETH: Many body localization

Anderson localization of non interacting particles:

Many body localization
(Basko et. al. 2006, Gomyi et. al. 2005)

\[
\text{Disorder strength} \quad \begin{cases} 
\text{Localized} \\
\text{Delocalized} \\
\text{Thermalizing}
\end{cases}
\]

\[
\begin{pmatrix}
\chi = 0 \\
\sigma = 0
\end{pmatrix}
\]

\[
\text{Non thermalizing}
\]
Dynamical Transitions between distinct localized states

RV and Altman, PRL (2013); RV and Altman, arXiv:1307.3256

Example: random transverse field Ising model ($\mathbb{Z}_2$ symmetry)

\[ H = \sum_i [J_i^x \sigma_i^x \sigma_{i+1}^x + h_i \sigma_i^z + J_i^z \sigma_i^z \sigma_{i+1}^z + \ldots] \]

"interaction"

Unitary evolution from a generic un-entangled initial state:

\[ e^{-iHt} | \Psi_0 \rangle \]

We study
1. Decay of local moments $\langle \sigma_i^z(t) \rangle$
2. Evolution of entanglement entropy

\[ S_A(t) = -Tr[\rho_A(t) \ln \rho_A(t)] \]
RG scheme for time evolution

\[ H = \sum \left( J_i^z \sigma^z_i \sigma^z_{i+1} + h_i \sigma^x_i + J_i^x \sigma^x_i \sigma^x_{i+1} + \ldots \right) \]

1. Select the fast degrees of freedom and solve for their evolution
2. Treat the slow degrees of freedom using time dependent perturbation theory
3. Average over the fast time-scale and obtain effective Hamiltonian for the slow degrees of freedom

Large field:

\[ J_L \quad J_R \]

\[ h_i = \Omega \]

\[ J_L J_R / \Omega \]
RG scheme for time evolution

\[ H = \sum_i \left[ J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots \right] \]

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Large field:

\[ J_L, J_R \]

\[ h_i = \Omega \]

\[ \rightarrow \]

\[ J_L J_R / \Omega \]
RG scheme for *time* evolution

\[ H = \sum_i \left[ J^z_i \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^x + J^x_i \sigma_i^x \sigma_{i+1}^x + \ldots \right] \]

1. Select the fast degrees of freedom and solve for their evolution
2. Treat the slow degrees of freedom using time dependent perturbation theory
3. Average over the fast time-scale and obtain effective Hamiltonian for the slow degrees of freedom

Large field:

\[ J_L \uparrow \uparrow J_R \downarrow \downarrow \quad \Rightarrow \quad \downarrow \downarrow \]

\[ h_i = \Omega \]

Large bond:

\[ J_L \uparrow \Omega \downarrow J_R \downarrow \quad \Rightarrow \quad J_L \uparrow \uparrow J_R \downarrow \downarrow \]

\[ h_L h_R / \Omega \]
Results from the RG Flow

\[ H = \sum_i \left[ J_i^x \sigma_i^x \sigma_{i+1}^x + h_i \sigma_i^z + J_i^z \sigma_i^z \sigma_{i+1}^z + \ldots \right] e^{-iHt} | \Psi_0 \rangle \]

Qualitative possible dynamical phases:

\[ J^z > h \]

“Glass”
Results from the RG Flow

\[ H = \sum_1 \left[ J^z_i \sigma^z_i \sigma^z_{i+1} + h_i \sigma^z_i + J^x_i \sigma^x_i \sigma^x_{i+1} + \ldots \right] e^{-iHt} | \Psi_0 \rangle \]

Qualitative possible dynamical phases:

- \( J^z > h \) "Glass"
- \( h > J^z \) "Paramagnet"
Results from the RG Flow

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\[ \langle \ln \left( \frac{1}{n^z} \right) \rangle \]

\[ \langle \ln \left( \frac{1}{J^{\tau \tau}} \right) \rangle \]

