Title: Equivalence of wave-particle duality to entropic uncertainty

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Abstract: <span>Interferometers capture a basic mystery of quantum mechanics: a single particle can exhibit wave behavior, yet that wave behavior disappears when one tries to determine the particle's path inside the interferometer. This idea has been formulated quantitatively as an inequality, e.g., by Englert and Jaeger, Shimony, and Vaidman, which upper bounds the sum of the interference visibility and the path distinguishability. Such wave-particle duality relations (WPDRs) are often thought to be conceptually inequivalent to Heisenberg's uncertainty principle, although this has been debated. Here we show that WPDRs correspond precisely to a modern formulation of the uncertainty principle in terms of entropies, namely the min- and max-entropies. This observation unifies two fundamental concepts in quantum mechanics. Furthermore, it leads to a robust framework for deriving novel WPDRs by applying entropic uncertainty relations to interferometric models (arXiv reference: 1403.4687).</span>
Equivalence of wave-particle duality to entropic uncertainty

*arXiv:1403.4687*

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Equivalence of wave-particle duality
to entropic uncertainty

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TU Delft
\[ f_2(\tilde{a}_1 - n b_2) \]

\[ \frac{w}{2\pi} \int b_2 da_1 \]

\[ \frac{-\pi d a_1}{2\pi} + \delta(S) - 0 \]

\[ -\frac{m}{2\pi} d b_2 + \delta(y) \]

\[ T = \epsilon k_\psi b_2 \]

\[ W_T = C \]

\[ \frac{\epsilon}{\eta} \]
Wave-particle duality

The transition (from no interference to interference) can even be seen with single electrons.

*Data from: “Controlled double-slit electron diffraction” Bach et al. NJP (2013)*
Wave-particle duality

The transition (from no interference to interference) can even be seen with single electrons.

*Data from* “*Controlled double-slit electron diffraction*” Bach et al. *NJP* (2013)

The great mystery:
Each kind of thing (bullet, electron, bacteria, ...) has the ability to exhibit wave behavior, i.e., produce interference. Likewise, each can exhibit particle behavior, i.e., have a well-defined path. But the two behaviors compete – you either get one or the other.
Wave-particle duality: big molecules
Wave-particle duality

While the behaviors are mysterious, we can get intuition for how they compete.
Wave-particle duality

*getting quantitative*

Simplification of double-slit:

Two-path interferometer for single photons (named after Mach and Zehnder).
Wave-particle duality

*getting quantitative*

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Fringe visibility

$$\mathcal{V} := \frac{p_{D_0}^{\text{max}} - p_{D_0}^{\text{min}}}{p_{D_0}^{\text{max}} + p_{D_0}^{\text{min}}}$$
Wave-particle duality

getting quantitative

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Two-path interferometer for single photons (named after Mach and Zehnder).

Fringe visibility

\[ \mathcal{V} := \frac{p^{D_0}_{\text{max}} - p^{D_0}_{\text{min}}}{p^{D_0}_{\text{max}} + p^{D_0}_{\text{min}}} \]

Path predictability (e.g. asymmetric BS₁)

\[ Z = \{ |0\rangle, |1\rangle \} \]
\[ \mathcal{P} := 2p_{\text{guess}}(Z) - 1 \]

probability of guessing Z correctly
Wave-particle duality

*getting quantitative*

Wooters, Zurek (1979)
Greenberger, Yasin (1988)
Englert (1996)

Wave-particle duality relation (WPDR):

\[ P^2 + V^2 \leq 1 \]

Full particle behavior → No wave behavior
Full wave behavior → No particle behavior

\[ V := \frac{p_{D_0}^{max} - p_{D_0}^{min}}{p_{D_0}^{max} + p_{D_0}^{min}} \]

Path predictability (e.g. asymmetric BS$_1$)

\[ Z = \{ |0\rangle, |1\rangle \} \]

\[ P := 2p_{guess}(Z) - 1 \]

*probability of guessing Z correctly*
Wave-particle duality

*getting quantitative*

Englert (1996)

Let $E$ be a (partial) which-path detector. $E$ could be gas of atoms whose internal state is sensitive to presence of photon.

