Abstract: The last decade has seen the impressive development of quantum information science, both in theory and in experiment. There are many measures that can be used to assess the achievements in the field: new algorithms, new applications and larger quantum processors, to name a few. The discovery of quantum algorithms has demonstrated the potential power of quantum information.

As pointed out by Bill some years ago, to realize this potential requires the ability to overcome the imprecision and imperfection inherent in physical systems.

Quantum error correction (QEC) has provided a solution, showing that errors can be corrected with a reasonable amount of resources as long as their rate is sufficiently small. Implementing QEC protocols remains one of the most important challenges in QIP.

In the experimental arena, the quest to build quantum processors that could outperform their classical counterparts has led to many blueprint proposals for potential devices based on NMR, electron spin resonance, ion traps, atom traps, optics, superconducting devices and nitrogen-vacancy centres, among others. Many have demonstrated not only the possibility of controlling quantum bits, but also the ability to do so in practice, showing the progression of quantum information science from the blackboard to the laboratory. My presentation will give an overview of some of the recent results in quantum information science on the way to implement quantum error correction. I will show how noise can be characterise efficiently when our goal is to find suitable quantum error correcting codes. I will show demonstrations of control to implement some quantum error correcting codes and finally how can noise be extracted through algorithmic cooling. I will comments on some challenges that need to be solved and a path towards implementing many round of quantum error correction.
Experimental Quantum Error Correction

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UnruhFest, Perimeter Institute, August 2015
April 13, 1983

Mr. R. LaFlamme
1334 de Repentigny
Quebec, P.Q.

Dear Mr. LaFlamme:

I have enclosed a few papers on some of the work I have done recently. My work has concentrated on black holes and quantum processes near them, and also on quantum effects in gravity wave detection.

Regarding the possibility of studying with me, I must first say that I will be on sabbatical this next year. In addition, I have had a number of students enquire about the possibility of working with me and will be somewhat selective in choosing my students. As I have not seen your record, I cannot say too much more except that your winning a 1967 Science scholarship is certainly a positive sign. In addition, I have a post-doctoral fellow who will be here next year (his name is Dr. A. Borde). I am sorry I cannot be more definite, but would be pleased to answer any questions you might have. I have asked our graduate committee to send you the standard application package.

Yours truly,

W.G. Unruh

WBU/20
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Experimental Quantum Error Correction

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Successes of Quantum Information Science

\[ \Psi(t_0) \xrightarrow{\text{\textbullet}} \Psi(t) \]

\[ -i \frac{\partial}{\partial t} \Psi = H \Psi \]

- Discovery of the power of quantum mechanics for information processing
- New language for quantum mechanics
- Discovery of how to control quantum systems
- Proof-of-concepts experiments
a large number of states is important), not only must the coupling be small, but the time taken in the quantum calculation must be less than the thermal time scale $\hbar/k_B T$. For longer times the condition on the strength of the coupling to the external world becomes much more stringent.

If the probability of doing a gate has success $(1 - p)$, the probability of doing successfully $n$ decrease exponentially as $(1 - p)^n$.
Threshold theorem

A quantum computation can be as long as required with any desired accuracy using a reasonable amount of resources as long as the noise level is below a threshold value

$P < 10^{-6}$, $4, 3, 2, ...$

Knill et al.; Science, 279, 342, 1998
Kitaev, Russ. Math Survey 1997
Aharonov & Ben Or, ACM press
Preskill, PRSL, 454, 257, 1998

Significance:

- imperfections and imprecisions are not fundamental objections to quantum computation
- it gives criteria for scalability
- its requirements are a guide for experimentalists
- it is a benchmark to compare different technologies
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Ingredients for FTQEC

- Parallel operations
- Good quantum control
- Ability to extract entropy
- Knowledge of the noise
  - No lost of qubits
  - Independent or quasi independent errors
  - Depolarising model
  - Memory and gate errors
  - ...
Ingredients for FTQEC

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  - Memory and gate errors
  - ...

