Title: Can Eigenstate Thermalization Breakdown without Disorder?

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Abstract: We describe a new diagnostic for many-body wavefunctions which generalizes the spatial bipartite entanglement entropy. By way of illustration, for a two-component wavefunction of heavy and light particles, a partial (projective) measurement of the coordinates of the heavy (but not light) particles is first performed, and then the entanglement entropy of the projected wavefunction for the light particles is computed. If the two-component wavefunction has a volume law entanglement entropy, yet the post measurement wavefunction of the light particles is disentangled with an area law entanglement, we refer to the original wavefunction as a "Quantum Disentangled State". This diagnostic can be generalized to include other partial measurements, such as measuring the charge, but not spin, for finite-energy density eigenstates of Fermion Hubbard-type model. Quantum disentanglement results if the post measurement spin-wavefunction has an area law entanglement entropy. Recent numerics searching for such Quantum Disentangled States in 1d Hubbard-type models will be discussed in detail.
Can Eigenstate Thermalization Breakdown without Disorder?

MPA Fisher
Workshop on Quantum Many-Body Dynamics
Perimeter Institute, May 15, 2014

- Thermal-vs-entanglement entropy and Eigenstate Thermalization (ETH)
- Breakdown of ETH? Yes: Many-Body-Localization w/ disorder

QUESTION:
Can ETH breakdown in disorder-free Quantum systems?

Maybe....

Describe:
- New Wavefunction Diagnostics, beyond entanglement entropy:
  (global) partial-measurement, followed by post-measurement entanglement
- Quantum Disentangled States: New “Thermal” states with “hidden” locality
Quantum (T=0) vs Classical (T>0) Phases:

Quantum Phases T=0 (ground states):
Characterized by symmetries, symmetry breaking.
But much richer:
Can have topological order, or emergent gapless excitations....
(Full) Classification will require universal properties of wavefons (entanglement, correlators), and excitations (anyons, Fermi/Bose surfaces, dynamical correlators)

Classical, T>0, phases characterized by:
- symmetries and conservation laws. (eg particle number)
- broken symmetries (eg as in a superfluid)
- Hydrodynamics, built from conserved densities and order parameters

Does all the rich quantum phenomena wash-out for T>0?
Quantum Statistical Mechanics

Canonical Ensemble, w/ heat bath
Thermal density matrix
\[ \hat{\rho}_{th} = \frac{1}{Z} e^{-\beta \hat{H}} \]

(Local) Observables, Hermitian ops
\[ \langle \hat{O} \rangle_{th} = Tr[\hat{\rho}_{th} \hat{O}] \]

Internal Energy (extensive)
\[ U = Tr[\hat{\rho}_{th} \hat{H}] \quad U/L^d > 0 \]

Microcanonical Ensemble: Isolated, single eigenstate
\[ \hat{H}|\psi\rangle = E|\psi\rangle \]
\[ \hat{\rho} = |\psi\rangle \langle \psi | \]

Spatial partition: Regions A ("system") and B ("environment")
Observables inside "system" A
\[ \langle \hat{O} \rangle_E = Tr[\hat{\rho} \hat{O}] \]

Equivalence of Canonical and Microcanonical Ensemble:
For "all" states at energy E that satisfy observables are equivalent
\[ \frac{E}{L^d} = \frac{U}{L^d} \]
\[ \langle \hat{O} \rangle_E = \langle \hat{O} \rangle_{th} \]
Quantum Statistical Mechanics

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\[ \langle \hat{O} \rangle_{th} = Tr[\hat{\rho}_{th} \hat{O}] \]

Internal Energy (extensive)
\[ U = Tr[\hat{p}_{th} \hat{H}] \quad U/L^d > 0 \]

Microcanonical Ensemble: Isolated, single eigenstate

\[ \hat{H}(\psi) = E(\psi) \]
\[ \hat{\rho} = \left| \psi \right\rangle \left\langle \psi \right| \]

Spatial partition: Regions A ("system") and B ("environment")

Observables inside "system" A
\[ \langle \hat{O} \rangle_E = Tr[\hat{\rho} \hat{O}] \]

Hydrodynamics, and thermalization (not discussed today)

Equivalence of Canonical and Microcanonical Ensemble:

For "all" states at energy E that satisfy observables are equivalent
\[ \frac{E}{L^d} = \left( \frac{U}{L^d} \right)_{th} \]
\[ \langle \hat{O} \rangle_E = \langle \hat{O} \rangle_{th} \]
Entropy: Thermal “versus” entanglement