**Stronger WPDR:**

$$D^2 + \mathcal{V}^2 \leq 1$$

Fringe visibility

$$\mathcal{V} := \frac{p_{D_0}^{D_0 \max} - p_{D_0}^{D_0 \min}}{p_{max}^{D_0} + p_{min}^{D_0}}$$

Path distinguishability

$$D := 2p_{guess}(Z|E) - 1$$

*probability of guessing $Z$ correctly given $E$ (i.e., given optimal measurement on $E$)*
Wave-particle duality

*getting quantitative*

Englert (1996)

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$$D^2 + V^2 \leq 1$$

**Fringe visibility**

$$V := \frac{p_{D_0}^{D_0_{\text{max}}} - p_{D_0}^{D_0_{\text{min}}}}{p_{D_0}^{D_0_{\text{max}}} + p_{D_0}^{D_0_{\text{min}}}}$$

**Path distinguishability**

$$D := 2p_{\text{guess}}(Z|E) - 1$$

*probability of guessing Z correctly given E (i.e., given optimal measurement on E)*
WPDRs

*Where do they come from?*

\[ P^2 + V^2 \leq 1 \]

\[ D^2 + V^2 \leq 1 \]

Is wave-particle duality a fundamental principle of quantum mechanics, or is it a corollary of some other principle?
WPDRs

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Is wave-particle duality a fundamental principle of quantum mechanics, or is it a corollary of some other principle?

“... Does not make use of Heisenberg’s uncertainty principle in any form”

Is it a consequence of position/momentum uncertainty principle?

\[ \Delta q \Delta p \geq \hbar / 2 \]
WPDRs

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\[ \mathcal{P}^2 + \mathcal{V}^2 \leq 1 \]
\[ \mathcal{D}^2 + \mathcal{V}^2 \leq 1 \]

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Is it a consequence of position/momentum uncertainty principle?

\[ \Delta q \Delta p \geq \hbar/2 \]

*This was intensely debated in 1990’s:*


*Looks to be inconclusive / still open to debate*
WPDRs

Where do they come from?

Consider: \( P^2 + V^2 \leq 1 \)
WPDRs

Where do they come from?

Consider: \( \mathcal{P}^2 + \mathcal{V}^2 \leq 1 \)

Several authors showed that this WPDR is equivalent to Robertson’s uncertainty relation for particular qubit observables:

\[ \Delta X \Delta Z \geq \frac{1}{2} \left| \langle \psi | [X, Z] | \psi \rangle \right| \]

**Qubit observables:**

\[ \hat{P} = \sigma_z \]
\[ \hat{V}_\phi = (\cos \phi) \sigma_x + (\sin \phi) \sigma_y \]

- Busch and Shilladay (2006)
- Bjork et al. (1999)
- Durr and Rempe (2000)
- Bosyk et al. (2013)
WPDRs

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**Qubit observables:**

\[ \hat{P} = \sigma_z \]

\[ \hat{V}_\phi = (\cos \phi) \sigma_x + (\sin \phi) \sigma_y \]

\[ (\Delta \hat{P})^2 = 1 - P^2 \]

\[ (\Delta \hat{V}_\phi)^2 = 1 - V^2 \cos^2(\theta - \phi) \]

Plugging into Robertson’s relation gives:

\[ (1 - P^2)[1 - V^2 \cos^2(\theta - \phi)] \geq P^2 V^2 \cos^2(\theta - \phi) + V^2 \sin^2(\theta - \phi) \]

Busch and Shilladay (2006)
Bjork et al. (1999)
Durr and Rempe (2000)
Bosyk et al. (2013)
WPDRs

Where do they come from?

So we have

\[ P^2 + V^2 \leq 1 \quad \iff \quad \Delta X \Delta Z \geq \frac{1}{2} |\langle \psi | [X, Z] | \psi \rangle| \]

\[ D^2 + V^2 \leq 1 \quad \iff \quad ?????? \]
WPDRs

*Where do they come from?*

So we have

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\[ D^2 + V^2 \leq 1 \quad \iff \quad ?????? \]

Note that distinguishability involves conditioning on system $E$. This is not so natural for standard deviation, but is quite natural for *entropies*. Could the $D$-$V$ relation be related to the *entropic* uncertainty principle?

\[ D := 2p_{\text{guess}}(Z | E) - 1 \]
WPDRs

Consider again: \( \mathcal{P}^2 + \mathcal{V}^2 \leq 1 \)

Bosyk et al. [Phys. Scr. (2013)] considered entropic uncertainty relations (EURs), of the form:

\[
H_q(P) + H_q(V) \geq \mathcal{B}_q
\]

for Renyi entropies:

\[
H_q(P) = \frac{1}{1-q} \ln \left[ \left( \frac{1+P}{2} \right)^q + \left( \frac{1-P}{2} \right)^q \right]
\]

\[
H_q(V) = \frac{1}{1-q} \ln \left[ \left( \frac{1+V}{2} \right)^q + \left( \frac{1-V}{2} \right)^q \right]
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They argue that such EURs are inequivalent to the P-V relation!
WPDRs

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They argue that such EURs are inequivalent to the P-V relation!

