The properties of a strange metal fermion model with infinite-range interactions turn out to be closely related to those of charged black holes with AdS2 horizons. I show that a microscopic computation of the ground state entropy density of the fermion model yields precisely the Bekenstein-Hawking entropy density of the black hole. The fermion model is UV finite and has no supersymmetry.
Bekenstein-Hawking entropy from strange metals

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\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{i,j;k,\ell} c_i^\dagger c_j^\dagger c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_{i} \langle c_i^\dagger c_i \rangle. \]
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{i,j;k,\ell} c_i^+ c_j^+ c_k c_\ell \]

\[ Q = \frac{1}{N} \sum_i \langle c_i^+ c_i \rangle. \]

\[-\langle f_i(\tau) f_i^+(\tau) \rangle \sim \begin{cases} -\tau^{-1/2}, & \tau > 0 \\ e^{-2\pi \mathcal{E} |\tau|^{-1/2}}, & \tau < 0 \end{cases}. \]
\[ P(w) = - \operatorname{Im} G(w) \]

\[ \frac{A^-}{\sqrt{w}} \quad \frac{A^+}{\sqrt{w}} \leq n\text{FL} \]
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Known 'equation of state' determines \( \mathcal{E} \) as a function of \( Q \)

Microscopic zero temperature entropy density, \( S \), obeys

\[ \frac{\partial S}{\partial Q} = 2\pi k_B \mathcal{E} \]
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{i,j,k,\ell} c_i^\dagger c_j^\dagger c_k c_\ell \]

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Einstein-Maxwell theory
+ cosmological constant

Horizon area \( A_h; \)
\[ \text{AdS}_2 \times R^d \]
\[ ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2 \]
Gauge field: \( A = (\mathcal{E}/\zeta)dt \)

Boundary area \( A_b; \)
charge density \( Q \)

\( \zeta = \infty \)

\( \zeta \)
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Boundary area \( A_b \): charge density \( Q \)

\[ \mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m\bar{\psi}\psi \]

\[-\langle \psi(\tau)\bar{\psi}(\tau) \rangle \sim \begin{cases} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi \mathcal{E}} |\tau|^{-1/2} & , \tau < 0. \end{cases} \]
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\[ -\langle f_i(\tau) f_i^\dagger(\tau) \rangle \sim \begin{cases} -\tau^{-1/2} e^{-2\pi \mathcal{E} |\tau|^{-1/2}}, & \tau > 0 \\ e^{-2\pi \mathcal{E} |\tau|^{-1/2}}, & \tau < 0. \end{cases} \]

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\( \text{AdS}_2 \times \mathbb{R}^d \)
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‘Equation of state’ relating \( \mathcal{E} \)
and \( Q \) depends upon the geometry
of spacetime far from the \( \text{AdS}_2 \)
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^\dagger c_j^\dagger c_k c_\ell \]

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\[ \zeta \]
\[ \vec{x} \]
\[ \mathcal{E} \]

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\[ -\left\langle \psi(\tau) \bar{\psi}(\tau) \right\rangle \sim \begin{cases} -\tau^{-1/2} & \tau > 0 \\ e^{-2\pi \mathcal{E}} |\tau|^{-1/2} & \tau < 0 \end{cases} \]

‘Equation of state’ relating \( \mathcal{E} \) and \( Q \) depends upon the geometry of spacetime far from the \( \text{AdS}_2 \)

Classical general relativity ('black hole mechanics') yields
\[
\frac{1}{A_b} \frac{\partial A_h}{\partial Q} = 8\pi G_N \mathcal{E}
\]
\[ H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,\ell=1}^{N} J_{ij;kl} c_i^+ c_j^+ c_k c_{\ell} \]

Einstein-Maxwell theory + cosmological constant

\[ \mathcal{L} = \bar{\psi} \Gamma^\alpha D_\alpha \psi + m \bar{\psi} \psi \]

Horizon area \( A_h \):
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\[ ds^2 = (d\zeta^2 - dt^2)/\zeta^2 + d\vec{x}^2 \]
Gauge field: \( A = (E/\zeta)dt \)

\[ \zeta = \infty \]

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\[ -\langle f_i(\tau) f_i^+(\tau) \rangle \sim \left\{ \begin{array}{ll} -\tau^{-1/2} & , \tau > 0 \\ e^{-2\pi \mathcal{E} |\tau|^{-1/2}} & , \tau < 0 \end{array} \right. \]

Boundary area \( A_b \):
charge density \( Q \)

\[ \text{Combination:} \]
\[ S = \frac{A_h}{4G_N A_b} \]

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'Equation of state' relating \( \mathcal{E} \) and \( Q \) depends upon the geometry of spacetime far from the AdS_2
\[ Z = \int dx_i \exp \left( - \int d\tau \left[ c_i^+ \frac{\partial c_i}{\partial \tau} + \frac{1}{2} J_{ij} c_i^+ c_j + \epsilon_i c_i + \mu_i c_i^+ c_i + \kappa_i c_i^2 \right] \right) \]

\[ c_i \rightarrow c_i^a, \quad a = 1 \ldots n \quad (n \rightarrow 0) \]

\[ \overline{Z}^n = \int dx_i \exp \left( - \int d\tau \left[ c_i^+ \frac{\partial c_i}{\partial \tau} - \frac{1}{N^2} \sum_{ij} c_i^+ c_j \right] \right) \]
\[ G(\bar{3}) = \frac{1}{z + \mu - \Sigma(\bar{3})} \]
\[ \Sigma(z) = J^2 G(z) G(-z) \]

**Confined uctariance**

We know
\[ G(\bar{3}) \sim \frac{1}{t^{3/2}} \]
\[ G(\bar{2}) \sim \frac{1}{t^{1/2}} \]
\[ \Sigma(z) \sim \frac{1}{t^{1/2}} \]
\[ \Sigma(3) \sim \frac{1}{t^{1/2}} \]

Constant of \[ \mu - \Sigma(0) \]
\[ \int d\tau_2 \ G(\tau_1, \tau_2) \Sigma(\tau_2, \tau_3) = \delta(\tau_1 - \tau_3) \]

\[ \Sigma(\tau_1, \tau_2) = \int^2 G(\tau_1, \tau_2) G(\tau_2 - \tau_1) \]

\[ \tau = f(\sigma) \]

\[ G(\tau_1, \tau_2) = \frac{\frac{g(\sigma)}{g(\sigma)'}}{\frac{f(\sigma)}{f(\sigma)'} + \frac{g(\sigma)}{g(\sigma)'}} \]

\[ \Sigma(\tau_1, \tau_2) = \frac{\frac{g(\sigma)}{g(\sigma)'}}{\frac{f(\sigma)}{f(\sigma)'} + \frac{g(\sigma)}{g(\sigma)'}} \]

\( f(\sigma) \) and \( g(\sigma) \) arbitrary
\[ G(3) = \sqrt{3} \mu - \Sigma(3) \]
\[ \Sigma(2) = \frac{1}{2} \left( G(2) - G(-2) \right) \]

**Conformal invariance**

- We know \[ G(3) \sim \frac{1}{\sqrt{3}} \]
- \[ G(2) \sim \frac{1}{\sqrt{12}} \]
- \[ \Sigma(2) \sim \frac{1}{3} \]

\[ \Sigma(3) - \Sigma(0) \sim \sqrt{3} \]

Consistency if \[ \mu = \Sigma(0) \]