Holographic Signatures of Cosmological Singularities

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Model

Consistent truncation of N=8 SUGRA to gravity in AdS$_4$ coupled to an $m^2 = -2$ scalar with potential

$$V(\phi) = -2 - \cosh(\sqrt{2}\phi)$$

In all asymptotically AdS solutions, the scalar falls off like

$$\phi = \frac{\alpha}{r} + \frac{\beta}{r^2}$$

Big Crunch solutions for alternative AdS boundary conditions

$$\beta(t, \Omega) = f \alpha^2(t, \Omega)$$
Euclidean Construction

First solve the Euclidean Einstein eq with SO(4) symmetry

\[ ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_3 \]

This yields Euclidean domain walls

\[ \phi = \frac{\alpha}{\rho} + \frac{\beta}{\rho^2} + \cdots \]

Define \( f \) by \( \beta = f \alpha^2 \). The restriction to the equator of \( S^3 \) defines initial data for a Lorentzian solution.
CFT Description

The 3D dual is ABJM theory, which has 8 scalars. With $\beta = 0$ boundary conditions the bulk scalar is dual to the $\Delta = 1$ operator

$$\mathcal{O} = Tr(\phi_1^2 - \phi_2^2)$$

Boundary conditions $\beta = f \alpha^2$ correspond to adding the potential term [Witten; Sever and Shomer, 2002]

$$\frac{f}{3} \int \mathcal{O}^3$$

Most potential energy is converted into particles while the field rolls down; no bounce. [Craps, TH, Turok, 2007]
Kasner - AdS

[Englehardt, TH, Horowitz, 2014]

Consider AdS$_5$ and no scalar,

$$ds^2 = \frac{1}{z^2} \left( \eta_{\mu \nu} dx^\mu dx^\nu + dz^2 \right)$$

Replace $\eta_{\mu \nu}$ with a Ricci flat Kasner metric,

$$ds^2 = -dt^2 + t^{2p_1} dx_1^2 + t^{2p_2} dx_2^2 + t^{2p_3} dx_3^2$$

$$\sum_i p_i = 1 = \sum_i p_i^2$$
Kasner - AdS

$ds^2 = \frac{H^2 t^2}{z^2} \left( -dt^2 + t^{2p_1} dx_1^2 + t^{2p_2} dx_2^2 + t^{2p_3} dx_3^2 + dz^2 \right)$

In terms of $\tau = \ln t$:

$ds^2 = -d\tau^2 + \sum_i e^{-2(1-p_i)H\tau} dy_i^2$

→ anisotropic de Sitter boundary

Dilation symmetry:

$z \rightarrow \lambda z, \quad t \rightarrow \lambda t, \quad x_i \rightarrow \lambda^{(1-p_i)} x_i$
Boundary two-point functions

We use spacelike bulk geodesics with endpoints on the boundary to compute two-point functions of CFT operators with large dimension $\Delta$:

$$\langle \psi | \mathcal{O}(x) \mathcal{O}(x') | \psi \rangle = e^{-m L_{\text{reg}}(x,x')}$$

where $L_{\text{reg}}$ is the regulated length.

Consider equal-time correlators for two points separated in $x=x_1$ direction only. Symmetries imply we can set $t=1$, $x_2=x_3=0$. 
Example: \( p_1 = -\frac{1}{4} \)

Geodesic equations can be solved analytically using \( w = t^{1/2} \) as the parameter,

\[
x(w) = \frac{4}{15} \sqrt{c + w(8c^2 - 4cw + 3w^2)}
\]

\[
z(w) = \frac{4}{3} \sqrt{c[w^3 - 1 + 3c(1 - w^2)]}
\]

At the boundary, \( t=1, \ z(1)=0 \), and \( L_{\text{bdy}} = 2x(1) \).

Boundary separation related to integration constant \( c \)

But..
Example: $p_1 = -1/4$

For fixed real $L_{\text{bdy}}$, 5 complex geodesics
Two-point correlator

\[ \mathcal{L}_{\text{reg}} = \ln \left[ -\frac{64}{9} c (1 + c)(2c - 1)^2 \right] \]

- For \( c \approx -1 \), \( L_{\text{bdy}} \) is small and \( L_{\text{reg}} = 2 \ln L_{\text{bdy}} \) so at short scales

\[ \langle \mathcal{O} \mathcal{O} \rangle = L_{\text{bdy}}^{-2\Delta} \]

- As \( c \to 0 \), \( L_{\text{reg}} \) again diverges while \( L_{\text{bdy}} \to \frac{1}{H_1} \). This leads to a pole at horizon size.

- For large \( L_{\text{bdy}} \) we have \( L_{\text{reg}} = \ln L_{\text{bdy}}^{8/5} \), which yields

\[ \langle \mathcal{O} \mathcal{O} \rangle \propto L_{\text{bdy}}^{-2\Delta/H_1} \]
Two-point correlator

\[ \text{Re}[e^{-L_{\text{reg}}}] \]
General Kasner exponents $p$

[Englehardt, TH, Horowitz, 2015]

- No pole at horizon scale for $p \geq 0$
- Always a pole at horizon scale for $p < 0$
- For large boundary separations:

$$\left\langle \mathcal{O}(\bar{x}) \mathcal{O}(-\bar{x}) \right\rangle \propto \mathcal{L}_{bdy}^{-2\Delta/(1-p)}$$
General Kasner exponents p

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**Interpretation of the pole at horizon size??**
Conjecture

Anisotropy effectively leads to dimensional reduction near past boundary, down to a 1+1 CFT

The two-point correlator in the geodesic approx. has a pole at horizon size only in the remaining (p<0) direction

Consider a 1+1 CFT after a quantum quench. Correlators of some operators show a large bump at `horizon size' which can be described in terms of EPR pairs of quasi-particles moving in opposite directions \[\text{[Calabrese & Cardy, 2006]}\]

Further, the entanglement entropy grows and then stabilizes. Is this the case in our system?
Happy Birthday Gary!!