Abstract: A variable speed of light (VSL) cosmology is developed with a spontaneous breaking of Lorentz invariance in the early universe. A non-minimal electromagnetic coupling to curvature and the resulting quantum electrodynamic vacuum polarization dispersive medium can produce $c >> c_0$ in the early universe, where $c_0$ is the measured speed of light today. Higher derivative curvature contributions to the effective gravitational action and quantum gravity vacuum polarization can produce a dispersive medium and a large increase in the speed of gravitational waves $c_g >> c_{g0}$ in the early universe, where $c_{g0}$ is the speed of gravitational waves today. The initial value problems of cosmology are solved: the horizon and flatness problems. The model predicts primordial scalar and tensor fluctuation spectral indices $n_s=0.96$ and $n_t=-0.04$, respectively. The BICEP2 observation of $r=0.2$ yields $r/n_t=-5$ which is close to the single-field inflationary consistency condition $r/n_t=-8$. 


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Primordial Perturbations

Quantum fluctuations...

"inflaton"

metric tensor

... imprinted onto cosmic scales

Inflation

Density perturbations

Primordial gravitational waves

\[ r \approx \frac{V[\phi]}{(4 \times 10^{16}\text{GeV})^4} \]

GUT-scale physics!?

J. Filippini - Primordial Gravitational Waves - May 16, 2014
based on maps shown by Bernard at ESLAB meeting

**Polarization Fraction**

**Apparent polarization fraction (p) at 353 GHz, 1° resolution**

Not CIB subtracted

\[ p = \frac{\sqrt{Q^2 + U^2}}{I} \]

p ranges from 0 to ~20%

Low p values in inner MW plane. Consistent with unpolarized CIB

Large p values in outer plane and intermediate latitudes

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Bernard J.P. ELSAB 2013 6
Newer corrected version of Planck polarization map. \( p \) = fraction of polarization. Polarization fraction due to dust (red regions) could be as much as 10 – 20%.
Forefront dust estimates compared to BICEP2 data

Is the BICEP2 signal foreground B-polarization contamination or is it a cosmological signal?
It shows the parameter space of inflationary models in the plane of the spectral index $n_s$ vs. the tensor-to-scalar ratio $r$. The yellow region is derived from Planck CMB temperature and WMAP polarization data, while the purple regions combine those with the BICEP2 data. Including BICEP gives a stronger constraint on the tensor modes, rather than a detection of $r=0$. (Mortensen and Seljak, 2014. See also Flauger, Hill and Spergel, 2014.)
The BICEP2 experimental result needs confirmation! We wait for Planck, STP, POLARBEAR, ACTpole and Keck Array and other data.

Foreground problem. **Potential foreground dust contamination.** Theoretical dust models used to determine dust contamination. Only 2 $\sigma$ elimination of dust problem by BICEP2 collaboration using theoretical dust models (focussed at one frequency 150 GHz).

However, if the result holds up with future experimental confirmation, it is a very important discovery. Confirms prediction of gravitational waves in GR. Reveals physics emanating from the big bang not hidden by the CMB surface of last scattering $\sim$ 380,000 yrs after the big bang.

The BICEP2 discovery can shed valuable information about which model of the very early universe solves the initial value problem of cosmology, and the mechanism for the growth of large structures.
2. Inflation

Pros: Inflation

Given that there is an initial condition for inflation to occur with \( a(t) \sim \exp(\text{H}t) \), then inflation models possess three key features:

(1) The horizon problem is solved. All events in spacetime can communicate causally allowing for the uniformity of the CMB Planck spectrum temperature.

(2) The flatness problem is solved. \( \Omega_0 \rightarrow 1 \) as the universe expands demanded by the Planck data to within a few percent.

(3) The wavelengths of tiny quantum seeds of matter fluctuations and gravitational waves are “stretched” by inflation to large values and can be observed in the CMB: \( \lambda_i \approx \exp(\text{H}t) \lambda_i \). Inflation produces an almost scale invariant, adiabatic Gaussian spectrum.