Entropy is “non-local”

Thermal entropy, w/ heat bath:

Number of states, extensive for $T > 0$

\[ S_{th} = -Tr[\tilde{\rho}_{th} \ln \tilde{\rho}_{th}] \]

\[ S_{th} \sim L^d \]

\[ \tilde{\rho}_{th} = \frac{1}{Z} e^{-\beta H} \]

Entanglement Entropy: Isolated, single eigenstate $|\psi\rangle$

\[ \hat{H}(\psi) = E(\psi) \]

\[ \hat{\rho} = |\psi\rangle \langle \psi| \]

Reduced density matrix in $A$

\[ \hat{\rho}_A = Tr_B(\hat{\rho}) \]

Entanglement entropy:

\[ S_A(L) = -Tr_A(\hat{\rho}_A \ln \hat{\rho}_A) \]
Entropy: Thermal “versus” entanglement

Entropy is “non-local”

Thermal entropy, with heat bath:
Number of states, extensive for T>0
\[ S_{th} = -Tr[\hat{\rho}_{th} \ln \hat{\rho}_{th}] \]
\[ S_{th} \sim L^d \]
\[ \hat{\rho}_{th} = \frac{1}{L} e^{-\hat{H}} \]

Entanglement Entropy: Isolated, single eigenstate
\[ \hat{H}(\psi) = E(\psi) \]
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Thermal entropy, w/ heat bath:

Number of states, extensive for $T>0$

\[
S_{th} = -Tr[\hat{\rho}_{th} \ln \hat{\rho}_{th}]
\]

\[
S_{th} \sim L^d \quad \hat{\rho}_{th} = \frac{1}{Z} e^{-\hat{H}}
\]

Entanglement Entropy: Isolated, single eigenstate $|\psi\rangle$

\[
\hat{H}(\psi) = E(|\psi\rangle)
\]

\[
\hat{\rho} = |\psi\rangle \langle \psi|
\]

Reduced density matrix in $A$ \[ \hat{\rho}_A = Tr_B(\hat{\rho}) \]

Entanglement entropy: \[ S_A(L) = -Tr_A(\hat{\rho}_A \ln \hat{\rho}_A) \]
Ex: Entanglement entropy in 1d spin-chain

1d $s=1/2$ $J_1-J_2$ chain

Ground state, Area-law:
$S_A \sim O(1)$

"High-energy" state, Volume law:
$S_A \sim \frac{L}{2} - |x - \frac{L}{2}|$

Maximum entropy state, Entropic (zero) local information:
$S_A/L_A \approx \ln 2$
Eigenstate Thermalization Hypothesis (ETH)

Josh Deutsch, Mark Srednicki

ETH = Microcanonical ensemble for E/L^d > 0 eigenstate
"equivalent", in region A, to canonical ensemble w/ heat bath

Eigenstates w/ (nearly) same energy have identical correlations inside A

Equivalence of (non-local) Thermal and entanglement entropies

$$S_A/L^d = S_{th}/L^d; \quad L \rightarrow \infty$$

Thermal entropy is state counting, entanglement entropy depends on the properties of the states!
**Eigenstate Thermalization Hypothesis (ETH)**

Josh Deutsch, Mark Srednicki

ETH = Microcanonical ensemble for $E/L^d > 0$ eigenstate equivalent, in region A, to canonical ensemble w/ heat bath

![Diagram](https://via.placeholder.com/150)

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Equivalence of (non-local) Thermal and entanglement entropies

$$S_A/L^d = S_{th}/L^d; \quad L \rightarrow \infty$$

Thermal entropy is state counting, entanglement entropy depends on the properties of the states!
Can ETH Break Down?

YES: In "Many-Body-Localized" (MBL) phase

Isolated, interacting, quantum particles in random potential can be in a Many-Body-Localized (MBL) phase

Eigenstates in MBL have area law entanglement entropy (even at E/L^d>0) while thermal entropy is extensive

\[ S_A \sim L^{d-1} \quad S_{th} \sim L^d \]

- Lack of thermalization: Particles don't serve as own thermal bath
- Conerved quantities (eg charge, or even energy) do not propagate, zero conductivity
- Entanglement does not "propagate", local quantum information cannot be transmitted

But... The random potential constitutes classical (frozen) set of degrees of freedom...
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But... The random potential constitutes classical (frozen) set of degrees of freedom...
Can ETH Breakdown w/out quenched disorder?

