Abstract: In this talk we will discuss the relation between the incompatibility of quantum measurements and quantum nonlocality. We show that any set of measurements that is not jointly measurable (i.e. incompatible) can be used for demonstrating EPR steering, a form of quantum nonlocality. This implies that EPR steering and (non) joint measurability can be viewed as equivalent. Moreover, we discuss the connection between Bell nonlocality and joint measurability, and give evidence that both notions are inequivalent. This suggest the existence of incompatible quantum measurements which are Bell local, similarly to certain entangled states which admit a local hidden variable model. Finally, we discuss applications of these results to problems in joint measurability, and for EPR steering using randomly chosen measurements.
Joint measurability, EPR steering, and Bell nonlocality

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Joint Measurability

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$
Joint Measurability

$$\Delta x \Delta p \geq \hbar / 2$$
Nonlocality

x

\[ \rightarrow \]

S

\[ \rightarrow \]

y

\[ \rightarrow \]

a

\[ \rightarrow \]

S

\[ \rightarrow \]

b
Separable states are local

Nonlocality

Entanglement  Incompatible Measurements
Compatible Measurements

- Quantum observables:

\[ A = A^\dagger, \quad B = B^\dagger \]
More general measurements

- POVM:

\[ A_a \geq 0, \quad \sum_a A_a = I \]

\[ B_b \geq 0, \quad \sum_b B_b = I \]
Pauli Measurements

\[ \sigma_z : \{ |0\rangle \langle 0|, |1\rangle \langle 1| \} \quad \sigma_x : \{ |+\rangle \langle +|, |−\rangle \langle −| \} \]
Noise Pauli Measurements

\[ \sigma_{Z,\eta} : \left\{ \eta |0\rangle \langle 0| + (1 - \eta) \frac{1}{2} ; \quad \eta |1\rangle \langle 1| + (1 - \eta) \frac{1}{2} \right\} \]
Noise Pauli Measurements

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EPR steering

ψ−
EPR steering

\[ \sigma_Z \]

\[ |0\rangle \quad \quad \quad \quad \quad |1\rangle \quad \quad \quad \quad \quad \quad p=1/2 \]
\[ |1\rangle \quad \quad \quad \quad \quad |0\rangle \quad \quad \quad \quad \quad \quad p=1/2 \]

\[ \sigma_X \]

\[ |+\rangle \quad \quad \quad \quad \quad |\pm\rangle \quad \quad \quad \quad \quad \quad p=1/2 \]
\[ |\pm\rangle \quad \quad \quad \quad \quad |+\rangle \quad \quad \quad \quad \quad \quad p=1/2 \]

Incompatible measurements + Entangled state $\rightarrow$ EPR steering
Assemblage

Bob’s system is completely described by an assemblage

$$\sigma_{a|x} = \text{tr}_A(\rho_{AB}A_{a|x} \otimes I)$$
Bob’s system is completely described by an *assemblage*:

\[ \sigma_{a|x} = \text{tr}_A (\rho_{AB} A_{a|x} \otimes I) \]

\[ \rho_{a|x} = \frac{\sigma_{a|x}}{\text{tr}(\sigma_{a|x})} \]
Unsteerable Assemblages

\[ \sigma_{a|x} = \sum_{\lambda} \pi(\lambda) \rho_{A}(a|x, \lambda) \rho_{\lambda} \]
Werner states (1989)
Some entangled states are EPR local!

- EPR Steering
- Entanglement
- No Joint Measurability
Local Incompatible Measurements??

IF SOME ENTANGLED STATES ARE LOCAL...

WHAT ABOUT INCOMPATIBLE MEASUREMENTS??
The precise result

Theorem
Let \( \{ A_a | x \} \) be a set of incompatible measurements. If Alice measures \( \{ A_a | x \} \) on her part of a pure entangled state, the resulting assemblage is steerable.
Applications

- Explore known results from the Steering community to get results for Joint Measurability
Applications

- Imagine that Alice wants to measure ALL projective measurements at the same time.
- Some white noise is accepted \((i.e. \eta M + (1 - \eta)I)\)
- How small should \(\eta\) be?
- Precisely 1/2!
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- More (tight) unsteerable states:

$$\rho_{UNS} = \frac{1}{2} \Phi_\theta + \frac{1}{2} I \otimes \rho_{B,\theta}$$
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$$\rho_{\text{UNS}} = \frac{1}{2} \Phi_\theta + \frac{1}{2} \frac{l}{2} \otimes \rho_{B,\theta}$$