\( \hbar_{\text{typ}} \to \text{const} \)
\( J^z_{\text{typ}} \to 0 \)
\( 2\Delta \)
Results from the RG Flow

\[ H = \sum \left[ J_i^x \sigma_i^x \sigma_{i+1}^x + h_i \sigma_i^z + J_i^z \sigma_i^z \sigma_{i+1}^z + \ldots \right] e^{-iHt} | \Psi_0 \rangle \]

Qualitative possible dynamical phases:

- \( J^z > h \) “Glass”
- \( h > J^z \) “Paramagnet”

At criticality:
Infinite randomness scaling

\[
\langle \ln \left( \frac{1}{\mathcal{R}} \right) \rangle^2 \quad \text{or} \quad \langle \ln \left( \frac{1}{\mathcal{R}} \right) \rangle^2
\]

“Paramagnet”

\[
\begin{align*}
\hat{h}_{\text{typ}} &\rightarrow \text{const} \\
\hat{J}_\text{typ}^z &\rightarrow 0 \\
\hat{J}_\text{typ}^x &\rightarrow \text{const} \\
\hat{h}_{\text{typ}} &\rightarrow 0
\end{align*}
\]
Results from the RG Flow

\[ H = \sum [J_i^z \sigma_i^z \sigma_{i+1}^z + h_i \sigma_i^z + J_i^x \sigma_i^x \sigma_{i+1}^x + \ldots] e^{-iHt} | \Psi_0 \rangle \]

Qualitative possible dynamical phases:

- \( J^z > h \): "Glass"
- \( h > J^z \): "Paramagnet"

\[ \langle \ln \left( \frac{1}{\ln} \right) \rangle^z \rightarrow 2\Delta \]

At criticality:
- Infinite randomness scaling
- Interactions irrelevant at the critical point:
  \[ J_{typ}^x \sim J_0^x e^{-\Gamma} \]
  \[ (\phi = \text{golden ratio}) \]

\[ \ln \left( \frac{1}{J_{typ}} \right) \sim \Gamma \equiv \log (\Omega_0 t) \]

\[ J_{typ}^z \rightarrow \text{const} \quad h_{typ} \rightarrow 0 \]
Result from the RG flow: spin decay

Critical point:
\[ \langle \sigma_i^z(t) \rangle \sim \frac{1}{\ln^{\phi} t} \]
\[ \phi = \frac{1 + \sqrt{5}}{2} \approx 1.618 \]
(golden ratio)

Saturation of local spin expectation. Glass order parameter!

\( \sigma_i^z \) has an overlap with an integral of motion in the glass
It breaks the \( Z_2 \) symmetry!
Result from the RG flow: spin decay

Critical point:

\[ \langle \sigma_i^z(r) \rangle \sim \frac{1}{\ln^{1-\phi} r} \]

\[ \phi = (1 + \sqrt{5})/2 \approx 1.618 \]

(golden ratio)

“Paramagnet”

\[ \langle \sigma_i^z(r) \rangle \sim \frac{\ln r}{r^\alpha} \]

Saturation of local spin expectation. Glass order parameter!

\( \sigma_i^z \) has an overlap with an integral of motion in the glass
It breaks the \( Z_2 \) symmetry!

“glass”:

\[ \langle \sigma_i^z(r) \rangle \sim \text{const} \sim \Delta^{1-\phi} \]
**RG result: Entanglement entropy growth**

Interaction is an irrelevant perturbation but has a dramatic effect on entanglement entropy growth.