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Cons: Inflation

(1) The initial conditions in the universe are not ‘natural’ for inflation models to produce enough inflation e.g. > 60 e-folds of inflation. Gibbons and Turok showed that the probability of producing an inflationary phase is exponential small (Gibbons and Turok, 2008). Using thermodynamics, Penrose demonstrated the same result (Penrose, 1985).

(2) The choices of potentials $V(\phi)$ for the inflaton field are many and any cosmological data observed can be fitted by some potential and its derivatives (lack of predictability: Ljjas, Steinhardt and Loeb 2013, 2014).

(3) There is significant fine-tuning of parameters to make inflation models work. The inflaton potential $V(\phi)$ must be flat, allowing for the slow roll of the $\phi$ field down towards the true vacuum producing enough e-folds of inflation. The parameters of an inflation model must be fine-tuned to a value near $10^{-15}$ to explain the CMB amplitude $\Delta T/T = 10^{-5}$.

(4) Pockets of spacetime inflate slower than others producing a non-uniform expansion, destroying the prediction of homogeneity as the universe expands and leading to eternal inflation – a MULTIVERSE!

(Ljjas, Steinhardt, Loeb 2013, 2014; Guth, Kaiser and Nomura 2014.)
Multiverse

Scientific American 2011
“Yet some proponents of inflation who celebrated the BICEP2 announcement already insist that the theory is equally valid whether or not gravitational waves are detected. How is this possible?”

“Scanning over all possible bubbles in the multiverse, everything that can physically happen does happen an infinite number of times. No experiment can rule out a theory that allows for all possible outcomes. Hence, the paradigm of inflation is unfalsifiable…”

“Taking this into account, it is clear that the inflationary paradigm is fundamentally untestable, and hence scientifically meaningless.”

Paul Steinhardt  Nature | Column: World View, 3 June 2014
3. Alternatives to inflation

Alternative models which can explain initial the conditions in cosmology have been proposed:

(1) Ekpyrotic model (Khoury, Ovrut, Steinhardt and Turok
(2) 2001...) Bouncing models avoiding a singularity at t=0 (Brandenberger, Easson, Peter, others 2000...). Hybrid bouncing and ekpyrotic models (Cia, Brandenberger 2012-14)

(3) String gas cosmology (Brandenberger, others 2000...)

Any alternative model must satisfy the key requirements: (i) It solves the horizon and flatness problems, (ii) A mechanism exists to stretch initial primordial quantum matter fluctuations and gravitational waves to super-horizon scales. The fluctuations must be almost scale invariant, adiabatic and Gaussian, (iii) If the BICEP2 discovery of B-mode relic gravitational waves is confirmed, then the model must produce a large enough amplitude for gravitational waves in the CMB.

We adopt the action: \[ S = S_G + S_\psi + S_\phi + S_M. \]

where

\[ S_G = \frac{1}{\kappa} \int d^4x \sqrt{-g} (R + 2\Lambda) \quad \kappa = 16\pi G/c_0^4 \]

\[ S_\psi = \int d^4x \sqrt{-g} \left[ -\frac{1}{4} B^{\mu\nu} B_{\mu\nu} - W(\psi_\mu) - \psi_\mu J^\mu \right] \]

\[ S_\phi = \frac{1}{\kappa} \int d^4x \sqrt{-g} \left[ \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) - V(\phi) \right] \]

\( \psi_\mu \) is a vector field, \( B_{\mu\nu} = \partial_\mu \psi_\nu - \partial_\nu \psi_\mu \), \( W(\psi_\mu) \) is a potential and \( J^\mu \) is a matter current density. Moreover, \( \phi \) is a dimensionless scalar field and \( V(\phi) \) is a potential. \( S_M \) is a matter action and \( c_0 \) is the measured speed of light today. The basic premise is that in the very early universe for a short duration of time \( c >> c_0 \) and \( c_g >> c_0 \).
The energy-momentum tensor is: 

\[ T_{\mu\nu} = T_{M\mu\nu} + T_{\psi\mu\nu} + T_{\phi\mu\nu} \]

\[ \frac{1}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}} = -\frac{1}{2} T_{M\mu\nu}, \quad \frac{1}{\sqrt{-g}} \frac{\delta S_\psi}{\delta g^{\mu\nu}} = -\frac{1}{2} T_{\psi\mu\nu}, \quad \frac{1}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} = -\frac{1}{2} T_{\phi\mu\nu} \]