and lots of qubits...
Progress in experimental QIP

- # of qubits vs time

Last updated August 2015

Adapted from Nature, 469(7335), 49-53, 2010
Progress in experimental QIP

- # of qubits vs time

- Increasing control of qubits

Table 1 | Current performance of various qubits

<table>
<thead>
<tr>
<th>Type of qubit</th>
<th>$T_2$ (ns)</th>
<th>Benchmarking (%)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Infrared photon</td>
<td>0.1</td>
<td>0.016</td>
<td>1</td>
</tr>
<tr>
<td>Trapped ion</td>
<td>15</td>
<td>0.48*</td>
<td>0.7*</td>
</tr>
<tr>
<td>Trapped neutral atom</td>
<td>3</td>
<td>5</td>
<td>507</td>
</tr>
<tr>
<td>Liquid molecule nuclear spins</td>
<td>2</td>
<td>0.01*</td>
<td>0.47*</td>
</tr>
<tr>
<td>$e^-$ spin in GaAs quantum dot</td>
<td>3 μs</td>
<td>5</td>
<td>43, 57</td>
</tr>
<tr>
<td>$e^-$ spins bound to $^{31}$P nuclei</td>
<td>0.6 s</td>
<td>5</td>
<td>49</td>
</tr>
<tr>
<td>$^{28}$Si nuclear spins in $^{28}$Si</td>
<td>25 s</td>
<td>5</td>
<td>50</td>
</tr>
<tr>
<td>NV centre in diamond</td>
<td>2 ms</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Superconducting circuit</td>
<td>4 μs</td>
<td>0.7*</td>
<td>10*</td>
</tr>
</tbody>
</table>

Measured $T_2$ times are shown, except for photons where $T_2$ is replaced by twice the hold-time (comparable to $T_1$) of a telecommunication-wavelength photon in fibre. Benchmarking values show approximate error rates for single or multi-qubit gates. Values marked with asterisks are found by quantum process or state tomography, and give the departure of the fidelity from 100%. Values marked with daggers are found with randomized benchmarking. Other values are rough experimental gate error estimates. In the case of photons, two-qubit gates fail frequently but success is heralded; error rates shown are conditional on a heralded success. NV, nitrogen vacancy.

Characterising noise

Usually we think of the circuit model: Prepare a state, compute, measure

$$|0\rangle \xrightarrow{R_{\pi}(\theta)} \xrightarrow{M} |0\rangle |1\rangle$$

Other possibility is to use only generators of the Clifford group (generated by Hadamard, Phase gate and CNOT), with state preparation and measurements in the computational basis:

$$|0\rangle \xrightarrow{e^{-i\frac{\pi}{2}Y}} \xrightarrow{e^{-i\frac{\pi}{2}X}} \xrightarrow{M} |0\rangle |1\rangle$$

and include the preparation of the magic state

$$\rho = \frac{1}{2} \mathbb{1} + \frac{1}{\sqrt{3}} (X + Y + Z)$$
Characterising noise for QIPs

How do we learn about the noise model?
- Assume first only for memory
- Focus on applying QEC
- Want to be efficient

1) Full process tomography

\[ \rho_f = \sum_j A_j \rho_i A_j^\dagger = \sum_{kl} \chi_{kl} P_k \rho_i P_l = \left( \cdots \right) \rho_i \]

2) Can we get instead coarse grained values of the quantum process such as probability of 0 error \((P_0)\), 1 error \((P_1)\), 2 errors\((P_2)\),... independent of which qubit is affected and the particular error (i.e. X/Y/Z).