But Maassen & Uffink (1988) proved an EUR that involves different \( q \)'s, for example,

\[
H_\infty(P) + H_{1/2}(V) \geq 1
\]

Our first result: This EUR is equivalent to the P-V relation!!!!
WPDRs

\[ H_\infty(P) + H_{1/2}(V) \geq 1 \]

INVITATION: Plug these formulas in to obtain \( P-V \) relation

\[ H_\infty(P) = 1 - \log(1 + P) \]
\[ H_{1/2}(V) = \log \left( 1 + \sqrt{1 - V^2} \right) \]
WPDRs

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So we have

\[ P^2 + V^2 \leq 1 \iff H_{\infty}(P) + H_{1/2}(V) \geq 1 \]
\[ D^2 + V^2 \leq 1 \iff ??????? \]
WPDRs

\[ H_\infty(P) + H_{1/2}(V) \geq 1 \]

INVITATION: Plug these formulas in to obtain P-V relation

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\[ H_{1/2}(V) = \log \left(1 + \sqrt{1 - V^2}\right) \]

So we have

\[ P^2 + V^2 \leq 1 \quad \iff \quad H_\infty(P) + H_{1/2}(V) \geq 1 \]
\[ D^2 + V^2 \leq 1 \quad \iff \quad \text{??????} \]

APOLOGY: In what follows, I will switch notation:

\[ H_\infty(P) \rightarrow H_{\min}(Z) \quad H_{1/2}(V) \rightarrow H_{\max}(W) \]
Goals of our work

1.) Unify a vast literature on WPDRs. Many complicated versions of WPDRs have been formulated, for exotic scenarios involving quantum beam splitters or quantum erasure or for alternative interferometers like the double slit. We show that all these WPDRs correspond to special cases of a single inequality.

2.) Show that WPDRs come from the uncertainty relation for the min- and max-entropies. Hence we unify the entropic uncertainty principle with the wave-particle duality principle.

3.) Provide a general, robust framework for discussing WPDRs and deriving novel WPDRs. We illustrate this by deriving a novel WPDR for a quantum beam splitter.
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2.) Show that WPDRs come from the uncertainty relation for the min- and max-entropies. Hence we unify the entropic uncertainty principle with the wave-particle duality principle.

3.) Provide a general, robust framework for discussing WPDRs and deriving novel WPDRs. We illustrate this by deriving a novel WPDR for a quantum beam splitter.

4.) Uncertainty relations can be applied in two different ways. We emphasize the distinction between preparation and measurement WPDRs.
Main Result

For a binary interferometer (i.e., two interfering paths), we identify particle and wave behaviors with the knowledge of complementary qubit observables:

which-path:  \[ Z = \{ |0\rangle, |1\rangle \} \]
which-phase:  \[ W = \{ |w_\pm\rangle \}, \quad |w_\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm e^{i\phi_0} |1\rangle) \]
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lack of particle behavior: \( H_{\text{min}}(Z|E_1) \)

lack of wave behavior: \( \min_{W \in XY} H_{\text{max}}(W|E_2) \)

\( E_1, E_2 \): some other quantum systems that help to reveal the behavior
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\( E_1, E_2 \) : some other quantum systems that help to reveal the behavior

**Our general WPDR:**

\[
H_{\min}(Z|E_1) + \min_{W \in XY} H_{\max}(W|E_2) \geq 1
\]

**Majority of WPDRs in literature are special cases of this relation.**
Revisiting Distinguishability-Visibility tradeoff

Recall scenario: photon interacts with $E$ inside interferometer
Revisiting Distinguishability-Visibility tradeoff