The variation of the action with respect to \( g^{\mu\nu}, \psi_\mu \) and \( \phi \) yields the field equations:

\[ G_{\mu\nu} = \kappa T_{\mu\nu} + \Lambda g_{\mu\nu} \]

\[ \nabla_\mu (B^{\mu\nu}) - \frac{\partial W(\psi_\mu)}{\partial \psi_\nu} = J^\nu \]

\[ g^{\mu\nu} \nabla_\mu \nabla_\nu \phi + \frac{\partial V(\phi)}{\partial \phi} = 0. \]

The energy-momentum tensor satisfies the conservation law:

\[ \nabla_\nu T^{\mu\nu} = 0. \]
As in inflation, the horizon problem is solved. The horizon scale is determined by

\[ d_H = c \alpha(t) \int_0^t \frac{dt'}{a(t')} . \]

Let us assume that at \( t = t_c \) we have \( c = c_0 \). Then, for \( t < t_c \) and \( c >> c_0 \) we get \( d_H \to \infty \) as \( c \to \infty \) and all points in the expanding spacetime for \( t < t_c \) will have been in causal communication.

Let us assume for simplicity \( \Lambda = 0 \). Then, the Friedmann equations are

\[ H^2 + \frac{K c^2}{a^2} = \frac{8\pi G c^2 \rho}{3c_0^2} \quad \frac{\ddot{a}}{a} = -\frac{4\pi G c^2}{3c_0^2} \left( \rho + \frac{3p}{c_0^2} \right) \]

where \( H = \dot{a}/a \), \( \rho = \rho_M + \rho_\Psi + \rho_\phi \).
VSL solves the flatness problem. We separate out the initial $\Omega_i - 1$ and the final $\Omega_0 - 1$ to give

$$\Omega_0 = 1 + \frac{c_i^2}{c_i^2} (\Omega_i - 1) \frac{a_i^2}{a_0^2}$$

When in an early phase:

$$c_i^2 \gg \frac{a_i^2}{a_0^2} (\Omega_i - 1)$$

then $\Omega_0 = 1$ to a high degree of accuracy.

Let us assume that $V(\phi) = 0$. The equation for the fluctuation “seed” field $\phi$ is

$$\ddot{\phi} + 3H \dot{\phi} = 0 \quad \text{with the solution:} \quad \dot{\phi} = \sqrt{12B} \left( \frac{a_*}{a} \right)^3$$

The Friedmann equation becomes:

$$H^2 + \frac{c^2 K}{a^2} = B^2 \left( \frac{a_*}{a} \right)^6$$

with the solution (neglecting the curvature for $c=c_0$) $a(t) = a_*(3Bt)^{1/3}$, $H(t) = \frac{1}{3t}$
The scalar field satisfies $\dot{\phi} \sim 1/a^3$ so that it has no observable effects in our present universe.

5. Electromagnetic and gravitational origin for $c >> c_0$ and $c_g >> c_{g0}$

Drummond and Hathrell (1980) applied generalized Maxwell equations to an FLRW spacetime, leading to a varying speed of light. They showed that photons propagate at a speed faster than $c_0$ when vacuum polarization in QED acts as a dispersive medium in the presence of tidal gravitational forces with a refractive index $n_{\text{refrac}}$.

