Title: Holographic mapping, quantum error correction code and sub-AdS locality

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Abstract: In recent years, tensor networks have been proposed as a useful framework for understanding holographic duality, especially the relation between quantum entanglement and space-time geometry. Most tensor networks studied so far are defined in the large scale compared with AdS radius. In this talk, I will describe a new tensor network approach which defines a holographic mapping that applies to a refined network with sub-AdS scale resolution, or even to a flat space. The idea of quantum error correction code plays an essential role in this approach. Using this new tensor network, we can study features of the bulk theory, such as how locality at sub-AdS scale emerges in a "low energy subspace" even though the whole theory is intrinsically nonlocal, as a quantum gravity theory should be.
Tensor networks, holographic mapping and sub-AdS locality

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Outline

• Tensor networks and holographic duality (MERA, exact holographic mapping, holographic code)
• New holographic mapping with error correction property: the bidirectional holographic code
• Properties of the new code
• Gauge invariance in the bulk
• Kinematics of the bulk theory
• Discussion

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Holographic duality

Holographic duality, or AdS/CFT (Maldacena ‘97, Witten ‘98, Gubser, Klebanov & Polyakov ‘98)

The extra dimension can be interpreted as energy scale. Bulk equation of motion
\[ \leftrightarrow \text{renormalization group flow.} \]
(E. Akhmedov, ’98, Heemskerk & Polchinski ‘10)

Application to condensed matter
(For a review, see S. Sachdev, Annual Review of Condensed Matter Physics 3, 9 (2012))
Holographic duality

- Holographic duality also relates **space-time geometry** with **quantum entanglement**.
- Bekenstein–Hawking formula
  \[ S = \frac{A}{4G_N} \]
  - **Black hole entropy**
  - **Black hole area**
- Ryu-Takayanagi formula (S. Ryu & T. Takayanagi PRL ‘06)
  \[ S = \frac{A}{4G_N} \]
  - **Entanglement entropy**
  - **Minimal surface area**

![Diagram showing holographic duality between QFT and gravity]
Tensor networks

- Multiscale Entanglement Renormalization Ansatz (MERA). A unitary tensor network describing critical states (Vidal ‘07)

- MERA has been proposed to be related to AdS/CFT (Swingle ‘10, Evenbly&Vidal ’11, Haegeman et al ‘11, Nozaki et al ’12, Hartman&Maldacena ‘13)

- How this relation exactly works remain an open question

Diagram:
- Many-body state $|\psi\rangle$ → Tensor network rep. → continuum limit?? → AdS
Tensor networks

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Exact holographic mapping

- An attempt to use tensor networks to describe a unitary mapping between boundary and bulk \( \text{XLQ 1309.6282} \)
- Redistribution of boundary degrees of freedom by their energy scale
Exact holographic mapping

- A “lossless” generalization of real space RG.
- Degrees of freedom at different energy scales are all kept, and they can entangle with each other.
Exact holographic mapping

- For the chosen holographic mapping, the geometry in the bulk can be determined by correlation functions.
- For two points $x, y$ at equal time, the distance can be obtained by
  \[ l_{xy} = I_0 e^{-d_{xy}/\xi} \]
- Or
  \[ d_{xy} = -\xi \log \frac{l_{xy}}{I_0} \]
- This is assuming short-ranged correlation in the bulk basis.
- Free fermion examples have been studied. (C H Lee & XLQ 1308.6831)
Difference between EHM and AdS/CFT

- There is a particular bulk direction (RG)
- A particular bulk coordinate system is chosen
- A bulk operator corresponds to a unique boundary operator
- No sub-AdS locality (only of order 1 bulk points in each AdS area)
- Partial solution of these differences: perfect tensors
  (Pastawski-Yoshida-Harlow-Preskill PYHP)

Almheiri, Dong, Harlow ‘15
Exact holographic mapping

- A “lossless” generalization of real space RG.
- Degrees of freedom at different energy scales are all kept, and they can entangle with each other.
Perfect tensors and holographic code