When might this happen?
Neutral atoms, isolated in box, prepared in many-body eigenstate, energy $E$

Vary energy $E$ (the "temperature")
- $T \sim 10^{-1,000} \text{ K, atomic-liquid}$
- $T > 10 \text{ eV, ionized-plasma}$

Question: Are eigenstates of atomic-liquid qualitatively the same as the ionized-plasma eigenstates?
Atomic-Liquid vs ionized-plasma?

Canonical answer: Atomic-liquid and ionized plasma are “equivalent”
- Atomic-liquid and ionized-plasma eigenstates both volume-law entangled
- Atoms weakly ionized (even at low T)
- Low density of ionized electrons “should” thermalize

But...
- How does one (or can one) define “ionized”?
- How to characterize thermalization of “ionized” electrons?

Can we:
- New waveon diagnostic ("a" characterization of ionization)
- New class of volume entangled (E>0) quantum states
  - "Quantum Disentangled States"
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New wavefcn diagnostic (“a” characterization of ionization)
New class of volume entangled ($E>0$) quantum states
- “Quantum Disentangled States”
Can ETH Breakdown w/out quenched disorder?

When might this happen?

Neutral atoms, isolated in box, prepared in many-body eigenstate, energy $E$

Vary energy $E$ (the "temperature")

- $T \sim 10^{-4} \text{ K, atomic-liquid}$
- $T > 10 \text{ eV, ionized-plasma}$

Question: Are eigenstates of atomic-liquid qualitatively the same as the ionized-plasma eigenstates?
Entanglement, locality, information and measurement

Alice and Bob share two s=1/2 particles

- Direct product: \( |\psi⟩ = |\uparrow⟩_A \otimes |\downarrow⟩_B \)
  - Bob's measurement, no effect on Alice. Alice has full local quantum information.
- Singlet state: \( |\psi⟩ = \frac{1}{\sqrt{2}} (|\uparrow⟩_A \otimes |\downarrow⟩_B - |\downarrow⟩_A \otimes |\uparrow⟩_B) \)
  - Bob's measurement affects Alice. Alice has no local quantum information about her spin - two spins are "entangled"

Entanglement entropy quantifies local quantum information

- \( \hat{\rho} = |\psi⟩⟨\psi| \) \( \rho_A = Tr_B(\hat{\rho}) \) \( S_A = -Tr_A(\rho_A \ln \rho_A) \)
- Direct product state – complete local info: \( S_A = S_B = 0 \)
- Entangled Singlet state – no local info: \( S_A = S_B = \ln(2) \)

(Local) Measurement induces disentanglement

- For singlet state Bob measures his spin:
  - \( |\psi⟩ = \frac{1}{\sqrt{2}} (|\uparrow⟩_A \otimes |\downarrow⟩_B - |\downarrow⟩_A \otimes |\uparrow⟩_B) \) \( \rightarrow \)
  - \( |\psi’⟩ = |\uparrow⟩_A \otimes |\downarrow⟩_B \)
  - \( S_A = S_B = \ln(2) \)
  - After measurement, direct product state
  - \( |\psi’⟩ = |\uparrow⟩_A \otimes |\downarrow⟩_B \) \( \rightarrow \)
  - \( S_A^f = S_B^f = 0 \)
Atomic-Liquid vs ionized-plasma?

Canonical answer: Atomic-liquid and ionized plasma are “equivalent”
- Atomic-liquid and ionized-plasma eigenstates both volume-law entangled
- Atoms weakly ionized (even at low $T$)
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But...
- How does one (or can one) define “ionized”?
- How to characterize thermalization of “ionized” electrons?

An:
- New wavefn diagnostic (“a” characterization of ionization)
- New class of volume entangled ($E>0$) quantum states
  - “Quantum Disentangled States”
Entanglement, locality, information and measurement

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- **Direct product:** $|\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B$

  - Bob’s measurement, no effect on Alice. Alice has full local quantum information.