The electromagnetic-gravity Lagrangian is

$$\mathcal{L}_{EM} = -\frac{1}{4} F^\mu_\nu F^\nu_\mu + \frac{1}{4} \kappa R F^\mu_\nu F^\nu_\mu + \frac{1}{2} \chi R^\mu_\rho F^\nu_\rho F^\nu_\mu + \frac{1}{4} \omega R^\mu_\nu_\rho_\sigma F^\nu_\rho F^\sigma_\nu + \zeta \nabla_\mu F^\mu_\nu \nabla_\sigma F^\sigma_\nu - j^\mu A^\mu$$

where $\kappa, \chi, \omega$ and $\zeta$ are constant coefficients.
Drummond and Hathrell derived the result

\[
k^2 = \frac{11 \xi^2}{c_0^2} \left( \frac{a^2 + K c_0^2}{a^2} - \frac{\ddot{a}}{a} \right) (k^\mu u_\mu)^2 = \frac{44 \xi^2}{c_0^2} \left[ \pi G \left( \rho + \frac{\rho}{c_s^2} \right) \right] (k^\mu u_\mu)^2
\]

\[
\xi^2/c_0^2 = 4.4 \times 10^{-47} \text{ sec}^2 \quad \xi^2 = \frac{\alpha}{\alpha_0 \pi} \lambda_c^2 \quad \alpha = e^2/\hbar c_0^2 \quad \lambda_c = h/m_e c_0
\]

For the radiation dominated phase \( w = 1/3, \ a \propto t^{1/2} \) we get

\[
c = \left( 1 - \frac{11}{2} \frac{t_*^2}{t^2} \right)^{-1/2} c_0 \quad t_* = \xi/c_0 = 6.6 \times 10^{-24} \text{ sec.}
\]

The speed of light \( c \to \infty \) as \( t \to \sqrt{11/2} t_* \). This occurs for

\[
t \gtrsim 1.5 \times 10^{-23} \text{ sec}
\]

This corresponds to an energy \( \sim 2 \times 10^5 \text{ TeV} \). The maximum energy of 14 TeV at the LHC corresponds to \( 4 \times 10^{-15} \text{ sec} \) after the big bang. A speed of light \( c \gtrsim 10^{30} c_0 \) will solve the horizon and flatness problems and stretch tiny quantum fluctuations to super-horizon size.
By adopting a coupling of the graviton to higher derivative curvature and the resulting tidal forces on the graviton, an induced quantum gravity (QG) vacuum polarization produces a dispersive medium with a refractive index and a large increase in the speed of gravitational waves. In the radiation dominated era \( \rho \propto 1/t^2 \) and near the Planck density \( \rho_{PL} = c_0^5/hG^2 = 5.2 \times 10^{93} \text{ g cm}^{-3} \) and the Planck time \( t_{PL} = 5.4 \times 10^{-43} \text{ sec} \) we get

\[
c_g = \left(1 - \frac{\zeta_g^2}{t^2}\right)^{-1/2} c_{g0}
\]

where \( \zeta_g \) is a constant determined by a QG vacuum polarization calculation, using the gravitational fine structure constant \( \alpha_g = Gm^2/hc_0 \). Near the Planck energy \( mc_0^2 = (hc_0^5/G)^{1/2} \) we will have \( \alpha_g \sim O(1) \) and QG vacuum polarization can be comparable in size to the QED vacuum polarization calculation. For \( t \gtrsim \zeta_g \) we can have a large increase in the speed of gravitational waves \( c_g \gtrsim 10^{30}c_{g0} \). This will stretch the wavelengths of gravitational waves \( \lambda_g = c_g/\nu_g \), permitting them to be detected by B-polarization experiments.
6. Primordial fluctuations and gravitational waves

In inflationary models the quantum fluctuations with amplitude $\delta\phi \sim H/2\pi$ are stretched by rapid expansion:

$$\lambda_f \propto a(t) \propto \exp(\text{H}t)$$

where $\text{H}$ is approximately constant. Short-wavelength fluctuations are quickly redshifted until their wavelengths are larger than the horizon size $R_H$.

In our VSL model, the wavelengths of the field fluctuations are stretched by the short duration large increase in the speed of light $c$:

$$\lambda_f = \frac{c}{\nu_f}$$

where $\nu_f$ is the frequency.
The redshifted wavelengths of the short-wave length quantum fluctuations become larger than the horizon size and are “frozen” in as classical scale invariant fluctuations. The amplitudes of quantum modes are calculated at the horizon crossing, when the wavelength of a mode is equal to \( a/k = c_0/H \).