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<td>$S(t)$</td>
<td>$\sim \ln t \Theta(t - t_{int})$</td>
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<td>Saturation in sys. of size $L$</td>
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$S_{\infty}(L)$
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- Universal log growth (Serbyn et. al., Oganesyan et. al., Bauer et al.)
- Enhanced evolution at the critical point (same as in random XXZ)
- Saturates to extensive value but less than thermal in finite system
- Absence of thermalization because of emergent conserved quantities
Delocalization due to distant resonances

Resonances between decimated sites can generate a slow mode that violates the integrals of motion

If $J_{\text{eff}} > \delta \Omega$

Are these resonances relevant near the random fixed points we found?
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\[ \Omega \quad J_{\text{eff}} \quad \Omega - \delta \Omega \]

If \( J_{\text{eff}} > \delta \Omega \)

Are these resonances relevant near the random fixed points we found?
Delocalization due to distant resonances

Typical distance between pairs that differ by $\delta \Omega$:

$$L_R = \frac{1}{\alpha_0 \delta \Omega}$$

Typical effective coupling mediated by the chain:

$$J_{\text{eff}} \sim \exp(-a\sqrt{L_R})$$

The resonance condition $J_{\text{eff}} > \delta \Omega$ leads to an equation for $L_R$:

$$\alpha_0 L_R e^{-a\sqrt{L_R}} > 1$$

No solution for $L_R$ at sufficiently strong disorder! (i.e. for $\alpha_0 < \alpha_s \sim 1$)
Delocalization due to distant resonances

Typical distance between pairs that differ by $\delta \Omega$:

$$L_R \approx \frac{\Omega}{\alpha_0 \delta \Omega}$$

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No solution for $L_R$ at sufficiently strong disorder! (i.e. for $\alpha_0 < \alpha_0^* - 1$)

- Resonances proliferate only below a critical disorder strength!
Outline

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  Dynamical quantum phase transition between MBL states

- RG theory for the MBL transition
  Surprising insight on the delocalization transition and the delocalized state
Coarse Grained Model of coupled blocks

The block parameters:

- $\Gamma_i$ - Decay rate through block $i$
- $g_i = \frac{\Gamma_i}{\Delta_i}$ - # of coupled levels
- $\Delta_i$ - Level spacing of block $i$
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\[
\Gamma_i \quad g_i \quad g_{12} \quad \Gamma_{12} \quad g_{12} \\
\Gamma_1 \quad g_1 \quad h_2 \quad g_2 \quad \Gamma_2 \\
\Gamma_3 \quad g_3 \quad g_{23} \quad \Gamma_3 \\
\Gamma_4 \quad g_4 \quad g_{34} \quad \Gamma_4 \\
\Gamma_5 \quad g_5 \quad g_{45} \quad \Gamma_5
\]
Coarse Grained Model of coupled blocks

The block parameters:

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$g_i = \frac{\Gamma_i}{\Delta_i}$ - # of coupled levels

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$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$
Coarse Grained Model of coupled blocks

The block parameters:

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- $g_i = \frac{\Gamma_i}{\Delta_i}$ - # of coupled levels
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$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$

Bath $E + \Delta E$ to Bath $E$
Renormalization Scheme

Pick fastest decay rate on bond $\Gamma_{12}$

$\Gamma_1 g_1 \Gamma_2 g_2 \Gamma_3 g_3 \rightarrow \tilde{\Gamma} \tilde{g} = ?$

$\Gamma_{12} g_{12} \Gamma_3 g_3$
Renormalization Scheme

Pick fastest decay rate on bond $\Gamma_{12}$

$\Gamma_1 \quad \Gamma_2 \quad \Gamma_3 \quad \Gamma_{12}$

$g_1 \quad g_2 \quad g_3 \quad g_{12}$

$\tilde{\Gamma} \tilde{g} =$?
Renormalization Scheme

Pick fastest decay rate on bond $\Gamma_{12}$

$\Gamma_1 \quad g_1 \quad \Gamma_2 \quad g_2 \quad \Gamma_3 \quad g_3 \quad \Gamma_{12} \quad g_{12} \quad \Gamma_3 \quad g_3$