- **Singlet state:** $|\psi\rangle = \frac{1}{\sqrt{2}} [ |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B ]$

  - Bob’s measurement affects Alice. Alice has no local quantum information about “her” spin. Two spins are “entangled”

Entanglement entropy quantifies local quantum information

- $\hat{\rho} = |\psi\rangle \langle \psi|$
- $\rho_A = Tr_B(\hat{\rho})$
- $S_A = Tr(\rho_A \ln \rho_A)$

Direct product state – complete local info:

- $S_A = S_B = 0$
- $S_{AB} = \ln(2)$

Entangled Singlet state – no local info:

- $S_A = S_B = \ln(2)$
- $S_{AB} = 0$

(Local) Measurement induces disentanglement

For singlet state Bob measures his spin:

- $|\psi\rangle = \frac{1}{\sqrt{2}} [ |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B ]$

  - After measurement, direct product state:
    - $S_A = S_B = \ln(2)$
    - $S'_A = S'_B = 0$
Entanglement, locality, information and measurement

Alice and Bob share two spin-1/2 particles

Direct product: \( |\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \)

Singlet state: \( |\psi\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B \right) \)

Entanglement entropy quantifies local quantum information

\( \rho = |\psi\rangle \langle \psi | \quad \rho_A = \text{Tr}_B(\rho) \quad S_A = -\text{Tr}_A(\rho_A \ln \rho_A) \)

Direct product state – complete local info: \( S_A = S_B = 0 \)
Entangled Singlet state – no local info: \( S_A = S_B = \ln(2) \)

(Local) Measurement induces disentanglement

For singlet state Bob measures his spin:

\( |\psi\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B \right) \rightarrow \begin{cases} 
|\psi'\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \\
|\psi''\rangle = |\downarrow\rangle_A \otimes |\uparrow\rangle_B 
\end{cases} 
\)

After measurement, direct product state

\( S_A = S_B = \ln(2) \rightarrow S'_A = S'_B = 0 \)
Entanglement, locality, information and measurement

Alice and Bob share two s=1/2 particles

Direct product:

\[ |\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \]

Singlet state:

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B) \]

Entanglement entropy quantifies local quantum information

\[ \hat{\rho} = |\psi\rangle \langle \psi| \quad \hat{\rho}_A = \text{Tr}_B(\hat{\rho}) \quad S_A = -\text{Tr}_A(\hat{\rho}_A \ln \hat{\rho}_A) \]

Direct product state – complete local info:

\[ S_A = S_B = 0 \]

Entangled Singlet state – no local info:

\[ S_A = S_B = \ln(2) \]

(Local) Measurement induces disentanglement

For singlet state Bob measures his spin:

\[ |\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B) \]

After measurement, direct product state

\[ S_A = S_B = \ln(2) \]

After measurement, direct product state

\[ S_A' = S_B' = 0 \]
Entanglement, locality, information and measurement

Alice and Bob share two spin-1/2 particles

Direct product: \( |\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \)

Singlet state: \( |\psi^\prime\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B \right) \)

Bob's measurement, no effect on Alice. Alice has full local quantum information.

Bob's measurement affects Alice. Alice has no local quantum information about "no" spin - two spins are "entangled".

Entanglement entropy quantifies local quantum information

\[ \hat{\rho} = |\psi\rangle \langle \psi | \quad \hat{\rho}_A = \text{Tr}_B(\hat{\rho}) \quad S_A = -\text{Tr}(\hat{\rho}_A \ln \hat{\rho}_A) \]

Direct product state – complete local info: \( S_A = S_B = 0 \)

Entangled Singlet state – no local info: \( S_A = S_B = \ln(2) \)

(Local) Measurement induces disentanglement

For singlet state Bob measures his spin:

\[ |\psi^\prime\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B \right) \]

After measurement, direct product state

\[ S_A = S_B = \ln(2) \quad \Rightarrow \quad S_A^\prime = S_B^\prime = 0 \]
Entanglement, locality, information and measurement

Alice and Bob share two s=1/2 particles

Direct product:

\[ |\psi\rangle = |\uparrow\rangle_A \otimes |\downarrow\rangle_B \]

Singlet state:

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B \right) \]

Entanglement entropy quantifies local quantum information

\[ \hat{S} = \langle \psi | \rho_A - T_A \rho_B | \psi \rangle \]

Direct product state - complete local info:

\[ S_A = S_B = 0 \]

Entangled Singlet state - no local info:

\[ S_A = S_B = \ln(2) \]

(Local) Measurement induces disentanglement

For singlet state Bob measures his spin:

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B \right) \]

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow\rangle_A \otimes |\downarrow\rangle_B \right) \]