Recall scenario: photon interacts with $E$ inside interferometer

Apply uncertainty relation at time $t_2$

$$H_{\text{min}}(Z)_{t_2} + \min_{W \in XY} H_{\text{max}}(W)_{t_2} \geq 1$$

$$P^2 + V^2 \leq 1$$
Revisiting Distinguishability-Visibility tradeoff

Recall scenario: photon interacts with $E$ inside interferometer

Apply uncertainty relation at time $t_2$

\[
H_{\min}(Z)_{t_2} + \min_{W \in XY} H_{\max}(W)_{t_2} \geq 1
\]

\[
\mathcal{P}^2 + \mathcal{V}^2 \leq 1
\]

\[
H_{\min}(Z|E)_{t_2} + \min_{W \in XY} H_{\max}(W)_{t_2} \geq 1
\]

\[
\mathcal{D}^2 + \mathcal{V}^2 \leq 1
\]
Operational meaning of entropies

Quantum key distribution

\[ H_{\text{min}}(Z|E_1) + \min_{W \in X \cup Y} H_{\text{max}}(W|E_2) \geq 1 \]

Used to prove security of QKD

- \( H_{\text{min}} \): randomness extraction
- \( H_{\text{max}} \): data compression
Operational meaning of entropies

Quantum key distribution

\[ H_{\min}(Z|E_1) + \min_{W \in XY} H_{\max}(W|E_2) \geq 1 \]

*Used to prove security of QKD*

- \(H_{\min}\): randomness extraction
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Operational meaning of entropies

**Quantum key distribution**

$$H_{\min}(Z|E_1) + \min_{W \in X Y} H_{\max}(W|E_2) \geq 1$$

*Used to prove security of QKD*

- $H_{\min}$: randomness extraction
- $H_{\max}$: data compression

**Guessing games**

classical-quantum state $\rho_{XB}$

$$H_{\min}(X|B) = - \log p_{\text{guess}}(X|B)$$

*When X is binary, we show that:*

$$H_{\max}(X|B) \leq \log \left(1 + \sqrt{1 - (2p_{\text{guess}}(X|B) - 1)^2}\right)$$
Complementary Guessing Games

Consider two games

**Game #1**: We ask Alice to guess which path the quanton takes, given that she has access to $E_1$. 

![Diagram of Complementary Guessing Games]
Complementary Guessing Games

Consider two games

**Game #1**: We ask Alice to guess which path the quanton takes, given that she has access to $E_1$. 

Which path ($Z$)?

Which phase ($W$)?
Complementary Guessing Games

Consider two games

**Game #1**: We ask Alice to guess which path the quanton takes, given that she has access to $E_1$.

**Game #2**: We ask Alice to guess which phase was applied to the quanton (0 or $\pi$), given that she has access to $E_2$.

*Our WPDR says that she cannot win both games.*

$$H_{\text{min}}(Z|E_1) + \min_{W \in XY} H_{\text{max}}(W|E_2) \geq 1$$
Complementary Guessing Games

Consider two games

**Game #1**: We ask Alice to guess which path the quanton takes, given that she has access to $E_1$.

**Game #2**: We ask Alice to guess which phase was applied to the quanton (0 or $\pi$), given that she has access to $E_2$.

*Our WPDR says that she cannot win both games.*

$$H_{\min}(Z \mid E_1) + \min_{W \in XY} H_{\max}(W \mid E_2) \geq 1$$

Related to winning probability for Game #1

Related to winning probability for Game #2
Complementary Guessing Games

Consider two games

**Game #1**: We ask Alice to guess which path the quanton takes, given that she has access to $E_1$.

**Game #2**: We ask Alice to guess which phase was applied to the quanton (0 or $\pi$), given that she has access to $E_2$.

$$H_{\min}(Z|E_1) + \min_{W \in XY} H_{\max}(W|E_2) \geq 1$$

**Generic measures**

$$D_g := 2p_{\text{guess}}(Z|E_1) - 1,$$

$$V_g := \max_{W \in XY}[2p_{\text{guess}}(W|E_2) - 1]$$

Rearrange into traditional WPDR form

$$D_g^2 + V_g^2 \leq 1$$
Preparation Uncertainty

Remark
We applied the preparation uncertainty relation at time $t_2$ to derive the WPDR. Preparation uncertainty restricts one’s ability to predict future measurements.
Preparation Uncertainty

Remark
We applied the preparation uncertainty relation at time $t_2$ to derive the WPDR. Preparation uncertainty restricts one’s ability to predict future measurements.

