Title: The non-superconducting states of the cuprates

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Abstract: The most interesting states of the cuprate compounds are not the superconductors with high critical temperatures. Instead, the novelty lies primarily in the higher temperature metallic "normal" states from which the superconductors descend, and in competing low temperature states with density wave order. I will review recent experimental and theoretical progress in understanding these states. The experimental evidence is compatible with the presence of a metal with topological order in the 'pseudogap' regime of low carrier density.
The non-superconducting states of the cuprates

Perimeter Institute, Waterloo
July 7, 2015
Subir Sachdev

Talk online: sachdev.physics.harvard.edu
Flavors of Quantum Matter

A. Ordinary quantum matter
   *Independent electrons, or pairs of electrons*

B. Topological quantum matter
   *Long-range quantum entanglement leads to sensitivity to spatial topology*

C. Quantum matter without quasiparticles
   *Hydrodynamics, memory functions, holography, and field theory*
Anti-ferromagnet with \( p \) holes per square

But relative to the band insulator, there are \( 1 + p \) holes per square
Figure: K. Fujita and J. C. Seamus Davis

Conventional
metal

Area enclosed by
Fermi surface = 1 + p
Ordinary quantum matter: the Fermi liquid (FL)

- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons (mod 2) = 1 + p.
Ordinary quantum matter: the Fermi liquid (FL)

- Fermi surface separates empty and occupied states in momentum space.
- Area enclosed by Fermi surface = total density of electrons \( \text{mod } 2 = 1+p \).
- Density of electrons can be continuously varied at zero temperature.
Fermi liquid (FL)

Topological argument for the area of Fermi surface

Put metal on a torus, adiabatically insert flux $\Phi = h/e$ through hole, and measure change in momentum. In a FL, we can assume the only low energy excitations are quasiparticles near the Fermi surface, and this leads to a non-perturbative proof of the Luttinger relation on the area enclosed by the Fermi surface.

Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density \( p \)

Evolution of the Hall Coefficient and the Peculiar Electronic Structure of the Cuprate Superconductors

Yoichi Ando, Y. Kurita, Seiki Komiya, S. Ono, and Kouji Segawa
PRL 92, 197001 (2004)

\[ R_H (10^{-3} \text{ cm}^3/\text{C}) \]

\[ \text{Temperature (K)} \]

\( T \)-independent Hall effect in a magnetic field of fermions of charge +e and density \( p \)
Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density $p$

Spectroscopic evidence for Fermi liquid-like energy and temperature dependence of the relaxation rate in the pseudogap phase of the cuprates

Seyed Iman Mirzaei, Damien Stoiser, Jason N. Hancock, Christophe Berthod, Antoine Georges, Eric van Heumen, Mun Y. Chan, Xiufeng Zhao, Yuan Li, Martin Greven, Neven Barić, and Dirk van der Marel

${\sigma}_{XX} \sim \frac{1}{(-i\omega + 1/\tau)}$

with $\frac{1}{\tau} \sim \omega^2 + T^2$

PNAS 110, 5774 (2013)
Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density $p$

In-Plane Magnetoresistance Obeys Kohler’s Rule in the Pseudogap Phase of Cuprate Superconductors

M. K. Chan,1,7 M. J. Veit,1 C. J. Dorow,1,9 Y. Ge,1 Y. Li,1 W. Tabis,1,2 Y. Tang,1 X. Zhao,1,3
N. Barišić1,4,5,6 and M. Greven1,8
PRL 113, 177005 (2014)

We report in-plane resistivity ($\rho$) and transverse magnetoresistance (MR) measurements for underdoped HgBa$_2$CuO$_{4+d}$ (Hg1201). Contrary to the long-standing view that Kohler’s rule is strongly violated in underdoped cuprates, we find that it is in fact satisfied in the pseudogap phase of Hg1201. The transverse MR shows a quadratic field dependence, $\rho_T/\rho_0 = a H^2$, with $a(T) \propto T^{-2}$. In combination with the observed $\rho \propto T^2$ dependence, this is consistent with a single Fermi-liquid quasiparticle scattering rate. We show that this behavior is typically masked in cuprates with lower structural symmetry or strong disorder effects.

