Title: Nonclassicality as the failure of noncontextuality

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Abstract: To make precise the sense in which nature fails to respect classical physics, one requires a formal notion of "classicality". Ideally, such a notion should be defined operationally, so that it can be subjected to a direct experimental test, and it should be applicable in a wide variety of experimental scenarios, so that it can cover the breadth of phenomena that are thought to defy classical understanding. Bell's notion of local causality fulfills the first criterion but not the second, because it is restricted to scenarios with two or more systems that are space-like separated. The notion of noncontextuality fulfills the second criterion, because it is applicable to any experiment (even those on a single system), but it is a long-standing question whether it can be made to fulfill the first. Previous attempts to experimentally test noncontextuality have all presumed certain idealizations that do not hold in real experiments, namely, noiseless measurements and exact operational equivalences. In this talk, I will describe how one can devise experimental tests that are free of these idealizations using an operational notion of noncontextuality that applies to both preparations and measurements. These new theoretical insights raise the bar significantly for any claim of an experimental demonstration of nonclassicality. They also provide the means of determining, for any phenomenon that is typically thought to defy classical explanation, which experimentally-testable features of that phenomenon, if any, conflict with the assumption of a noncontextual model.
Nonclassicality as the failure of noncontextuality

Robert Spekkens
Perimeter Institute

Information Theoretic Foundations of Quantum Theory
PI, May 12, 2015
What we want in a notion of nonclassicality

Subject to direct experimental test

Applicable to a broad range of physical scenarios
What we want in a notion of nonclassicality

Subject to direct experimental test

Applicable to a broad range of physical scenarios

Failure of local causality ✔

X
What we want in a notion of nonclassicality

- Subject to direct experimental test
- Applicable to a broad range of physical scenarios

- Failure of local causality: ✓
- Failure of noncontextuality: ❌
- Failure of noncontextuality: ✓
### What we want in a notion of nonclassicality

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<th>Subject to direct experimental test</th>
<th>Applicable to a broad range of physical scenarios</th>
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<td>Failure of local causality</td>
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Operational theory

\[ p(X|M, P) \]
Operational theory

$p(X|M, P)$

Ontological model of an operational theory

$\lambda \in \Lambda$  Ontic state space  $\lambda$ causally mediates between $P$ and $M$
Operational theory

\[ p(X|M, P) \]

Ontological model of an operational theory

\[ \lambda \in \Lambda \quad \text{Ontic state space} \quad \lambda \text{ causally mediates between } P \text{ and } M \]
Operational theory

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Ontological model of an operational theory

\[ \lambda \in \Lambda \quad \text{Ontic state space} \]

\[ P \leftrightarrow \mu(\lambda|P) \]

\[ M \leftrightarrow \xi(X|M, \lambda) \]

\( \lambda \) causally mediates between \( P \) and \( M \)
Operational theory

\[ p(X|M, P) \]

Ontological model of an operational theory

\[ \lambda \in \Lambda \quad \text{Ontic state space} \]

\[ P \leftrightarrow \mu(\lambda|P) \quad M \leftrightarrow \xi(X|M, \lambda) \]

\[ p(X|M, P) = \int \xi(X|M, \lambda) \mu(\lambda|P) \, d\lambda \]

\( \lambda \) causally mediates between \( P \) and \( M \)
An ontological model of an operational theory is \textit{noncontextual} if

Operational equivalence of two experimental procedures \quad \Rightarrow \quad Equivalent representations in the ontological model

Operational equivalence classes of preparations
Operational equivalence classes of preparations
Operational equivalence classes of preparations

\[ e(P) = e(P') \]

\[ \forall M : p(X|P, M) = p(X|P', M) \]
Example from quantum theory

Different density op's

$\rho$ $\rho'$
Example from quantum theory

\[ \frac{1}{2} I = \frac{1}{2} |0\rangle \langle 0| + \frac{1}{2} |1\rangle \langle 1| \]

\[ \frac{1}{2} I = \frac{1}{2} |+\rangle \langle +| + \frac{1}{2} |-\rangle \langle -| \]
Preparation noncontextual model

\[ \mu(\lambda) \]

\[ \lambda \]
Preparation contextual model

\[ \mu(\lambda | P) \rightarrow \lambda \]