After measurement, direct product state

\[ S_A = S_B = \ln(2) \]

\[ S_{A'} = S_{B'} = 0 \]
Measurements in extended systems

Ex: Heisenberg model, 2-leg ladder with $s=1/2$

$$\hat{\mathcal{H}} = \sum_{i} J_{ij} \hat{S}_{i} \cdot \hat{S}_{j} + \ldots$$

An eigenstate with volume law entanglement:

$$S_{A} = -\text{Tr}_{A} [\hat{\rho}_{A} \ln \hat{\rho}_{A}] = s L^{d}$$

$$\hat{\rho}_{A} = \text{Tr}_{B} |\Psi\rangle \langle \Psi|$$

Full Measurement: Measure spin ($S_{z}$) on every site, find

$$|\Psi\rangle \rightarrow |\Psi'\rangle = |\bar{S}\rangle$$

(a direct product state)

Post-measurement if fully disentangled

$$S_{A} = s L^{d} \rightarrow S'_{A} = 0$$

(Global) Partial Measurement:

Measure spins on one leg of ladder only, leg 2, but NOT on leg 1

$$|\Psi\rangle \rightarrow |\Psi'\rangle = |S_{1}\rangle \bar{S}_{2}$$

What is spatial entanglement entropy of post-partial measurement if?
Measurements in extended systems

Ex: Heisenberg model, 2-leg ladder with $s=1/2$

\[
\mathcal{H} = \sum_{i} J_{ij} \vec{S}_i \cdot \vec{S}_j + \ldots
\]

Eigensate w/ volume law entanglement:

\[
S_A = -\text{Tr}_A [\hat{\rho}_A \ln \hat{\rho}_A] = s L^d
\]

\[
\hat{\rho}_A = \text{Tr}_D [\Psi \langle \Psi |]
\]

Full Measurement: Measure spin ($S_z$) on every site, find \{\vec{S}\} (a direct product state)

\[
|\Psi\rangle \rightarrow |\Psi'\rangle = |\vec{S}\rangle
\]

Post-measurement w/ fully disentangled

\[
S_A = s L^d \rightarrow S'_A = 0
\]

(Global) Partial Measurement:

Measure spins on one-leg of ladder only, leg 2, but NOT on leg 1

\[
|\Psi\rangle \rightarrow |\Psi_1'\rangle = |S_1\rangle \vec{S}_2
\]

What is spatial entanglement entropy of post-partial measurement w/ ??
New wavefcn diagnostic: "Post-measurement entanglement"

Illustrate w/ Atomic-liquid

\[ E > 0 \text{ volume-law wavefcn, } \psi(R, r) \]

1. Measure positions of nuclei, but not electrons, (partial measurement) find \( B \)

2. Consider wavefcn of (unmeasured) electrons,

\[ \psi_e(r) \sim \psi(R, r) \]

3. Compute (spatial) bi-partition entanglement entropy of electron wavefcn (w fixed nuclei coordinates, \( R \))

4. Repeat (i)-(iii) and average, gives post-measurement entanglement entropy

\[ S^{R_R}_{\psi} \]

Can repeat w/ \( R \leftrightarrow r \)

Interest: Size scaling of \( S^{R_R}_{\psi} \)
New wavefcn diagnostic: “Post-measurement entanglement”

Illustrate w/ Atomic-liquid

1. Measure positions of nuclei, but not electrons, (partial measurement) find $R$.

2. Consider wavefcn of (unmeasured) electrons,

   $\psi(r) \sim \psi(R, r)$

3. Compute (spatial) bi-partition entanglement entropy of electron wavefcn (w/ fixed nuclei coordinates, $R$).

4. Repeat (i)-(iii) and average, gives post-measurement entanglement entropy

   $S_A^{r/R}$

Can repeat w/ $R \leftrightarrow r$ $S_A^{R/r}$

Interest: Size scaling of $S_A^{r/R}$
Scaling of post-measurement entanglement entropy?

**Volume Law:** \( S_A^r/R \sim L^d \)

Electrons entangled (thermalized) even after measuring nuclei positions.

**Fully Thermalized Eigenstate**

**Area Law:** \( S_A^r/R \sim L^{d-1} \)

Measuring nuclei positions has disentangled ("localized") electrons

**"Quantum Disentangled Eigenstate"**
Scaling of post-measurement entanglement entropy?

Volume Law: \[ S_A^r/R \sim L^d \]
Electrons entangled (thermalized) even after measuring nuclei positions.