- do process tomography and coarse grain \(\rightarrow\) not efficient
- find an efficient protocol, coarse graining \(\Rightarrow\) symmetrise
Characterising noise for QIPs

Coarse graining \(\mapsto\) imposing a symmetry
- error independent of a particular qubit use permutations: \(\pi_S\)
- average over \(X/Y/Z\) \(\rightarrow\) “twirl” average over \(SU(2)^{\otimes n}\)

\[
\rho_f = \sum_{kl} \chi_{kl} \int d\mu(U) U U^\dagger P_k U \rho_i U U^\dagger P_i^\dagger U
\]

\[
\rho_f = \sum_{kl} \chi_{kl} \sum_{\alpha} C_{\alpha}^\dagger P_k C_{\alpha} \rho_i C_{\alpha}^\dagger P_i^\dagger C_{\alpha}
\]

where \(C_{\alpha}\) belongs to the Clifford group \(\sim SP\) with
\(P = \{1, X, Y, Z\}, S = \{e^{-i\pi/4}X, e^{-i\pi/4}Y, e^{-i\pi/4}Z\}\)

\[
\rho_f \approx \sum_{kl} \chi_{kl} \sum_{\alpha} C_{\alpha}^\dagger P_k C_{\alpha} \rho_i C_{\alpha}^\dagger P_i^\dagger C_{\alpha}
\]

Dankert. C..et.al.. PRA 80. 012304. 2009.
Characterising noise protocol

- start with the state $|000\ldots\rangle$,
- implement the symmetrisation group and the Clifford group
- measure how many bits have been flipped.
- Repeat $\rightarrow P_0, P_1, \ldots$

see Emerson et al. Science 317, 1893, 2007
Experimental result

Noise Characterization - NMR results

<table>
<thead>
<tr>
<th>#</th>
<th>Map Description</th>
<th>Krauss operators ${A_i}$</th>
<th>$k_m$</th>
<th>$p_0$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Engineered: $\mathbf{p} = [0, 1, 0]$.</td>
<td>$\frac{1}{2} {Z_1, Z_2}$</td>
<td>288</td>
<td>0.000</td>
<td>0.991</td>
<td>0.009</td>
<td>0.007</td>
</tr>
<tr>
<td>2</td>
<td>Engineered: $\mathbf{p} = [0, 0, 1]$.</td>
<td>${Z_1, Z_3}$</td>
<td>288</td>
<td>0.001</td>
<td>0.004</td>
<td>0.996</td>
<td>0.004</td>
</tr>
<tr>
<td>3</td>
<td>Engineered: $\mathbf{p} = [1/4, 1/2, 1/4]$.</td>
<td>${\exp(i {Z_1 + Z_2})}$</td>
<td>288</td>
<td>0.254</td>
<td>0.495</td>
<td>0.250</td>
<td>0.019</td>
</tr>
<tr>
<td>4</td>
<td>Engineered: $\mathbf{p} = [0, 1, 0]$.</td>
<td>$\frac{1}{2} {Z_1, Z_3, Z_2}$</td>
<td>432</td>
<td>0.01</td>
<td>0.99</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>Natural noise (a)</td>
<td>unknown</td>
<td>432</td>
<td>0.44</td>
<td>0.45</td>
<td>0.10</td>
<td>0.01</td>
</tr>
<tr>
<td>6</td>
<td>Natural noise (b)</td>
<td>unknown</td>
<td>432</td>
<td>0.84</td>
<td>0.15</td>
<td>0.01</td>
<td>0.00</td>
</tr>
</tbody>
</table>

TABLE I: Summary of experimental results.

Emerson, RL et al., Science 317, 1893, 2007
Adapt the idea to benchmark Clifford gates

\[ |0\rangle^{\otimes n} \quad \overset{C_i \quad \mathcal{E} \quad C_i^\dagger}{\longrightarrow} \quad \hat{M}_j \]

\[ |0\rangle^{\otimes n} \quad \overset{C_i \quad \mathcal{E} \quad \mathcal{U} \quad \mathcal{U}^\dagger \quad C_i^\dagger}{\longrightarrow} \quad \hat{M}_j \]

\[ |0\rangle^{\otimes n} \quad \overset{C_i \quad \tilde{\mathcal{U}}}{\longrightarrow} \quad f(\hat{M}_j, C_i, \mathcal{U}) \]