To measure $P$ or $D$, one removes the second beam splitter (BS$_2$) and tries to predict which detector clicks.

$$D := 2p_{\text{guess}}(Z|E)_{t_2} - 1$$
Measurement Uncertainty

Preparation uncertainty
- Fixed input state; complementary output measurements

Measurement uncertainty
- Fixed output measurement; complementary input ensembles:
\[ Z_i = \{|0\rangle, |1\rangle\} \quad W_i = \{|w_\pm\rangle\} \quad |w_\pm\rangle = (|0\rangle \pm e^{i\phi}|1\rangle) / \sqrt{2} \]
Measurement Uncertainty

Preparation uncertainty
- Fixed input state; complementary output measurements

Measurement uncertainty
- Fixed output measurement; complementary input ensembles:

$$Z_i = \{ |0\rangle, |1\rangle \} \quad W_i = \{ |w_\pm\rangle \} \quad |w_\pm\rangle = (|0\rangle \pm e^{i\phi}|1\rangle)/\sqrt{2}$$

Guessing game
The $Z_i$ states are generated by Bob flipping a coin and blocking either the top or bottom arm depending on flip outcome. Alice tries to guess Bob’s coin flip, given $E$ and given which detector clicks, denoted by $C$.

$$D_i := 2p_{\text{guess}}(Z_i|EC') - 1$$
Measurement Uncertainty

Preparation uncertainty
- Fixed input state; complementary output measurements

Measurement uncertainty
- Fixed output measurement; complementary input ensembles:
  \[ Z_i = \{|0\rangle, |1\rangle\} \quad W_i = \{|w_\pm\rangle\} \quad w_\pm = (|0\rangle \pm e^{i\phi}|1\rangle)/\sqrt{2} \]

Retrodiction

Guessing game
The \( Z_i \) states are generated by Bob flipping a coin and blocking either the top or bottom arm depending on flip outcome. Alice tries to guess Bob’s coin flip, given \( E \) and given which detector clicks, denoted by \( C \).

\[ D_i := 2p_{\text{guess}}(Z_i|EC') - 1 \]
Preparation vs. Measurement
Uncertainty

Output distinguishability
\[
\mathcal{D} := 2p_{\text{guess}}(Z|E)_{t_2} - 1
\]

Output visibility
\[
\mathcal{V} := \frac{p_{D_0}^{D_0 \text{ max}} - p_{D_0}^{D_0 \text{ min}}}{p_{D_0}^{D_0 \text{ max}} + p_{D_0}^{D_0 \text{ min}}}
\]

Input distinguishability
\[
\mathcal{D}_i := 2p_{\text{guess}}(Z_i|EC') - 1
\]

Input visibility
\[
\mathcal{V}_i := \max_{W \in XY} \left( p_{W+|D_0} - p_{W-|D_0} \right)
\]
Preparation vs. Measurement Uncertainty

Output distinguishability

\[ D := 2p_{\text{guess}}(Z|E)_{t_2} - 1 \]

Output visibility

\[ V := \frac{p_{D_0}^{D_0} - p_{D_0}^{D_0}}{p_{\text{max}}^{D_0} + p_{\text{min}}^{D_0}} \]

Input distinguishability

\[ D_i := 2p_{\text{guess}}(Z_i|EC) - 1 \]

Input visibility

\[ V_i := \max_{W \in XY} (p_{W+|D_0} - p_{W-|D_0}) \]

“Preparation” WPDR

\[ D^2 + V^2 \leq 1 \]

Addresses question of how well Alice can prepare a state with low uncertainty in \( Z \) and \( W \).

“Measurement” WPDR

\[ D_i^2 + V_i^2 \leq 1 \]

Addresses question of how well Alice can jointly measure Bob’s \( Z \) and \( W \) observables.
Novel WPDRs
Preparation vs. Measurement Uncertainty

Output distinguishability
\[ D := 2p_{\text{guess}}(Z|E)_{t_2} - 1 \]

Output visibility
\[ \nu := \frac{p_{D_0}^{\text{max}} - p_{D_0}^{\text{min}}}{p_{D_0}^{\text{max}} + p_{D_0}^{\text{min}}} \]

Input distinguishability
\[ D_i := 2p_{\text{guess}}(Z_i|EC') - 1 \]

Input visibility
\[ \nu_i := \max_{W \in XY} \left( p_{W+|D_0} - p_{W-|D_0} \right) \]
Preparation vs. Measurement
Uncertainty

Output distinguishability
\[ D := 2p_{\text{guess}}(Z|E)_{t_2} - 1 \]

Output visibility
\[ V := \frac{p^{D_0}_{\text{max}} - p^{D_0}_{\text{min}}}{p^{D_0}_{\text{max}} + p^{D_0}_{\text{min}}} \]

“Preparation” WPDR
\[ D^2 + V^2 \leq 1 \]
Addresses question of how well Alice can prepare a state with low uncertainty in Z and W.