Fused Thermalized Eigenstate

Fully thermalized, fully thermalized w/ volume laws

Area Law: \[ S_A^r/R \sim L^{d-1} \]
Measuring nuclei positions has disentangled ("localized") electrons

"Quantum Disentangled Eigenstate"
Scaling of post-measurement entanglement entropy?

Volume Law: \[ \frac{S_A}{R} \sim L^d \]
Electrons entangled (thermalized) even after measuring nuclei positions.

Fully Thermalized Eigenstate

Area Law: \[ \frac{S_A}{R} \sim L^{d-1} \]
Measuring nuclei positions has disentangled ("localized") electrons

"Quantum Disentangled Eigenstate"
Quantum Disentangled State for atomic fluid

An atomic fluid eigenstate is Quantum-Disentangled if,

Measuring (heavy) nuclei, disentangles (light) electrons, Area law: \( S_A^{r/R} \sim L^{d-1} \)

Measuring (light) electrons does not disentangle (heavy) nuclei, Volume law: \( S_A^{R/r} \sim L^d \)

Quantum Disentangled States: General defn

Volume law entangled eigenstate (E > 0)

\[ S_A^\psi \sim L^d \]

Perform (some) global partial-measurement, wf of un-measured d.o.f.

\[ |\Psi\rangle \rightarrow |\psi\rangle \quad |\psi\rangle \rightarrow S_A^{\psi_0} \]

Entanglement-entropy of post-measurement wf

Quantum Disentangled State: post-measurement wf has area law

Partial Measurement induces full locality

\[ S_A^{\psi_0} \sim L^{d-1} \]

Thermal State: wf post-measurement still entangled (non-local, volume law)

\[ S_A^{\psi_0} \sim L^d \]
Do Quantum Disentangled States exist?
(as eigenstates of generic Hamiltonians)

Fermion Hubbard models: Partial measurements

\[ |\psi\rangle \quad H = -i \sum_{\langle ij\rangle} (\hat{c}_i^\dagger \hat{c}_j + h.c.) + \sum_{\langle ij\rangle} n_i n_j + ... \]

(1) Measure Charge on each site, but not spin

\[ |\psi\rangle \rightarrow \mathcal{P}|\psi\rangle = |\psi'\rangle \]

After measurement, "spin-wf" \[ |\psi'\rangle \]

Post-measurement entanglement entropy of "spin-wf" \[ S_{s/c} \]

(2) Measure Spin on each site, but not charge

\[ |\psi\rangle \rightarrow \mathcal{P}|\psi\rangle = |\psi'\rangle \]

After measurement, "holon/dualon" charge wf \[ |\psi'\rangle \]

Post-measurement entanglement entropy of "charge-wf" \[ S_{c/s} \]
Do Quantum Disentangled States exist?
(as eigenstates of generic Hamiltonians)

Fermion Hubbard models: Partial measurements
\[ |\Psi\rangle \]

\[ H = -t \sum_{\langle i,j \rangle} (c_i \dagger c_j + h.c.) = \sum_{\langle i,j \rangle} \tau_{ij} - \sum a_i a_i^\dagger + \ldots \]

1. Measure Charge on each site, but not spin
   \[ |\Psi\rangle \rightarrow \mathcal{P}|\Psi\rangle = |\psi\rangle \]
   After measurement, "spin-wf" \[ |\psi\rangle \]
   A \[ \times \]
   B

Post-measurement entanglement entropy of "spin-wf" \[ S_{S/C} \]

2. Measure Spin on each site, but not charge
   \[ |\Psi\rangle \rightarrow \mathcal{P}|\Psi\rangle = |\psi\rangle \]
   After measurement, "holon/doublon" charge wf \[ |\psi\rangle \]
   A \[ \times \]
   B

Post-measurement entanglement entropy of "charge-wf" \[ S_{C/S} \]
“Doublons” in the Hubbard Model

Near-neighbor Hubbard, $H$, in any dimension

$$\hat{H} = -t \sum_{ij} (\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Order-by-order in $t/U$, can perform a unitary transformation on $H$ which eliminates terms coupling states w/ differing number of doubly occupied sites

$$H' = e^{iS} H e^{-iS}$$

$H'$ is "block-diagonalized" into decoupled "doublon-sectors"

Example: At leading order in $t/U$

$$iS = (T_1 - T_{-1})/U + O(t/U)^2$$

Terms in kinetic energy which change number of doubly occupied sites by $\pm 1$

In sector w/ zero doublons, at leading order in $t/U$, get t-J model

$$H' = H_{t,J}$$
QDL in Hubbard Model?