$$\rho_{xx} \sim \frac{1}{\tau} (1 + a H^2 \tau^2 + \ldots)$$

with $$\frac{1}{\tau} \sim T^2$$
Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density $p$

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$$\rho_{xx} \sim \frac{1}{\tau} \left( 1 + aH^2\tau^2 + \ldots \right)$$

with $\frac{1}{\tau} \sim T^2$
Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size $p$?
Can we have a metal with no broken translational symmetry, and with long-lived electron-like quasiparticles on a Fermi surface of size $p$?

Answer: Yes.
There can be a Fermi surface of size $p$, but it must be accompanied by topological order, in a “fractionalized Fermi liquid”.

At $T=0$, such a metal must be separated from a Fermi liquid (with a Fermi surface of size $1+p$) by a quantum phase transition.
Note: relative to the fully-filled band insulator, there are \(1 + p\) holes per square.

Anti-ferromagnet with \(p\) holes per square.
Fractionalized Fermi liquid (FL*)

\[ | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \]

Realizes a metal with a Fermi surface of area \( p \) co-existing with "topological order".

A fermionic “dimer” describing a “bonding” orbital between two sites

Realizes a metal with a Fermi surface of area $p$ co-existing with “topological order”

Topological order

Place pseudogap metal on a torus; obtain "topological" states nearly degenerate with the ground state: change sign of every dimer across red line
Topological order

\[ | \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle \]

Place pseudogap metal on a torus; obtain "topological" states nearly degenerate with the ground state; change sign of every dimer across the red line.
Topological order

Place pseudogap metal on a torus; to change overall sign, a pair of “spinons” have to be moved globally around a circumference of the torus.
Topological order

\[ \left\langle \uparrow \downarrow \right\rangle - \left\langle \downarrow \uparrow \right\rangle \]

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Topological argument for the area of Fermi surface

Put metal on a torus, adiabatically insert flux $\Phi = \hbar/e$ through hole, and measure change in momentum. In a FL, we can assume the only low energy excitations are quasiparticles near the Fermi surface, and this leads to a non-perturbative proof of the Luttinger relation on the area enclosed by the Fermi surface.

Quantum dimer model with bosonic and fermionic dimers

Connection to the $t-t'-t''-J$ model:

- $t_1 = -(t + t')/2$
- $t_2 = (t - t')/2$
- $t_3 = -(t + t' + t'')/4$

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978
Quantum dimer model with bosonic and fermionic dimers

Dispersion and quasiparticle residue of a single fermionic dimer for $J = V = 1$, and hopping parameters obtained from the $t$-$J$ model for the cuprates, $t_1 = -1.05$, $t_2 = 1.95$ and $t_3 = -0.6$, on a $8 \times 8$ lattice.

M. Punk, A. Allais, and S. Sachdev, arXiv:1501.00978
“Back side” of Fermi surface is suppressed for observables which change electron number in the CuO$_2$ layer.
A new metal — a fractionalized Fermi liquid (FL*) — with electron-like quasiparticles on a Fermi surface of size $\rho$?
Quantum dimer model with bosonic and fermionic dimers

Connection to the $t-t'-t''-J$ model:

$$t_1 = -(t + t')/2$$
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Electrical and optical evidence for Fermi surface of long-lived quasiparticles of density $\rho$

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M. K. Chan,¹ M. J. Veit,¹ C. J. Dorow,¹,² Y. Ge,¹ Y. Li,¹ W. T. T. Li,¹,² Y. Tang,¹ X. Zhao,¹,³
N. Barisic,¹,⁴,⁵,⁶ and M. Greven¹,³
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$$\rho_{xx} \sim \frac{1}{T} \left( 1 + aH^2T^2 + \ldots \right)$$

with $\frac{1}{T} \sim T^2$
Unconventional density wave (DW):
Bose condensation of particle-hole pairs

\[ \langle c^\dagger_\alpha(r_1)c_\alpha(r_2) \rangle \]
\[ = \left[ P(r_1 - r_2) \right] \times \Psi_{DW} \left( \frac{r_1 + r_2}{2} \right) e^{iQ \cdot (r_1 + r_2)/2} \]

Disordered uni-directional charge density waves ("stripes") with wavelength ≈ 4 lattice sites?