\[ \mu(\lambda | P') \rightarrow \lambda \]
Operational equivalence classes of measurements
Example from quantum theory

\[ \{ |\psi_1\rangle\langle\psi_1|, I - |\psi_1\rangle\langle\psi_1| \} \]

\[ I - |\psi_1\rangle\langle\psi_1| = |\psi_2\rangle\langle\psi_2| + |\psi_3\rangle\langle\psi_3| \]

\[ I - |\psi_1\rangle\langle\psi_1| = |\psi_2\rangle\langle\psi_2|' + |\psi_3\rangle\langle\psi_3|' \]
\[ e(P) = e(P') \]
\[ \forall M : p(X|P, M) = p(X|P', M) \]

Preparation noncontextuality

\[ \mu(\lambda|P) = \mu(\lambda|P') \]

\[ e(M) = e(M') \]
\[ \forall P : p(X|P, M) = p(X|P, M') \]

Measurement noncontextuality

\[ \xi(X|\lambda, M) = \xi(X|\lambda, M') \]
Noncontextuality is an analogue of Leibniz’s principle of the identity of indiscernables:

The ontological identity of operational indiscernables
Noncontextuality is an analogue of Leibniz’s principle of the identity of indiscernables:

The ontological identity of operational indiscernables

Noncontextuality is an analogue of Einstein’s strong equivalence principle

An information-theoretic equivalence principle
Noncontextuality is an analogue of Leibniz’s principle of the identity of indiscernables:

The ontological identity of operational indiscernables

Noncontextuality is an analogue of Einstein’s strong equivalence principle

An information-theoretic equivalence principle

Noncontextuality is an analogue of the principle of no fine-tuning used in causal inference

To achieve context-independence at the operational level while having context-dependence at the ontological level requires fine-tuning
Obstacles to a direct experimental test of noncontextuality

Obstacle #1: How to contend with noise?
Obstacle #2: How to contend with inexactness of operational equivalences?
Obstacles to a direct experimental test of noncontextuality

Obstacle #1: How to contend with noise?
Obstacle #2: How to contend with inexactness of operational equivalences?

Joint work with:
Ravi Kunjwal, Matt Pusey (theory)
Mike Mazurek, Kevin Resch (experiment)
Obstacle #1: How to contend with noise?

Previous no-go results

- measurement noncontextuality
- and
- outcome determinism

for projective measurements

contradiction
Obstacle #1: How to contend with noise?

Previous no-go results

measurement noncontextuality

and

outcome determinism for projective measurements

contradiction

It turns out that

preparation noncontextuality

and

Facts about projective measurements

outcome determinism for projective measurements
Obstacle #1: How to contend with noise?

Previous no-go results

measurement noncontextuality

and

outcome determinism

for projective measurements

It turns out that

preparation noncontextuality

and

Facts about projective measurements

And therefore:

universal noncontextuality

and

Facts about projective measurements

contradiction

outcome determinism for projective measurements

contradiction
Obstacle #1: How to contend with noise?

Recast as:

universal noncontextuality
and
Certain operational equivalences
Perfect correlations

And then as:

universal noncontextuality
and
Certain operational equivalences

Degree of correlation above some bound

contradiction
A noncontextuality inequality robust to experimental noise
\[ e(M_*) = e(\text{coin flip}) \]
\[ p(X = 0, 1|M_*, P) = \frac{1}{2}, \quad \forall P \in \mathcal{P}. \]

Measurement noncontextuality

\[ \xi(X = 0, 1|M_*, \lambda) = \frac{1}{2}, \quad \forall \lambda \in \Lambda \]
\[ e(M_*) = e(\text{coin flip}) \]

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Measurement noncontextuality

\[ \xi(X = 0, 1|M_*, \lambda) = \frac{1}{2}, \quad \forall \lambda \in \Lambda \]
\[ e(P_1) = e(P_2) = e(P_3) \]
\[ p(X|M, P_1) = p(X|M, P_2) = p(X|M, P_3) \quad \forall M \in \mathcal{M} \]
\[ \mu(\lambda|P_1) = \mu(\lambda|P_2) = \mu(\lambda|P_3) \quad \forall \lambda \in \Lambda \]