Deep in localized phase $g_{12}, g_{23} < 1$

$$\tilde{\Gamma} = \frac{\Gamma_{12} \Gamma_{23}}{\Gamma_2} \quad \tilde{g} = \frac{g_{12} g_{23}}{g_2}$$

Deep in delocalized phase $g_{12}, g_{23} > 1$ (diffusive transport)

$$\frac{1}{(l_1 + l_2 + l_3)\Gamma} = \frac{1}{(l_1 + l_2)\Gamma_{12}} + \frac{1}{(l_2 + l_3)\Gamma_{23}} \quad (\text{length} \sim \sqrt{\text{time}})$$
Renormalization Scheme

Pick fastest decay rate on bond $\Gamma_{12}$

$\Gamma_1 \quad g_2 \quad \Gamma_2 \quad g_3 \quad \Gamma_3 \quad \Gamma_{12}$

Deep in localized phase $g_{12}, g_{23} < 1$

Deep in delocalized phase $g_{12}, g_{23} > 1$ (diffusive transport)

$$\tilde{\Gamma} = \frac{\Gamma_{12} \Gamma_{23}}{\Gamma_2}$$

$$\tilde{g} = \frac{g_{12} g_{23}}{g_2}$$

$$\frac{1}{(l_1 + l_2 + l_3)\Gamma} = \frac{1}{(l_1 + l_2)\Gamma_{12}} + \frac{1}{(l_2 + l_3)\Gamma_{23}}$$

(length $\sim \sqrt{\text{time}}$)
Preliminary Flow Results

Griffiths phase

\[ P(l_{\text{int}}) \sim e^{-d_{\text{int}}} \]

\[ t(l_{\text{int}}) \sim e^{d_{\text{int}}} \]
The Many-Body Localization Transition

Based on entanglement subadditivity: (T. Grover, arXiv 1405.1471)

Localized  Ergodic

Localized  Delocalized Non-Ergodic  Ergodic

Disorder strength

Our theory gives the second option
The Many-Body Localization Transition

Based on entanglement subadditivity: (T. Grover, arXiv 1405.1471)

Localized  Ergodic

Localized  Delocalized Non-Ergodic  Ergodic

Disorder strength

Our theory gives the second option
Preliminary Flow Results

Griffiths phase

\[ P(I_{im}) \sim e^{-t_{im}} \]
\[ f(I_{im}) \sim e^{t_{im}} \]

Wide distribution of decay times for \( a < b \)

L - \log t \sim \frac{4}{2}

L \sim t^\alpha

L \sim \sqrt{t}
Preliminary Flow Results

Griffiths phase

\[ P(I_{mm}) \sim e^{-d_{mm}} \]

\[ t(I_{mm}) \sim e^{d_{mm}} \]

Wide distribution of decay times for \( a<b \)
The Many-Body Localization Transition

Based on entanglement subadditivity: (T. Grover, arXiv 1405.1471)

Localized    Ergodic

Localized    Delocalized Non-Ergodic    Ergodic

Disorder strength

Our theory gives the second option
Scaling in the localized phase

\[ L \sim (\log t)^{\alpha} \]

\[ \Phi \sim \text{Golden ratio} \]

Entanglement entropy

Assuming \( S \sim L(t) \)

- Localized phase: \( S \sim \log t \)
- At criticality: \( S \sim (\log t)^{2/\Phi} \)?
“Microscopic” Derivation

- Each block is a random matrix
- Fixed bandwidth $W$, $\Gamma_i = \Delta_i$ and $g_i = 1$
- Matrix element between neighboring blocks

$$\langle a', b' | \hat{J}_{ij} | a, b \rangle = J_{ij} (1 - \delta_{a' a})(1 - \delta_{b' b})x \quad x \sim N(\mu = 0, \sigma^2 = 1)$$
“Microscopic” Derivation