See also

Unconventional density wave (DW):
Bose condensation of particle-hole pairs

\[ \langle c_\alpha^\dagger(r_1)c_\alpha(r_2) \rangle = [P(r_1 - r_2)] \times \Psi_{DW} \left( \frac{r_1 + r_2}{2} \right) e^{iQ \cdot (r_1 + r_2)/2} \]

**Unconventional density wave (DW):**
Bose condensation of particle-hole pairs

\[
\langle c_\alpha^\dagger(r_1) c_\alpha(r_2) \rangle
= \left[ \mathcal{P}(r_1 - r_2) \right] \times \Psi_{DW} \left( \frac{r_1 + r_2}{2} \right) e^{iQ \cdot (r_1 + r_2)/2}
\]

Density wave **form factor** (internal particle-hole pair wavefunction)

\[
\mathcal{P}(r) = \int \frac{d^2k}{4\pi^2} \mathcal{P}(k) e^{i\mathbf{k} \cdot \mathbf{r}}
\]

Time-reversal symmetry requires \( \mathcal{P}(\mathbf{k}) = \mathcal{P}(-\mathbf{k}) \).

We expand (using reflection symmetry for \( Q \) along axes or diagonals)

\[
\mathcal{P}(\mathbf{k}) = \mathcal{P}_s + \mathcal{P}_s'(\cos k_x + \cos k_y) + \mathcal{P}_d(\cos k_x - \cos k_y)
\]

Conventional CDW order: s-form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\beta} \rangle$ for $i = j$, and $i, j$ nearest neighbors.

$$P_{ij} = \left[ \int_k \mathcal{P}(k) e^{ik(\mathbf{r}_i - \mathbf{r}_j)} \right] e^{iQ \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.}$$

$\mathcal{P}(k) = 1$ and $Q = 2\pi(1/4, 0)$

$Q = (\pi/2, 0)$
Unconventional DW order: $s'$-form factor

Plot of $P_{ij} = \langle c_{i\alpha}^\dagger c_{j\alpha} \rangle$ for $i = j$, and $i, j$ nearest neighbors.

\[
P_{ij} = \left[ \int_{k} \mathcal{P}(k) e^{i k \cdot (r_i - r_j)} \right] e^{i Q \cdot (r_i + r_j)/2} + \text{c.c.}
\]

\[
\mathcal{P}(k) = e^{i \phi} \left[ \cos(k_x) + \cos(k_y) \right] \quad \text{and} \quad Q = 2\pi (1/4, 0)
\]

$Q = (\pi/2, 0)$
Unconventional DW order: $s + s'$-form factor

Plot of $P_{ij} = \langle c_{io}^\dagger c_{j\alpha} \rangle$ for $i = j$, and $i, j$ nearest neighbors.

$$P_{ij} = \left[ \int_k \mathcal{P}(k)e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)} e^{iQ \cdot (\mathbf{r}_i + \mathbf{r}_j)/2} + \text{c.c.} \right]$$

$$\mathcal{P}(k) = e^{i\phi} \left[ 1/2 + \cos(k_x) + \cos(k_y) \right] \quad \text{and} \quad Q = 2\pi (1/4, 0)$$

X-ray observations indicate strong $s'$ component in LBCO

David Hawthorn, Waterloo

Predicted $d$ form factor observed in STM measurements on BSCCO, Na-CCCO.

d-form factor is peaked at the pseudogap energy, and does not disperse as a function of wavevector.