Preparation noncontextuality
\[ e(P_1) = e(P_2) = e(P_3) \]
\[ p(X|M, P_1) = p(X|M, P_2) = p(X|M, P_3) \quad \forall M \in \mathcal{M} \]

\[ \mu(\lambda|P_1) = \mu(\lambda|P_2) = \mu(\lambda|P_3) \quad \forall \lambda \in \Lambda \]
\[ t \in \{1, 2, 3\} \]

\[ P_t = \begin{array}{c}
\begin{array}{c}
P_{t,0}
\end{array}
\end{array} \]

\[ (1/2, 1/2) \]

\[ P_{t,1} \]

\[ \mu(\lambda|P_t) = \frac{1}{2} \mu(\lambda|P_{t,0}) + \frac{1}{2} \mu(\lambda|P_{t,1}) \]
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\[ \mu(\lambda|P_t) = \frac{1}{2} \mu(\lambda|P_{t,0}) + \frac{1}{2} \mu(\lambda|P_{t,1}) \]

\[ \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda|P_{1,b}) = \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda|P_{2,b}) = \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda|P_{3,b}) \]
\[ \xi(X = b | M_*, \lambda) = \frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b | M_t, \lambda) \]
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\[ \frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b| M_t, \lambda) = \frac{1}{2} \]
Quantum example
Def'n of average degree of correlation

\[ A \equiv \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(X = b | M_t, P_{t,b}) \]

Theorem: For any \( P_{1,0}, P_{1,1}, P_{2,0}, P_{2,1}, P_{3,0}, P_{3,1} \)
\( M_1, M_2, M_3 \)

If \( e(P_1) = e(P_2) = e(P_3) \)
\( e(M_\ast) = e(\text{coin flip}) \)

Then universal noncontextuality implies

\[ A \leq \frac{5}{6} \quad \text{A noncontextuality Inequality} \]
Recall \( A \equiv \frac{1}{6} \sum_{t \in \{1, 2, 3\}} \sum_{b \in \{0, 1\}} p(X = b | M_t, P_{t,b}) \)

Recall \( p(X = b | M_t, P_{t,b}) = \sum_{\lambda} \xi(X = b | M_t, \lambda) \mu(\lambda | P_{t,b}) \)
Recall \[ A \equiv \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} p(X = b| M_t, P_{t,b}) \]

Recall \[ p(X = b| M_t, P_{t,b}) = \sum_{\lambda} \xi(X = b| M_t, \lambda) \mu(\lambda| P_{t,b}) \]

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A = \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} \sum_{\lambda} \xi(X = b|M_t, \lambda) \mu(\lambda|P_{t,b})
\]

\[
\xi(X = b|M_t, \lambda) \leq \eta(M_t, \lambda)
\]

where \( \eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b'|M_t, \lambda) \).

\[
A \leq \frac{1}{3} \sum_{t \in \{1,2,3\}} \sum_{\lambda \in \Lambda} \eta(M_t, \lambda) \left( \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda|P_{t,b}) \right)
\]
\[ A = \frac{1}{6} \sum_{t \in \{1,2,3\}} \sum_{b \in \{0,1\}} \sum_{\lambda} \xi(X = b | M_t, \lambda) \mu(\lambda | P_{t,b}) \]

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\[ \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda | P_{1,b}) = \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda | P_{2,b}) = \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda | P_{3,b}) \equiv \nu(\lambda) \]
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\[ A \leq \frac{1}{3} \sum_{t \in \{1,2,3\}} \sum_{\lambda \in \Lambda} \eta(M_t, \lambda) \left( \frac{1}{2} \sum_{b \in \{0,1\}} \mu(\lambda|P_{t,b}) \right) \]

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\[ A \leq \sum_{\lambda \in \Lambda} \left( \frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right) \nu(\lambda) \]
\[ A \leq \sum_{\lambda \in \Lambda} \left( \frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right) \nu(\lambda) \]
\[ A \leq \max_{\lambda \in \Lambda} \left( \frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right) \]

where \( \eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b' | M_t, \lambda) \).
\[ A \leq \max_{\lambda \in \Lambda} \left( \frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right) \]

where \( \eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b'|M_t, \lambda) \).