For an inflationary epoch of 60 e-folds of inflation of the cosmic scale \( a \), the initial wave lengths \( \lambda_i \) are stretch by

\[
\lambda_0 \sim \exp(\mathcal{N})\lambda_i
\]

where \( \mathcal{N} \) is the number of e-folds.

In the VSL model, the initial fluctuations are stretch by an equivalent amount:

\[
\lambda_0 \sim Q\lambda_i
\]

where \( Q \gtrsim 10^{30} \).
The fluctuations are of two kinds: scalar matter quantum fluctuations and gravitational waves. The scalar fluctuations $\delta \phi(t, x)$ satisfy the minimally-coupled KG equation where we take the potential $V(\phi) = 0$.

$$\frac{d^2 \delta \phi_k}{dt^2} + 3H \frac{d \delta \phi_k}{dt} + \frac{c_0^2 k^2}{a^2} \delta \phi_k = 0, \quad \delta \phi(x) = (2\pi)^{-2/3} \int d^3k \exp[-ik \cdot x] \delta \phi_k$$

Substituting the solution: $a(t) = a_*(3Bt)^{1/3}$, $H(t) = \frac{1}{3t}$ we get

$$\frac{d^2 \delta \phi_k}{dt^2} + \frac{1}{t} \frac{d \delta \phi_k}{dt} + \frac{c_0^2 k^2}{a_*^2 (3Bt)^{2/3}} \delta \phi_k = 0 \quad \text{with the solution}$$

$$\delta \phi_k(y) = A_1 J_0(y) + A_2 Y_0(y) \quad y = \frac{c_0 k}{2a_* H_\ell} \left( \frac{a}{a_*} \right)^2 \quad H_\ell = (3B)^{2/3}/3a_*$$

We define $H_\ell = c_0/\ell_0$ where $\ell_0$ is the length at which the normalized plane wave solution is in its ground state:

$$\delta \phi_k(t_k) = \left( \frac{\kappa c_0^2 h}{(2\pi a_k)^3 \omega_k} \right)^{1/2} \cos(\omega_k t_k - k \cdot x + \delta) \quad \omega_k = \frac{c_0 k}{a_k}$$

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The scale at which the fluctuation mode exists is $a_k = k\ell_0$ for which $y_k = \gamma k^3$, where $\gamma = \ell^3_0/2a^3_\ast$. Keeping only the dominant contribution as $y \to 0$ we get

$$\delta\phi_k \approx \left( \frac{9k\ell_0^2\hbar}{(2\pi a_k)^332\omega_k} \right)^{1/2} \cos(\omega_k t_k - k \cdot x + \delta) \ln(y_k) J_0(y_k)$$

The scalar field fluctuation spectrum is for the Planck length:

$$\ell_{\text{PL}} = (G\hbar/c_0^3)^{1/2} \sim 10^{-33} \text{ cm.}$$

$$P_{\delta\phi} = \frac{9}{2(2\pi)^3} \left( \frac{\ell_{\text{PL}}^2}{\ell_0^2} \right) \ln^2(y_k) \quad P_R(k) = \left( \frac{H^2}{\dot{\phi}^2} \right) P_{\delta\phi} = \frac{3}{8(2\pi)^3} \left( \frac{\ell_{\text{PL}}^2}{\ell_0^2} \right) \ln^2(y_k)$$

$$P_R(k) = 2\pi^2 A_s \left( \frac{k}{k_\ast} \right)^{n_s - 1}$$

The Bessel function $Y_0(y_k)$ is logarithmically divergent as $y_k \to 0$.

The scalar mode spectral index (tilt) is given by

$$n_s = 1 + \frac{d}{d \ln k} \frac{P_R}{\ln(y_k)} \quad n_s = 1 + \frac{6}{\ln(y_k)} \quad \alpha_s = \frac{dn_s}{d \ln k} \quad \alpha_s = -\frac{1}{2} (1 - n_s)^2$$
In the inflationary models the derivation of the power spectrum and the spectral index depends sensitively on the shape of the potential $V(\phi_{\text{inflaton}})$ and its derivatives. The condition of a slow-roll potential is required to produce enough e-folds of inflation. This is not the case in our VSL model derivation of the power spectrum and the spectral index. It does not depend sensitively on the shape of the potential $V(\phi)$. This reduces the VSL model dependence and associated fine-tuning problems.