- $\eta \leq 5/12 \implies$ all POVMs can be measured
  (Barrett 2002 + Quintino et al 2015 + this work -> Pusey 2015)
Applications

- Alice and Bob share a two qubit Werner state
- What is the probability of Bob having an steering assemblage when Alice perform random measurements?
Applications

\[ p(\eta) = \sqrt{2\eta^2 - 1} / \eta^4 \]

Bell Inequalities

Bell Nonlocality can be witnessed by Bell Inequalities

\[ CHSH = \langle A_0B_0 \rangle + \langle A_0B_1 \rangle + \langle A_1B_0 \rangle - \langle A_1B_1 \rangle \leq 2 \]
Bell Inequalities

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\[ \langle A_xB_y \rangle := p(a = b | xy) - p(a \neq b | xy) \]
Incompatible measurements and Bell Nonlocality

- There may exist incompatible Bell local measurements …
Incompatible measurements and Bell Nonlocality

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- Evidence 1: Maximally entangled state $\implies$ incompatible Bell local measurements
Incompatible measurements and Bell Nonlocality

- There may exist incompatible Bell local measurements ...
- Evidence 1: Maximally entangled state $\implies$ incompatible Bell local measurements
- Evidence 2: Hollow triangle measurements + full-correlation type cannot show bell nonlocality
  
  $\langle A_x B_y \rangle := p(a = b | xy) - p(a \neq b | xy)$

![Diagram of M1, M2, and M3 forming a hollow triangle]
Incompatible measurements and Bell Nonlocality

\[
B \Gamma = \sum_{xy} \gamma_{xy} \langle A_x B_y \rangle + \sum_x \alpha_x \langle A_x \rangle + \sum_y \beta_y \langle B_y \rangle \leq 1
\]

\[
\langle A_x B_y \rangle := p(a = b|x,y) - p(a \neq b|x,y)
\]

\[
\langle A_x \rangle := p(a = 1|x) - p(a = -1|x)
\]

\[
\langle B_y \rangle := p(b = 1|y) - p(b = -1|y)
\]

Evidence 3: The Pauli Hollow triangle measurements does not violate many inequalities:

\[
\sum_y |\beta_y| \leq \frac{1 - \eta B \Gamma_{C^2}}{1 - \eta} \implies \text{No Bell Violation}
\]
More evidences

Evidence 4: SDP optimisations on known Bell inequalities

|                                  | $\{M'_a|_x(\eta)\}$   | $\{M''_a|_x(\eta)\}$   |
|----------------------------------|------------------------|-------------------------|
| Pairwise JM (CHSH violation)     | $1/\sqrt{2} \approx 0.7071$ | 0.5858                  |
| Triplewise JM                    | $1/\sqrt{3} \approx 0.5774$ | 0.4226                  |
| Bell violation ($n = 3$): $I_{3322}$ | 0.8037                 | 0.6635                  |
| Bell violation ($n = 4$): $I_{3422}^4$ | 0.8522                 | 0.7913                  |
|                                  | $I_{3422}^2$           | 0.8323                  | 0.5636                  |
|                                  | $I_{3422}^3$           | 0.8188                  | 0.6795                  |
| Bell violation ($n = 5$): $I_{3522}$ | 0.7786                 | 0.5636                  |

$$M''_{1|1}(\eta) := \eta |0\rangle\langle 0|, \quad M''_{1|2}(\eta) := \eta |+\rangle\langle +|, \quad M''_{1|3}(\eta) := \eta |Y_+\rangle\langle Y_+|$$
Incompatible measurements and Bell Nonlocality

Conjecture

There exists a set of non jointly measurable measurements that can never lead to Bell inequality violation.
Conclusions

- Better understanding on the relation between quantum measurements and nonlocality
- Conceptually: How to interpret JM in terms of EPR steering (vice versa!)
- Applications: Some theorems for JM can be translated to Nonlocality (vice versa!)
Thank you!

- EPR Steering
- Entanglement
- No Joint Measurability