Recall \[ \frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b|M_t, \lambda) = \frac{1}{2} \]
$$A \leq \max_{\lambda \in \Lambda} \left( \frac{1}{3} \sum_{t \in \{1,2,3\}} \eta(M_t, \lambda) \right)$$

where

$$\eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b'|M_t, \lambda).$$

Recall

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where \( \eta(M_t, \lambda) \equiv \max_{b' \in \{0,1\}} \xi(X = b'|M_t, \lambda) \).

Recall

\[ \frac{1}{3} \sum_{t \in \{1,2,3\}} \xi(X = b|M_t, \lambda) = \frac{1}{2} \]

\[ A \leq \frac{1}{3}(1 + 1 + \frac{1}{2}) = \frac{5}{6} \]
Quantum violation

\[ A = 1 \]

Robust to noise
Obstacle #2: Lack of exact operational equivalence

\[ e(P) = e(P') \]
\[ \forall M : p(X|P, M) = p(X|P', M) \]

Preparation noncontextuality

\[ \mu(\lambda|P) = \mu(\lambda|P') \]

Measurement noncontextuality

\[ e(M) = e(M') \]
\[ \forall P : p(X|P, M) = p(X|P, M') \]

\[ \xi(X|\lambda, M) = \xi(X|\lambda, M') \]
A remaining issue:
How to verify that a given set of operations is
tomographically complete?

\[ e(P) = e(P') \]
\[ \forall M : p(X|P, M) = p(X|P', M) \]

Preparation noncontextuality

\[ \mu(\lambda|P) = \mu(\lambda|P') \]

\[ e(M) = e(M') \]
\[ \forall P : p(X|P, M) = p(X|P, M') \]

Measurement noncontextuality

\[ \xi(X|\lambda, M) = \xi(X|\lambda, M') \]

This is the new frontier for experimental tests of noncontextuality
A = 0.99709 ± 0.00007
violating the noncontextual bound by 2300σ
Significance for characterizing nonclassicality
Epistemically restricted classical theories
Can recover Gaussian quantum mechanics

Bartlett, Rudolph, RWS, 2011

Can recover the stabilizer theory of qudits

RWS, arXiv:1409.5041
## Categorizing nonclassical phenomena

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<th>Those not arising in epistemically restricted classical theories</th>
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<td>Certain aspects of items on the left</td>
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<td>No perfect state discrimination</td>
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<td>Pre and post-selection effects</td>
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<td>Key distribution</td>
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<tr>
<td>Others...</td>
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Weakly nonclassical                                          Strongly nonclassical
Operational theories

\[ p(X|M, P) \]

Classical

Nonclassical

“Nonsimplicial”
Nonsimplicial but noncontextual

Remote steering
Einstein 1935; Caves-Fuchs-Schack 2000; Harrigan-RWS 2010

Three box paradox
Leifer-RWS 2004

Quantum multiplexing
RWS 2004

No error-free discrimination of nonorthogonal states
RWS 2004

Nonzero probability of wavepacket tunneling through a barrier
Bartlett-Rowe 1999

Failure of noncontextuality

Failure of preparation
noncontextuality = BI violation
Bell 1964; Barrett 2006 unpublished;
Liang-RWS-Wiseman 2010

Anomalous weak values
Pusey 2015

Probability of success in parity-oblivious multiplexing
RWS-Buzacott-Kheenn-Pryde-Toner 2008

Precise tradeoff of probability of discrimination with nonorthogonality
RWS-Wolfe (work in progress)

Precise dependence of tunneling probability on wavepacket width
RWS (work in progress)
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<th>Failure of noncontextuality</th>
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<tr>
<td>Teleportation</td>
<td>???</td>
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<td>No cloning</td>
<td>???</td>
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<tr>
<td>Various quantum information</td>
<td>???</td>
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<td>processing protocols</td>
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<td>Interference phenomena</td>
<td>???</td>
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<td>Leifer-RWS unpublished</td>
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<td>Quantum vacuum phenomena</td>
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<td>Existence of path integral</td>
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<td>expression for unitary dynamics</td>
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<td>Koh-Penney-RWS (unpublished)</td>
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<td>Thermodynamic phenomena</td>
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