Abstract: The theory of causal fermion systems is an approach to describe fundamental physics. It gives quantum mechanics, general relativity and quantum field theory as limiting cases and is therefore a candidate for a unified physical theory. Instead of introducing physical objects on a preexisting space-time manifold, the general concept is to derive space-time as well as all the objects therein as secondary objects from the structures of an underlying causal fermion system. The dynamics of the system is described by the causal action principle.

I will give a non-technical introduction, with an emphasis on conceptual issues related to information theory.
Overview: limiting cases

Causal fermion system
- abstract mathematical framework
- quantum geometry, causal action

continuum limit

description in the continuum limit
- Dirac fields
- strong and electroweak gauge fields
- gravitational field

- fermion field: second-quantized
- bosonic field: classical
Overview: limiting cases

Causal fermion system

\[ \text{continuum limit} \]

description in the continuum limit

\[ \text{microscopic mixing} \]

- also second-quantized bosonic field
- loop diagrams, renormalization, … (work in progress)

relativistic quantum field theory

Example: curved space-time

$t \quad \bar{x}$

$F \quad \mathcal{F} \subseteq L(\mathcal{H})$

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$M := \text{supp } \rho$
Advantage of general framework:
- "Spinors on singular spaces . . .",
- UV-regularized space-times,
  similar to talk by Achim Kempf.

UV-regularized = bandlimited

regularized objects are considered as fundamental objects
framework for doing analysis and geometry in this setting
Information theoretic point of view

Consider the setting from perspective of information theory:
Let $(\rho, \mathcal{F}, \mathcal{H})$ be a causal fermion system,
Encodes plenty of information:

- $x \in \mathcal{F}$ has eigenvalues
- operator products $xy$ has eigenvalues
- integrate quantities over space-time,

$$\int_{\mathcal{F}} \cdots d\rho$$

Connection to information theory remains to be developed:

- Right now: no operational point of view
- No definitions of entropy, temperature, ... 

Next: Try bring the information into a useful form.
Causal structure

Let \( x, y \in M \). Then
\[
\begin{align*}
  x \cdot y &\in L(H) \\
  \text{rank} &\leq 2n \\
  \text{in general not self-adjoint: } (x \cdot y)^* &= y \cdot x \neq x \cdot y
\end{align*}
\]
thus non-trivial complex eigenvalues \( \lambda_1^{xy}, \ldots, \lambda_m^{xy} \).

Definition (causal structure)
The points \( x, y \in \mathcal{I} \) are called
\[
\begin{cases}
  \text{timelike separated} & \text{if } \lambda_1^{xy}, \ldots, \lambda_m^{xy} \text{ are all real} \\
  \text{spacelike separated} & \text{if } \lambda_1^{xy}, \ldots, \lambda_m^{xy} \text{ are all non-real} \\
  \text{lightlike separated} & \text{otherwise}
\end{cases}
\]
and \( |\lambda_i^{xy}| = |\lambda_j^{xy}| \) \( \forall i, j \).
Causal action principle

Lagrangian: \[ \mathcal{L}[A_{ij}] = \frac{1}{4n} \sum_{ij=1}^{2n} \left( |A_{ij}^{xy}| - |A_{ij}^{yx}| \right)^2 \geq 0 \]

Action: \[ S = \int_{x,y \in M} \mathcal{L}[A_{ij}] \, d\mu(x) \, d\mu(y) \]

Minimize \( S \) under variations of \( \mu \), impose suitable constraints. Gives mathematically well-defined variational principles.

- Lagrangian is compatible with causal structure, i.e.
  - \( x, y \) spacelike separated \( \Rightarrow L(x, y) = 0 \)
  - "points with spacelike separation do not interact"
What does causality mean?