Bi-partite Hubbard, half-filling

$$\hat{\mathcal{H}} = -i \sum_{ij} (\hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Charge/spin duality between positive and negative U Hubbard

$$\hat{c}_{j\uparrow} \rightarrow \hat{c}_{j\uparrow}, \quad \hat{c}_{j\downarrow} \rightarrow (-1)^{j} \hat{c}_{j\downarrow}$$

$$\hat{\mathcal{H}}(U) \rightarrow \hat{\mathcal{H}}(-U)$$

Duality between charge/spin and spin/charge entanglement entropies

$$S_{c/s}(U) = S_{s/c}(-U)$$

Implication: Free Fermions are not in a QDL phase!

$$S_{c/s}(0) = S_{s/c}(0) \sim L^d$$

Large $U>0$: Measuring spin almost determines charge, since doublons are rare. Possible charge-disentangled QDL?

Large $U<0$: Measuring charge almost determines state since mostly Cooper pairs. Possible spin-disentangled QDL?
“Doublons” in the Hubbard Model

Near-neighbor Hubbard, $H$, in any dimension

$$\hat{H} = -t \sum_{ij} (\hat{c}^\dagger_{i\sigma} \hat{c}_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Order-by-order in $t/U$, can perform a unitary transformation on $H$ which eliminates terms coupling states with differing number of doubly occupied sites

$$H' = e^{iS} H e^{-iS}$$

$H'$ is “block-diagonalized” into decoupled “doublon-sectors”

Example: At leading order in $t/U$

$$iS = (T_1 - T_{-1}) / U + O(t/U)^2$$  \( T_{\pm 1} \) Terms in kinetic energy which change number of doubly occupied sites by ±1

In sector with zero doublons, at leading order in $t/U$, get t-J model  \( H' = H_{t,J} \)
“Doublons” in the Hubbard Model

Near-neighbor Hubbard, $H$, in any dimension

$$\hat{H} = -t \sum_{ij} (\hat{c}_{i\alpha}^\dagger \hat{c}_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}$$

Order-by-order in $t/U$, can perform a unitary transformation on $H$ which eliminates terms coupling states w/ differing number of doubly occupied sites

$$H' = e^{iS} H e^{-iS}$$

$H'$ is "block-diagonalized" into decoupled "doublon-sectors"

Example: At leading order in $t/U$

$$iS = (T_1 - T_{-1})/U + O(t/U)^2 \quad T_{\pm 1}$$

Terms in kinetic energy which change number of doubly occupied sites by $+1$

In sector w/ zero doublons, at leading order in $t/U$, get t-J model

$$H' = H_{t,J}$$
"Doublons" at half-filling

At half-filling get spin-model

$$H' = \sum_{ij} S_i \cdot S_j + \sum_{ijkl} (P_{ijkl} + h.c.) + ...$$

$$J_n \sim t(t/U)^{n-1}$$

Naively, can transform spin-model eigenstates into Hubbard eigenstates with the unitary,

$$|\psi_{Hubbard} \rangle = e^{iS} |\psi_{spin} \rangle$$

Questions:
- Does the t/U expansion converge for ground state of spin-model?
- For thermal eigenstates of spin-model??
- Is the operator S local in space (for U>0)? Is exp(S) a "local unitary"?
- Role of integrability of 1d Hubbard model?
- Generalization to 1d non-integrable model, eg wf n.n. interaction, V

"If" a volume law Hubbard eigenstate can be transformed via a local unitary into a (volume law) spin-wf, then a spin-measurement of wf will disentangle, giving an area law post-measurement charge-entanglement entropy (ie it is a QDL state)

$$S_{c/s} \sim L^{d-1}$$
“Doublons” at half-filling

At half-filling get spin-model

\[ H = J \sum_{ij} S_i \cdot S_j + J_x \sum_{ijkl} (P_{ijkl} + h_{ijkl}) + \ldots \]

\[ J_n \sim t(t/U)^{n-1} \]

Naively, can transform spin-model eigenstates into Hubbard eigenstates with the unitary,

\[ |\psi_{Hubbard}\rangle = e^{iS} |\psi_{spin}\rangle \]

Questions:
- Does the t/U expansion converge
  - for ground state of spin-model?
  - for thermal eigenstates of spin-model??
- Is the operator S local in space (for U=1)? Is exp(iS) a "local unitary"?
- Role of integrability of 1d Hubbard model?
- Generalization to 1d non-integrable model, eg with n.n. interaction, V

"If" a volume law Hubbard eigenstate can be transformed via a local unitary into a (volume law) spin-wf, then a spin-measurement of wf will disentangle, giving an area law post-measurement charge-entanglement entropy (ie it is a QDL state)

\[ S_{C/S} \sim L^{d-1} \]
Large U “Bands” and Mixing?