6-leg tensor  $T_{a_1 a_2 a_3 a_4 a_5 a_6}$

$$T_{a_1 a_2 a_3 a_4 a_5 a_6} T_{b_1 b_2 b_3}^{a_4 a_5 a_6} = \delta_{a_1 b_1} \delta_{a_2 b_2} \delta_{a_3 b_3}$$

And all permutations

- Holographic code and holographic state (PYHP)
- RT formula for single interval
- Bulk points are equivalent. No special point or direction
- Error correction property of the operator mapping
- The code and the state are different networks
- No sub-AdS physics
Goal of the current work

• We want to define a tensor network that does the following

• 1. It defines a unitary mapping between boundary theory and bulk theory

• 2. The bulk is isotropic. No point is special. No direction is special.

• 3. It defines both holographic state and holographic code. Certain low energy operators are mapped to the boundary with the error correction property.

• 4. Bulk geometry can be defined for scale < AdS radius.
Definition of the new tensor network

- Consider a 5-leg tensor $T_{\alpha\beta\gamma\delta}^A$, $\alpha, \beta, \gamma, \delta = 1, 2, ..., D$, $A = 1, 2, ..., D^4$ with the following properties:
  - 1. $T$ is unitary from $\alpha\beta\gamma\delta$ to $A$
  - 2. For a subspace $A = n = 1, 2, ..., D^2$, $T^n$ is unitary from any two of $\alpha\beta\gamma\delta$ to the rest.
  - 3. From the same subspace, there is a unitary mapping from $n\alpha$ to $\beta\gamma\delta$ and all 4 permutations.
Why are these conditions natural?

- A generalization of Page’s theorem
- Asymptotically satisfied for $D^2$ randomly selected orthogonal states at large $D$
- Rewriting property 2: any two sites have $\rho_2 = \frac{1}{D^2} I$
- Rewriting property 3: for any single site operator $O$, $\langle n | O | m \rangle = \delta_{nm} tr(O) \frac{1}{D}$
- These conditions are idealized version of eigenstate thermalization hypothesis (ETH), which requires (1) the reduced density matrix of subsystem size $< 2L$ is thermal; (2) the off-diagonal matrix element $\langle n | O | m \rangle \sim 0$ vanishes exponentially for simple operators.
An explicit realization of the required tensor

- Take the 4-qutrit code $[403]^3_3$. This code is a state of 4 qutrits $|00\rangle$ satisfying $S_i |00\rangle = |00\rangle$
- $\{S_{1,2,3,4}\} = \{ZZZI, ZZ^{-1}IZ, XXXI, XX^{-1}IX\}$
- Define two operators $A = XXZZ^{-1}, B = ZZ^{-1}XX$, and define $|nm\rangle = A^nB^m|00\rangle$, $n, m = 0, 1, 2$
- These 9 states satisfy properties 2,3.
- We can easily then expand the space to 81 dimensional and realize property 1.
Definition of the tensor network

- Building a network using these tensors

- For any geometry, this defines an isometry from boundary to bulk

- Boundary is mapped unitarily to a subspace of bulk
• We can define a refined AdS tiling with arbitrarily many points per AdS volume (although the symmetry is not enhanced).
Properties of the mapping I: gauge invariance

• Bulk sites are redundant
• Boundary states are mapped to a subspace of the bulk
• This subspace can be defined by “gauge invariant conditions”
Properties of the mapping I: gauge invariance

- In general, the gauge transformations are defined by $g_{ij}$ for each bulk link. ($g_{ij} = g_{ji}^{-1}$)
- $g_{ij}$ defines $W_i = W[g_{ij}]$
- Boundary Hilbert space is mapped to bulk states satisfying $W_i |\psi\rangle = |\psi\rangle$ for all $i$
- This is a “higher form gauge symmetry”
- Consequence of the gauge symmetry: no local operators.
- Bulk operator with support in the interior becomes trivial for gauge invariant states
Properties of the mapping I: gauge invariance

- Nontrivial operators must live in a region bounding the boundary
- This gauge invariance is an analog of general covariance.