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  \[
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  x \sim N(\mu = 0, \sigma^2 = 1)
  \]
- Decay rate of two blocks given by Fermi golden rule
  \[
  \Gamma_{12} = \frac{2\pi J_{12}^2}{\Delta_{12}} \\
  g_{12} = \frac{\Gamma_{12}}{\Delta_{12}} \\
  \Delta_{12} = \frac{2W}{(N_1N_2)} = 2\Delta_1\Delta_2/W
  \]
“Microscopic” Derivation

- Each block is a random matrix
- Fixed bandwidth $W$, $\Gamma_i = \Delta_i$ and $g_i = 1$
- Matrix element between neighboring blocks

$$\langle a', b' \mid \hat{J}_{ij} \mid a, b \rangle = J_{ij} (1 - \delta_{a' a}) (1 - \delta_{b' b}) x \quad x \sim \mathcal{N}(\mu = 0, \sigma^2 = 1)$$

- Decay rate of two blocks given by Fermi golden rule

$$\Gamma_{12} = 2\pi \frac{J_{12}^2}{\Delta_{12}} \quad \Delta_{12} = 2W/(N_1 N_2) = 2\Delta_1 \Delta_2 / W$$

$$g_{12} = \frac{\Gamma_{12}}{\Delta_{12}}$$
3 Block Decay Rate

Generalized Fermi golden rule: \((g_{12} \cdot g_{23} < 1)\)

T Matrix

\[
T = J + J \frac{1}{E_i - H_0 + i\eta} J + J \frac{1}{E_i - H_0 + i\eta} J + \ldots
\]

\[
\Gamma = 2\pi |\langle f | T | i \rangle|^2 \rho(E_i)
\]
3 Block Decay Rate

Generalized Fermi golden rule: \((g_{12} g_{23} < 1)\)

\[
T = J + J 
\frac{1}{E_i - H_0 + i\eta} J + J
\frac{1}{E_i - H_0 + i\eta} J + J
\frac{1}{E_i - H_0 + i\eta} J + \ldots
\]

\[
\Gamma = 2\pi \langle \langle f | T | i \rangle \rangle^2 \rho(E_i)
\]

For 3 blocks:
\[
\hat{J} = \hat{J}_{12} + \hat{J}_{23}
\]

\[
\eta = \Gamma_2
\]

\[
\langle f | T | i \rangle = \sum_m \langle f | (J_{12} + J_{23}) | m \rangle \frac{1}{E_i - E_m + i\eta} \langle m | (J_{12} + J_{23}) | i \rangle
\]
3 Block Decay Rate (2)

\[ \Gamma_{12} g_{12} \Gamma_{31} g_{31} \rightarrow \Gamma_{12} g_{12} \Gamma_{31} g_{31} \]

Generalized Fermi golden rule: \((g_{12}, <1,g_{31}>1)\)

The decay of blocks 2 and 3: \(\Gamma_{23} = 2\pi \frac{\Gamma_{23}^2}{\Delta_{23}}\)
Open Questions

• The critical point
  – What is the flow at the critical point?
  – Verify infinite randomness scaling
  – Extract universal exponents
• Entanglement entropy
  – Logarithmic evolution?
  – Enhanced evolution at the critical point?
  – Volume law entropy in the Griffiths phase?
• …
Open Questions

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Summary

1. Dynamical quantum phase transitions between distinct localized states.
   - Formulation of RG for time evolution
   - Universal dynamical description the “phases” and the critical point
   - Conserved quantities
   - Logarithmic evolution of entanglement entropy

2. Dynamical RG for the many-body localization transition
   - Intermediate Griffiths phase with anomalous diffusion
   - Derivation of RG rules using generalized Fermi golden rule
Summary

1. Dynamical quantum phase transitions between distinct localized states.
   - Formulation of RG for time evolution
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   - Conserved quantities
   - Logarithmic evolution of entanglement entropy

2. Dynamical RG for the many-body localization transition
   - Intermediate Griffiths phase with anomalous diffusion
   - Derivation of RG rules using generalized Fermi golden rule