Density wave (DW) order at low $T$ and $p$

Pseudogap

$T$ (K)

$T$ (K)

AF

PG

SM

DW

FL

B

dSC + DW

dSC

Na-CCOC

0.4 nm
Pairing “glue” for d-wave superconductivity from antiferromagnetic fluctuations

\[ \langle c^\dagger_{k\alpha} c^\dagger_{-k\beta} \rangle = \epsilon_{\alpha\beta} \Delta (\cos k_x - \cos k_y) \]

Density wave instability of large Fermi surface (FL) leads to an incorrect "diagonal" wavevector

\[
\langle c_k^{\dagger} c_{k-Q/2,\alpha} c_{k+Q/2,\alpha} \rangle = \mathcal{P}_d (\cos k_x - \cos k_y)
\]

Fermi surface of a fractionalized Fermi liquid (FL*)

universal constraints on transport

hydrodynamics

few conserved quantities

memory matrix

perturbative limit

appropriate microscopics for cuprates

holography

matrix large N theory; non-perturbative computations

long time dynamics; "renormalized IR fluid" emerges

Electrical transport at a strongly-coupled critical theory 
without particle-hole symmetry, 
with a conserved momentum $P$

$$\sigma = \sigma_Q + \frac{Q^2}{M} \pi \delta(\omega)$$

with $Q \equiv \chi_{J_\perp, P_x}$ and $M \equiv \chi_{P_x, P_x}$ thermodynamic response functions

Obtained in hydrodynamics, holography, and by memory functions

Electrical transport at a strongly-coupled critical theory without particle-hole symmetry, with an almost conserved momentum $P$, and an applied magnetic field $B$

$$\sigma_{xx} = \frac{(\tau_L^{-1} - i\omega) M \sigma_Q + Q^2 + B^2 \sigma_Q^2}{Q^2 B^2 + ((\tau_L^{-1} - i\omega) M + B^2 \sigma_Q)^2} M \left( \frac{1}{\tau_L} - i\omega \right),$$

$$\sigma_{xy} = \frac{2(\tau_L^{-1} - i\omega) M \sigma_Q + Q^2 + B^2 \sigma_Q^2}{Q^2 B^2 + ((\tau_L^{-1} - i\omega) M + B^2 \sigma_Q)^2} B Q.$$

Obtained in hydrodynamics, holography, and by memory functions

M. Blake and A. Donos, PRL 114, 021601 (2015)
Electrical transport at a strongly-coupled critical theory without particle-hole symmetry, with an almost conserved momentum $P$, and an applied magnetic field $B$.

\[
\sigma_{xx} = \frac{(\tau_L^{-1} - i\omega)M\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau_L^{-1} - i\omega)M + B^2\sigma_Q)^2}M\left(\frac{1}{\tau_L} - i\omega\right),
\]
\[
\sigma_{xy} = \frac{2(\tau_L^{-1} - i\omega)M\sigma_Q + Q^2 + B^2\sigma_Q^2}{Q^2B^2 + ((\tau_L^{-1} - i\omega)M + B^2\sigma_Q)^2}BQ.
\]

Obtained in hydrodynamics, holography, and by memory functions.

M. Blake and A. Donos, PRL 114, 021601 (2015)
Conclusions

1. Predicted d-form factor density wave order observed in the non-La hole-doped cuprate superconductors.

2. Proposed the pseudogap metal is a fractionalized Fermi liquid (FL*): a Fermi liquid co-existing with topological order.

3. Can we experimentally detect possible "topological order" in the pseudogap metal? (topological order is directly linked to Fermi surface size)

4. Hydrodynamic, memory-function, holographic, and field-theoretic approaches to transport without quasiparticles
d-form factor is peaked at the pseudogap energy, and does not disperse as a function of wavevector.
A tensor product state approach to spin-1/2 square J1-J2 Heisenberg model:

evidence for deconfined quantum criticality

Zheng-Cheng Gu

Perimeter Institute

Collaborator:

Dr. Ling Wang (Caltech)
Prof. Xiao-Gang Wen (MIT)
Prof. F. Verstraete (U. of Vienna)
Deconfined Quantum critical point (QCP) in cuprates?