\[ \alpha \] 
Identity operator on the boundary

\[ \equiv \] 
nontrivial operators on the boundary
Properties of the mapping II: RT formula

- For bulk product state $\prod_x |n_x\rangle$, $n_x = 1, 2, ..., D^2$, RT formula applies to a single region if the network has non-positive curvature. (PYHP)

- An isometry is defined from $\gamma_A \rightarrow A$ and also $\gamma_A \rightarrow \overline{A} \Rightarrow S_A = \gamma_A \log D$ given by EPR pairs across the geodesic $\gamma_A$

- These are states corresponding to a classical geometry.

- Generic superposition of these states do not satisfy RT formula.
Properties of the mapping III: bulk “low energy” operators

• For “low energy excitations” with most of bulk points at state $|0\rangle$, few-point bulk operators can be mapped to boundary by an isometry

• Example:
More examples for flat space

Allowed configurations with isometry defined

Disallowed configurations with *possibly* no isometry

- Isometries exist for many configurations
- For any 3 sites, there is an isometry to the boundary.
- Rules for “allowed configurations” are non-local.
Kinematics in flat space holography

- Allowed configurations (bulk points with isometry to the boundary) can be counted iteratively.

- Criteria of an allowed configuration: Starting from a rectangle at the boundary, it is possible to shrink this rectangle to a point through smaller rectangles, while crossing only 1 or 0 bulk selected points in each step.

- Percentage of allowed $n$-point configurations

\[ \frac{A(n,L)}{\binom{L^2}{n}} = f \left( \frac{n}{\sqrt{L}} \right) \]

- For $n < n^* = \alpha \sqrt{L}$, almost all points appear independent.
Explicit calculation of allowed configurations

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• For a given stabilizer code such as the qutrit code, we can directly calculate the configurations

• In general, the “disallowed” configurations may or may not define isometries

• The arrow drawing result gives a lower bound of $f(n, L)$

• For the qutrit code, the result is much better: numerically $n_c = 2L - 1$ rather than $\propto \sqrt{L}$
Quantum information interpretation of this tensor network

- The bulk-to-boundary isometries are error correction codes
- Different bulk site configurations defined different error correction codes.
- The “low energy” subspace consists of many code subspaces.
Quantum information interpretation of this tensor network

- For a fixed set of bulk points, the “low energy operators” form an algebra.
- For the whole bulk, low energy operators do not form a closed algebra, as expected for a continuous theory.
- $\psi(x), \psi(x_1)\psi(x_2), \ldots, \psi(x_1)\psi(x_2) \ldots \psi(x_n)$

![Diagram of holographic code and qubits](image)
Different schemes seem to be converging

- With bulk state fixed to $\prod_x |0_x\rangle$, the tensor network defines a state for each bulk region.
- Convex deformation of a region corresponds to an isometry between the corresponding two states.
- An explicit realization of the “surface-state correspondence” (Miyaji-Takayanagi 1503.03542)
Different schemes seem to be converging

- Partial order is defined between regions that are related by convex moves. This seems to be a generalization of the kinematic space story (Czech-Lampropoulos-McCandlish-Sully)
Entropy of a bulk region

- Following Miyaji-Takayanagi We can consider a disk as a limit of polygon
- A pure state is defined on the polygon
- Entanglement entropy of an edge $e$ with its complement is $S = \gamma \log D$ (for any convex region with $e$ at its boundary)
- Area of a bulk region $A = \sum_i \gamma_i$ is a coarse-graining entropy.
- The coarse-graining entropy is maximal.
- When bulk qubits are entangled, this entropy is modified.
Summary and open questions

• We proposed a new holographic mapping, the bidirectional holographic code. It defines a unitary mapping between boundary and bulk.

• Bulk theory is intrinsically nonlocal.

• However, there is a low energy subspace with emergent bulk locality.

• Open questions
  • 1. Hamiltonian and conserved quantities. Lorentz symmetry. A space-time approach?
  • 2. Continuum limit.
  • 3. Spaces without boundary such as de-Sitter space.