\( f(\hat{M}_j, C_i, \mathcal{U}) \) is a measurement in a different classical basis we can then find \( P_0, P_1, \ldots \) in the gate \( \mathcal{U} \).
Errors in Clifford gates

Use malonic acid in solid state

One qubit can be benchmarked using the Knill procedure:

and Clifford gates using the new procedure

Note: the difference between b) and c) is improving the pulse ("fixing")

Moussa, Silva, Lafamme PRL 109, 100503 (2012)
Benchmarking gates

**Diagram:**
- State Prep.
- C C C C C C R
- Meas.

**Table:**

<table>
<thead>
<tr>
<th>System</th>
<th>Error Rate</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superconducting</td>
<td>0.0008</td>
<td>Nature 508, 501 (2014)</td>
</tr>
<tr>
<td>Ion Trap (single)</td>
<td>0.00002</td>
<td>PRA 83 030303(R) (2011)</td>
</tr>
<tr>
<td>Neutral atoms</td>
<td>0.00014</td>
<td>NJP 12, 113007 (2010)</td>
</tr>
<tr>
<td>Liquid-State NMR</td>
<td>0.00013</td>
<td>NJP 11 013034 (2009)</td>
</tr>
<tr>
<td>Superconducting</td>
<td>0.011</td>
<td>PRL 102 090502 (2009)</td>
</tr>
<tr>
<td>Ion Trap (crystal)</td>
<td>0.0008</td>
<td>QIC 9 920 (2009)</td>
</tr>
<tr>
<td>Ion Trap (single)</td>
<td>0.00482</td>
<td>PRA 77 012307 (2008)</td>
</tr>
<tr>
<td>ESR</td>
<td>0.007</td>
<td>Laflamme/Morton</td>
</tr>
</tbody>
</table>
### Benchmarking gate

**Two qubit comparison**

**Summary table**

<table>
<thead>
<tr>
<th>System</th>
<th>Error Rate</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superconducting</td>
<td>0.006</td>
<td>Nature 508, 501 (2014)</td>
</tr>
<tr>
<td>Ion Trap</td>
<td>0.069/0.162</td>
<td>PRL 108, 260503 (2012)</td>
</tr>
<tr>
<td>Liquid-State NMR</td>
<td>0.005</td>
<td>NJP 11 013034 (2009)</td>
</tr>
<tr>
<td>Neutral Atoms</td>
<td>0.27</td>
<td>PRL 104, 010503 (2010)</td>
</tr>
<tr>
<td>Ion trap</td>
<td>0.007</td>
<td>Nat. Phys. 4 463 (2008)</td>
</tr>
<tr>
<td>NV centre</td>
<td>0.11</td>
<td>Science 320 1326 (2008)</td>
</tr>
<tr>
<td>ESR</td>
<td>0.05</td>
<td>Nature 455 1085 (2008)</td>
</tr>
<tr>
<td>Linear Optics</td>
<td>0.10</td>
<td>PRL 93 080502 (2004)</td>
</tr>
</tbody>
</table>
QEC Experimental implementation

Experimental Quantum Error Correction


1998:
T2: H = 3s, C1 = 1.1s, C2 = 0.6s
DE: 0.85 - 1.10t + O(t^2)
EC: 0.79 - 0.09t + O(t^2)

T2: H = 1.7s, C1 = 1.18s, C2 = 0.45s
DE: 0.99 - 0.436t + O(t^2))
EC: 0.98 - 0.017t + O(t^2)
Demonstration of Sufficient Control for Two Rounds of Quantum Error Correction in a Solid State Ensemble Quantum Information Processor