Input distinguishability
\[ D_i := 2p_{\text{guess}}(Z_i|EC) - 1 \]

Input visibility
\[ V_i := \max_{W \in XY} \left( p_{w+|D_0} - p_{w-|D_0} \right) \]

“Measurement” WPDR
\[ D_i^2 + V_i^2 \leq 1 \]
Addresses question of how well Alice can jointly measure Bob’s Z and W observables
Preparation vs. Measurement Uncertainty

Output distinguishability
\[ D := 2p_{\text{guess}}(Z|E)_{t_2} - 1 \]

Output visibility
\[ V := \frac{p_{D_0}^{D_0} - p_{D_0}^{D_0}}{p_{\max}^{D_0} + p_{\min}^{D_0}} \]

Input distinguishability
\[ D_i := 2p_{\text{guess}}(Z_i|EC) - 1 \]

Input visibility
\[ V_i := \max_{W \in XY} \left( p_{w+|D_0} - p_{w-|D_0} \right) \]

"Preparation" WPDR
\[ D^2 + V^2 \leq 1 \]
Addresses question of how well Alice can prepare a state with low uncertainty in Z and W.

"Measurement" WPDR
\[ D_i^2 + V_i^2 \leq 1 \]
Addresses question of how well Alice can jointly measure Bob's Z and W observables.
Example: quantum beam splitter
Example: quantum beam splitter

Feeding in a polarization superposition means that BS₂ is in a superposition of “absent” and “present”.

\[ \rho_P^{(2)} = |\psi_P^{(2)}\rangle\langle\psi_P^{(2)}| \]

\[ |\psi_P^{(2)}\rangle = \cos \alpha |H\rangle + \sin \alpha |V\rangle \]
Example: quantum beam splitter

WPDR was experimentally tested:

$$D_i^2 + V^2 \leq 1$$
Example: quantum beam splitter

WPDR was experimentally tested:

\[ D_i^2 + V^2 \leq 1 \]
Example: quantum beam splitter

WPDR was experimentally tested:

\[ D_i^2 + \nu^2 \leq 1 \]

This relation is untight!
Preparation vs. Measurement Uncertainty

Output distinguishability
\[ D := 2p_{\text{guess}}(Z|E)_{t_2} - 1 \]

Output visibility
\[ V := \frac{p_{D_0}^{\text{max}} - p_{D_0}^{\text{min}}}{p_{D_0}^{\text{max}} + p_{D_0}^{\text{min}}} \]

“Preparation” WPDR
\[ D^2 + V^2 \leq 1 \]
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\[ D_i := 2p_{\text{guess}}(Z_i|EC) - 1 \]

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\[ V_i := \max_{W \in XY} (p_{w+|D_0} - p_{w-|D_0}) \]

“Measurement” WPDR
\[ D_i^2 + V_i^2 \leq 1 \]
Addresses question of how well Alice can jointly measure Bob’s Z and W observables.
Example: quantum beam splitter

WPDR was experimentally tested:

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\[ D_i^2 + V^2 \leq 1 \]

Our framework easily provides a tight relation that captures beam splitter’s coherence. We condition distinguishability on the final polarization.

\[ (D_i^P)^2 + V^2 \leq 1 \]
Binary interferometers

Two interfering paths

Double Slit

Which position ($W$)?

Which slit ($Z$)?

|0⟩

|1⟩

$E_1$

$E_2$

$\varepsilon$

$L$

source
Binary interferometers

Two interfering paths

Double Slit

Which position (W)?

Which slit (Z)?

source

|0⟩

|1⟩

L

E₁

E₂

y
Binary interferometers

Two interfering paths

Double Slit

Which position (W)?
Which slit (Z)?

Franson

Which path, (LL) or (SS)?
Which phase, $\phi = \phi_A + \phi_B$?
Final Remarks

- WPDRs are EURs in disguise.
  Namely, the EUR for the min- and max-entropies applied to qubits.
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