DQCP in spin models

Theory: beyond Landau's paradigm

\[ \mathcal{L}_z = \sum_{a=1}^{N} |(\partial_\mu - ia_\mu) z_a|^2 + s|z|^2 + u (|z|^2)^2 + \kappa (\epsilon_{\mu\nu\kappa} \partial_\nu a_\kappa)^2 \]

\[ Q = \frac{1}{4\pi} \int d^2 r \hat{n} \cdot \nabla \times \nabla \hat{n}. \]

On square lattice, Q always changes by four, thus the instanton effect becomes dangerous irrelevant!


Numerical: J-Q model (simulated by using QMC)

\[ H = J \sum_{\langle ij \rangle} S_i \cdot S_j - Q \sum_{\langle ijk \rangle} (S_i \cdot S_j - \frac{1}{4})(S_k \cdot S_l - \frac{1}{4}) \]

**DQCP in spin models**

**Theory: beyond Landau's paradigm**

\[ \mathcal{L}_z = \sum_{a=1}^{N} |(\partial_\mu - ia_\mu) z_a|^2 + s|z|^2 + u(\frac{|z|^2}{2})^2 + \kappa(\epsilon_{\mu\nu\kappa}\partial_\nu a_\kappa)^2 \]

\[ Q = \frac{1}{4\pi} \int d^2 r \hat{n} \cdot \partial_x \hat{n} \times \partial_y \hat{n}. \]

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Relevant spin model for cuprates

**J1-J2 model:**

\[ H = J_1 \sum_{\langle ij \rangle} S_i \cdot S_j + J_2 \sum_{\langle\langle ij \rangle\rangle} S_i \cdot S_j \]

**Landau Paradigm: a meanfield phase diagram**

\[ J_2 / J_1 = 0.5 \]
Recent progress

A recent work by using density matrix renormalization group (DMRG) algorithm claims a Z2 spin liquid

Hong-Chen Jiang, Hong Yao, Leon Balents, Phys. Rev. B 86, 024424 (2012)
Another work by using SU(2) symmetry DMRG algorithm suggests a different result

Shou-Shu Gong, Wei Zhu, D. N. Sheng, Olelexi I. Motrunich, Matthew P. A. Fisher
Landau paradigm: meanfield approach

- The key concept is to find an ideal trial wave function, e.g., for a spin $\frac{1}{2}$ system:
  $$|\Psi_{\text{trial}}\rangle = \otimes \left( u^{\uparrow} | \uparrow \rangle_i + u^{\downarrow} | \downarrow \rangle_i \right)$$

- After minimizing the energy, we can find various symmetry ordered phases.

\[
H = -\sum_{\langle ij \rangle} \sigma^z_i \sigma^z_j - h \sum_i \sigma^x_i
\]

- $h_{MF}^c = 4$
- $\beta_{MF}^c = 0.5$
- $\langle \sigma_z \rangle \propto |h - h_c|^\beta$
Tensor Product State approach

Mean-field states: $\uparrow \rightarrow u^\uparrow; \quad \downarrow \rightarrow u^\downarrow \quad m_1 \quad m_2 \quad m_3 \quad m_4 \quad m_5 \quad m_6 \quad m_7$

$\Psi(\{m_i\}) = u^{m_1} u^{m_2} u^{m_3} u^{m_4} \cdots; \quad m_i = \uparrow, \downarrow$
Tensor Product State approach

Mean-field states: $\uparrow \rightarrow u^\uparrow$; $\downarrow \rightarrow u^\downarrow$; $m_1 \ m_2 \ m_3 \ m_4 \ m_5 \ m_6 \ m_7$

$\Psi(\{m_i\}) = u^{m_1} u^{m_2} u^{m_3} u^{m_4} \cdots$; $m_i = \uparrow, \downarrow$

MPS/DMRG (the most powerful method in 1D):

$\Psi(\{m_i\}) = \text{Tr} [A^{m_1} A^{m_2} A^{m_3} A^{m_4} \cdots ]$; $m_i = \uparrow, \downarrow \rightarrow A^\uparrow$; $\downarrow \rightarrow A^\downarrow$