- Start w/ infinite U
- Spin-sector
  \[ E_0 = 0 \]
- Band w/ a single doublon/holon
  \[ E_1 = E_0 + U \]
- Perturb in hopping \( t \), effective spin model
  \[ H_s = J_2 \sum_{ij} \vec{S}_i \cdot \vec{S}_j \quad J_2 \sim \frac{t^2}{U} \]

Does the spin band “mix” with doublon/holon bands in the same energy range?
Numerics on 1d “Hubbard” chains

\[ \mathcal{H} = -t \sum_{i,\sigma} (\hat{c}_{i\sigma}^\dagger \hat{c}_{i+1\sigma} + \text{h.c.}) + U \sum_i \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + V \sum_i \hat{n}_i \hat{n}_{i+1} \]

- Repulsive interaction, U>0, at half-filling
- n.n. repulsion, V, destroys integrability
- Compute "all" eigenstates from ED

First: Extract mean-double occupancy for each eigenstate
(at many different values of U, V=0,3)

\[ D = \langle \hat{n}_{j\uparrow} \hat{n}_{j\downarrow} \rangle \]
Full and post-measurement entanglement entropy

10 site Hubbard chain
U=n. repulsion, V=3

Full Entanglement entropy

\[ S(L/2) \]

Post-spin-measurement entropy of "charge-wf"

\[ S_c/s(L/2) \]

"Spin-band" -
Post-spin-measurement, "charge-wf" low entanglement.

"Charge-band"
Co-existing “entropy-bands”?

\[ S(L/2) \]

\[ E/L \]

“Charge-band”

“Spin-band”

Conjecture: Breakdown of ETH, co-existing “hot spin-states” w/ “cold charge states”
Entropy scaling for 3 eigenstates

Full Entropy $S(x)$
Post-measurement entropies $S_{f/S}(x)$ $S_{C/S}(x)$

Ground state
Area-law

High entropy state from "charge-band"
Volume-law

High entropy state from "spin-band"
Volume-law

Possible "spin-band" of Quantum-disentangled states, area law charge entropy post-spin-measurement
Entropy scaling for 3 eigenstates

Full Entropy $S(x)$

Post-measurement entropies

$S_{S/C}(x)$

$L = 10, U = 4, V = 3$

$S_{C/S}(x)$

Area-law

Volume-law

Possible “spin-band” of Quantum-disentangled states, area law charge entropy post-spin-measurement
**Focus on 3 eigenstates**

10 site Hubbard chain
n.n. repulsion V=3

High entropy state,
charge-band

High entropy state
spin-band

Bound state

Full Entanglement entropy

\[ S(L/2) \]

Post-spin-measurement entropy of "charge-wf"

\[ S_{C/S}(L/2) \]

\[ L=10, U=4, V=3 \]
Summary

Possible “Hidden” Locality in (some) “Thermal” eigenstates

New diagnostic for many-body wf: **Partial-measurement**, followed by post-measurement entanglement entropy of un-measured dof

**Quantum Disentangled States**: Volume-law eigenstates w/ post-measurement area-law entanglement entropy – **“induced-locality”**

**Possible Breakdown of ETH** (“atomic” and “ionized” states co-exist at same energies)

Challenge:

Identify Hamiltonians which exhibit Quantum-disentangled eigenstates? (eg heavy/light particles)

Dynamics/transport in “Low-entanglement” “atomic” bands?

Other diagnostics to reveal hidden structure in volume-law entangled states?
Challenge:

- Identify Hamiltonians which exhibit Qual (eg heavy/light particles)

Dynamics/transport in “Low-entanglement”

Other diagnostics to reveal hidden structure
Focus on 3 eigenstates

- 0 site Hubbard chain
- U-n. repulsion V=3
- high entropy state, charge-band
- high entropy state spin-band
- ground state

Full Entanglement entropy \( S(L/2) \)

Post-spin-measurement entropy of "charge-wf" \( S_{c/s}(L/2) \)