Osama Moussa,1,2* Jonathan Baagh,1,3 Colm A. Ryan,1,2 and Raymond Laflamme1,2,4

FIG. 1. Shown are the implemented quantum circuits for: (a) labeled C1, C2, and Cm. Results for two rounds of quantum error correction, labeled C1, C2, and Cm. Labels that partially modulate the system are identified by “.” The top row is the C1, C2, and Cm labels. (b) The implemented quantum circuit for two rounds of the 3-bit code (black network), showing the encoding, decoding, and error correction step. The two qubits shown in black are the two qubits from the 3-bit code that are monitored after each round of error correction. After the two-qubit block, qubits that are restored to its initial state, while the top two qubits carry information about which error has occurred; (c) the procedure for two rounds: U_p prepares X, Y, or Z inputs, and U_i = [H, XI, IX, XX] toggles between the different syndrome subspaces; i.e., the experiment is repeated 4 times, cycling through the different U_i, and then the results are added, similar to a standard phase cycling procedure.

PRL 107, 160501 (2011) PHYSICAL REVIEW LETTERS

week ending
14 OCTOBER 2011

kH 2 C1 C2 Cm
6.380 0.297 0.780
-0.025 1.533 1.050
0.042 -5.650

(2.1)
FIG. 4 (color online). Summary of experimental results for the partial decoupling map: the system evolves under the natural Hamiltonian as well as 70 kHz decoupling fields that partially modulate the heteronuclear interactions (between the carbons and protons). Shown (on left) are the single-qubit entanglement fidelities in the cases where no encoding is employed (blue dots); or one round of the 3-bit code (red crosses); or two rounds of the 3-bit code (black asterisks), where the interaction interval is split to two equal intervals. The dashed lines are quadratic fits to the data and are included to guide the eye. Also shown (on right) is the signal after one round of error correction as distributed over the various error-syndrome subspaces. In this case, the dominant errors are phase flips on the top and bottom qubits, which are encoded on $C_1$ and $C_m$, respectively.
Algorithmic cooling with a heat bath

Energy and disorder of a 3 qubit system
(at equilibrium)

000: \( (1-p)^3 \)
001: \( p(1-p)^2 \)
010: \( p^2(1-p) \)
011: \( p^2(1-p) \)
100: \( p(1-p)^2 \)
101: \( p^2(1-p) \)
110: \( p^2(1-p) \)
111: \( p^3 \)

Algorithmic cooling with a heat bath

Qubit System
- Target qubit
- Scratch qudit
- Spin-1/2
- Spin-$\epsilon$
- $d = 2^{n'}$
- $n'$ qubits

Heat-Bath
- $m$ qubits
- Reset qubits

$\epsilon_1^\infty = \frac{(1 + \epsilon_b)^{md} - (1 - \epsilon_b)^{md}}{(1 + \epsilon_b)^{md} + (1 - \epsilon_b)^{md}}$

N Rodriguez-Briones, RL PRL to appear

Moussa, MSc thesis 2005

Graph: Scaled Heat-bath polarization $\epsilon_1^\infty$ vs $d$
Multiple Rounds of Algorithmic Cooling

- By using heat-bath able to surpass Shannon/Soresnsen bound of 1.5X heat-bath polarization


C. Ryan et al. A spin based heat engine: multiple rounds of algorithmic cooling; Ryan et al. PRL 100, 140501, 2008

**Polarization Boost w.r.t. heat-bath**

<table>
<thead>
<tr>
<th>Compression Step</th>
<th>C₂</th>
<th>C₁</th>
<th>Cₘ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.39</td>
<td>0.47</td>
<td>0.49</td>
</tr>
<tr>
<td>2</td>
<td>1.56</td>
<td>0.68</td>
<td>0.71</td>
</tr>
<tr>
<td>3</td>
<td>1.64</td>
<td>0.76</td>
<td>0.79</td>
</tr>
<tr>
<td>4</td>
<td>1.69</td>
<td>0.79</td>
<td>0.84</td>
</tr>
</tbody>
</table>
Conclusion

In order to implement quantum error correction, we will need

- Good knowledge of the noise
- Good quantum control
- Ability to extract entropy
- Parallel operations

Recent experiments have demonstrated these elements individually but we need to pull them together.

It is only the beginning of experimental QEC.