We now turn to the spectrum of gravitational waves. The tensor perturbation can be expressed as

$$ds^2 = a^2[c^2d\eta^2 - (\delta_{ij} + 2h_{ij})dx^idx^j]$$

$$h_{ij} = \int \frac{d^3k}{(2\pi)^3/2} \sum_{\lambda=1}^{2} \psi_{k,\lambda}e_{ij}(k, \lambda \exp(ik \cdot x))$$

$$\langle \psi_{k,\lambda}, \psi_{1,\lambda}^{*} \rangle = 2\pi^2 P_{\delta\psi} \delta(k - 1)$$

$$h_{+,\times}^{\text{tensor}} = \phi_{+,\times} = \text{scalar}$$

The transverse and longitudinal modes of polarization obey the scalar wave equation at the horizon and super-horizon:

$$\frac{d^2 \phi_{+,\times,k}}{dt^2} + 3H \frac{d\phi_{+,\times,k}}{dt} + \frac{c_s^2 k^2}{a^2} \phi_{+,\times,k} = 0$$
If we now adopt the BICEP2 observed value for the $r = \text{tensor/scalar ratio}$, $r = 0.2^{+0.07}_{-0.05}$ (for zero or small dust foreground), then we get $n_t/r = -5$ which is close to the single-field inflationary consistency condition $r/n_t = -8$ determined by the slow-roll parameter $\epsilon = -\dot{H}/H^2$, which is related to the equation of state $p = \epsilon - \rho$. We have $r = 16\epsilon$ and $n_t = -2\epsilon$ giving $r/n_t = -8$, which is satisfied irrespective of the form of the single-field inflationary potential.

7. Conclusions

Our VSL model solves the horizon and flatness problems.

It can account for the observed fluctuations in the CMB as stretched superhorizon quantum fluctuations that are almost scale invariant, adiabatic and Gaussian. It can also stretch relic gravitational wave wavelengths to an observable size in the CMB.

It can predict power spectra and spectral indices for scalar quantum fluctuations and gravitational waves. The gravitational tensor spectral index is red $n_t = -0.04$ and we get for the preliminary BICEP2 result $r/n_t = -5$, which is close to the single-field self-consistency condition $r/n_t = -8$.  

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Since the ordered phase in the spontaneous symmetry breaking of Lorentz invariance is at a much lower entropy than the symmetry restored disordered phase and due to the existence of a domain determined by the direction of the vev, $< 0 | \psi_\mu | 0 >$, a natural explanation is given for the cosmological arrow of time and the origin of the second law of thermodynamics. The ordered state of low entropy in the symmetry broken phase with $c >> c_0$, becomes a state of high entropy in the symmetry restored, disordered phase with $c = c_0$ (JM (1992-3)).

The spontaneous symmetry breaking of the gravitational vacuum leads to a manifold with the structure $O(3) \times \mathbb{R}$, in which time appears as an absolute external time parameter. The vev $< 0 | \psi_\mu | 0 >$ points in a chosen direction of time to break the symmetry of the vacuum creating an arrow of time.
The power spectra results are not sensitive to the shape of a potential \( V(\phi) \) and its derivatives, as in the case of inflationary models. The flatness problem solution produces a uniformity as the universe expands avoiding the non-uniformity generated by chaotic or eternal inflation.

The lack of significant fine-tuning in a VSL model can avoid the need for a multiverse scenario as seems inevitable in standard inflationary models.

Independent confirmation of the B-polarization observation of relic gravitational waves is needed. If the BICEP2 result holds up this will be a very important discovery in cosmology. Hopefully, it can help to distinguish standard inflation models from alternative models such as a VSL model.

A measurement of the tensor spectral index \( n_s \) is needed to verify inflation models, the VSL model or any other competing model of early universe cosmology.