- There are causal relations:
  - distinction space-like, time-like
  - direction of time
- Locality holds:
  Space-time regions with space-like separation have independent dynamics

BUT
- relation "lies in the future of" not necessarily transitive
- no causation
Inherent structures

- Spinors
  \[ S_\kappa := \mathcal{X}(\mathbb{H}) \subset \mathcal{H} \quad \text{``spin space''}, \quad \text{dim } S_\kappa \leq 2n \]
  \[ \langle u | v \rangle_x = -(u \cdot v)_x \quad \text{``spin scalar product''}, \]
  is indefinite of signature \((\leq n, \leq n)\)

- Space of one-particle wave functions
  \[ \psi : x \in M \mapsto \psi(x) \in S_\kappa \quad \text{``wave function''} \]

  wave functions form Krein space \((\mathcal{H}, (\cdot | \cdot))\):

  \[ \langle \psi | \phi \rangle := \int_M \langle \psi(x) | \phi(x) \rangle_x \, d\mu(x) \quad \text{indefinite inner product} \]
  \[ \| \psi \|^2 := \int_M |\psi(x)\rangle_x |\psi(x)\rangle_x \, d\mu(x) \quad \text{norm, induces topology} \]
**Inherent structures**

- Physical wave functions and the fermionic operator

\[ \psi(x) = \pi_x \psi \quad \text{with } \psi \in \mathcal{H} \]

physical wave function

\[ P(x, y) = \pi_x y \rightarrow S_y \rightarrow S_x \]

“kernel of fermionic operator”

\[ = - \sum_{i=1}^{f} \psi_i(x) \psi_i(y) \]

where \( \psi_i \) basis of \( \mathcal{H} \)

The fermionic operator was indeed the starting point:

“The Principle of the Fermionic Projector”
AMS/SP Statistics in Advanced Math. 35 (2006)
Inherent structures

- **Physical wave functions and the fermionic operator**

  \[ \psi(x) = \pi_x \psi \quad \text{with } \psi \in \mathcal{H} \quad \text{physical wave function} \]

  \[ P(x, y) = \pi_x y : S_y \to S_x \quad \text{"kernel of fermionic operator"} \]

  \[ = - \sum_{i=1}^{f} |\psi_i(x)\rangle \langle \psi_i(y)| \quad \text{where } \psi_i \text{ basis of } \mathcal{H} \]

  The fermionic operator was indeed the starting point:

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Inherent structures

- Geometric structures
  - \( P(x, y) \) \( S_y \rightarrow S_x \) yields relations between spin spaces.
  - Using a polar decomposition (\ldots) one gets:
    \[
    D_{x,y} S_y \rightarrow S_x \text{ unitary} \quad \text{"spin connection"}
    \]
  - tangent space \( T_x \) carries Lorentzian metric,
  \[
  \nabla_{x,y} T_y \rightarrow T_x \quad \text{corresponding "metric connection"}
  \]
  - a distinguished time direction
  - holonomy of connection gives curvature
    \[
    R(x, y, z) = \nabla_{x,y} \nabla_{y,z} \nabla_{z,x} \rightarrow T_x \rightarrow T_x.
    \]
Underlying physical principles

- Pauli exclusion principle:
  Choose orthonormal basis $\psi_1, \ldots, \psi_N$ of $\mathcal{H}$. Set
  $$\psi = \psi_1 \wedge \cdots \wedge \psi_N.$$ 
  gives equivalent description by Hartree-Fock state

- local gauge principle:
  freedom to perform local unitary transformations.

- the "equivalence principle":
  symmetry under "diffeomorphisms" of $M$
  (note: $M$ merely is a topological measure space)

  locality, causality and time direction are emergent
Underlying physical principles

- Pauli exclusion principle:
  Choose orthonormal basis $\psi_1, \ldots, \psi_n$ of $\mathcal{H}$. Set
  $\psi = \psi_1 \wedge \cdots \wedge \psi_n$.

  gives equivalent bosonization by Hartree-Fock state.

- local gauge principle:
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  (note: $M$ merely is a topological measure space)

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The causal action principle in the continuum limit

Specify vacuum:

- Choose $\mathcal{H}$ as the space of all negative-energy solutions, hence "Dirac sea"

\[ \omega \]

\[ \hat{k} \]

Dirac sea

Antiparticles

Fixes length scale ("Compton length")

- Introduce ultraviolet regularization, "quantum geometry"
  Fixes length scale ("Planck length")
The causal action principle in the continuum limit

- specify vacuum as sum of Dirac seas,

\[ P(x, y) = \sum_{\beta=1}^{g} P_{\beta}(x, y) \]

\[ P_{\beta}(x, y) = \int \frac{d^4 k}{(2\pi)^4} \, \delta(k^2 - m^2) \, \Theta(-k^0) \, e^{-ik(x-y)} \]

\( \beta \) labels "generations" of elementary particles

- Dynamical equations only if three generations \( (g = 3) \)
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