$A_{\alpha}^{m_i} : \alpha \longrightarrow \beta$
Tensor Product State approach

Mean-field states: \[ \uparrow \rightarrow u^\uparrow; \quad \downarrow \rightarrow u^\downarrow \quad m_1 \quad m_2 \quad m_3 \quad m_4 \quad m_5 \quad m_6 \quad m_7 \]

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\[ \Psi (\{ m_i \} ) = \text{Tr} [A^{m_1} A^{m_2} A^{m_3} A^{m_4} \cdots]; \quad m_i = \uparrow, \downarrow \quad \uparrow \rightarrow A^\uparrow; \quad \downarrow \rightarrow A^\downarrow \]

TPS: \[ \uparrow \rightarrow T_{lru}^\uparrow; \quad \downarrow \rightarrow T_{lru}^\downarrow \]  
(F. Verstraete and J. I. First 2004)
Properties of TPS:

- Entanglement entropy satisfies area law

\[ S(\rho_L) = \alpha L \quad \text{(F. Verstraete et al.)} \]

\[ |\Psi_0\rangle = \prod_{\text{link}} |I\rangle \quad |I\rangle = \sum_{l=1}^{D} |ll\rangle \]

\[ |\Psi_{TPS}\rangle = \prod_i P_i |\Psi_0\rangle \quad P_i = T_{lrud}^{m_i} |m_i\rangle \langle lrud| \]
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- TPS faithfully represent non-chiral topologically ordered states

(Z.C. Gu, etal., PRB, 2008, O. Buerschaper, etal., PRB, 2008)
Ground state energy

- We use an imaginary time evolution algorithm to find the variational ground state.

\[
|\Psi_{GS}\rangle = \lim_{\tau \to -\infty} e^{-\tau H} |\Psi_0\rangle = \lim_{N \to -\infty} e^{-N\delta \tau H} |\Psi_0\rangle
\]

- \( D=9 \)

The TPS energy is lower than the best VMC energy!
Ground state energy

- We use an imaginary time evolution algorithm to find the variational ground state.

$$\left| \Psi_{GS} \right\rangle = \lim_{\tau \to -\infty} e^{-\tau H} \left| \Psi_0 \right\rangle = \lim_{N \to -\infty} e^{-N\delta\tau H} \left| \Psi_0 \right\rangle$$

- $D=9$

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- D=9

The TPS energy is lower than the best VMC energy!
AF order parameter:

(b)

(a)

Spin-Spin Correlation:

(a)

(b)
AF order parameter:

Spin-Spin Correlation:
Dimer-Dimer Correlation:

\[ C^*_{dx}(r, 0) = C_{dx}(r, 0) - C_{dx}(r - 1, 0), \quad D_x(x, y) = S_{(x,y)} \cdot S_{(x+1,y)}, \]

\[ C^*_{dx}(r, r) = C_{dx}(r, r) - C_{dx}(r - 1, r), \quad D_y(x, y) = S_{(x,y)} \cdot S_{(x,y+1)} \cdot \]

\[ C_{dx}(r_x, r_y) = \frac{1}{N} \sum_{x,y} D_x(x, y) D_x(x + r_x, y + r_y), \]

\[ C_{dy}(r_x, r_y) = \frac{1}{N} \sum_{x,y} D_y(x, y) D_y(x + r_x, y + r_y). \]
The critical exponents are intrinsically close to the DQCP behavior observed in other systems, e.g., J-Q model.
The universal scaling function

\[ C(L/2, L/2) = L^{-1-\eta} f(L^{1/\nu}(g_c - g)/g_c). \]

The critical exponents are intrinsically close to the DQCP behavior observed in other systems, e.g., J-Q model.

\( U(1) \) spin liquid is unstable, a VBS order with exponentially small amplitude might develop at long wave length.
Dimer-Dimer Correlation:

\[ C_{dx}^{*}(r, 0) = C_{dx}(r, 0) - C_{dx}(r - 1, 0). \]
\[ C_{dx}^{*}(r, r) = C_{dx}(r, r) - C_{dx}(r - 1, r). \]
\[ D_{x}(x, y) = S_{(x,y)} \cdot S_{(x+1,y)}. \]
\[ D_{y}(x, y) = S_{(x,y)} \cdot S_{(x,y+1)}. \]

\[ C_{dx}(r_x, r_y) = \frac{1}{N} \sum_{x,y} D_{x}(x, y) D_{x}(x + r_x, y + r_y). \]

\[ C_{dy}(r_x, r_y) = \frac{1}{N} \sum_{x,y} D_{y}(x, y) D_{y}(x + r_x, y + r_y). \]
**Single parameter variational approach**

**A D=3 TPS description of short range RVB state**

\[
\mathcal{P}_1 = \sum_{k=1}^{4} (|\uparrow\rangle \langle 0|_k + |\downarrow\rangle \langle 1|_k) \otimes (222|_k
\]

\[
|S\rangle = |01\rangle - |10\rangle + |22\rangle
\]

\[
|\Psi\rangle_{s-RVB} = \prod_{V} \mathcal{P}_1 \prod_{B} |S\rangle
\]
**Single parameter variational approach**

A D=3 TPS description of short range RVB state

\[ P_1 = \sum_{k=1}^{4} (|\uparrow\rangle\langle 0|_k + |\downarrow\rangle\langle 1|_k) \otimes (222|_k) \]

\[ |S\rangle = |01\rangle - |10\rangle + |22\rangle \]

Including longer range RVB

\[ P_2 = \sum_{i \neq j \neq k \neq l} (|\uparrow\rangle\langle 0|_i + |\downarrow\rangle\langle 1|_i) \otimes (2|_j \otimes \langle \epsilon|_{kl} \]

\[ |\Psi\rangle_{s-RVB} = \prod_V P_1 \prod_B |S\rangle \]
Topological sectors and variational energy

Similar to the short range RVB state, we can define four different topological sectors.

![Images of topological sectors](image)

Vison sectors

$$|\Psi(\pm)\rangle \equiv |\Psi\rangle_{G_h=1} \pm |\Psi\rangle_{G_h=-1}$$

Best variational energy at $c=0.35$

- $E=-0.4862$/site on cylinder/stripe geometry
- Comparing to $E=-0.494$/site with $D=9$

![Graphs of energy vs. charge density](image)
Correlation functions

Dimer-dimer correlation shows different behaviors on cylinder and torus! (Challenges to DMRG)
Entanglement entropy

Both dimer-dimer correlation and entanglement entropy indicate gapless spin liquid behaviors!

\[ S_2(L) = a_1 L - \frac{1}{2} \ln L + b_1 \]

\[ S_2(L) = a_2 L + b_2 \]

\[ b_2 = -0.68(1) \]

A linear fitting leads to a negative constant close to ln2

All these results can be understood as vanishing of same sublattice pairing in our variational ansatz, which describes a U(1) spin liquid
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- Cylinder
- Stripe
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Including longer range RVB

\[ P_2 = \sum_{i \neq j \neq k \neq l} (|\uparrow\rangle\langle 0|_i + |\downarrow\rangle\langle 1|_i) \otimes \langle 2|_j \otimes \langle \epsilon |_{kl} \]

\[ |\Psi\rangle_{s-RVB} = \prod_{V} P_1 \prod_{B} |S\rangle \]

A single-parameter ansatz

\[ P = P_1 + \epsilon P_2 \]
Discussions and future directions:

Other variational approach

- The best Schwinger VMC approach also predicts gapless U(1) spin liquid.
- The best Slave boson VMC approach predicts gapless Z2 spin liquid with a very small vison gap.
- In general, symmetric spin liquid must be gapless if it is close to AF state.
Properties of the variational state

Longer range RVB configurations can be generated through quantum teleportation

- Sign convention for bond singlets

Longer range RVB decays exponentially