Title: TBA

Date: Aug 20, 2015  04:00 PM

URL: http://pirsa.org/15080077

Abstract: TBA
AdS/CFT, Quantum Gravity & Entanglement Workshop

Montreal, Sept 14-16, 2015
Register at http://www.crm.umontreal.ca/2015/Gravity15/
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Entanglement holography

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based on
1508.xxxxx with Jan de Boer, Rob Myers and Yasha Neiman
Holography

AdS/CFT provides the best understood example of the holographic principle.

QG theory in (d+1)-dim = QM theory in d-dim

Within AdS/CFT, emergent direction $z$ is related to energy scale in a dual QFT:

$$ds^2 = \frac{1}{z^2} dz^2 + \frac{1}{z^2} \eta_{\mu\nu} dx^\mu dx^\nu \text{ with } (z, x^\mu) \rightarrow (\lambda z, \lambda x^\mu)$$

Bulk: Wilsonian RG-flow with $z$ playing the role of (the energy scale)$^{-1}$.

The coarse-graining direction is spacelike.
MERA is an example of real-space RG-flow

MERA has “Lorentzian causal structure”. The coarse-graining direction is “timelike”.

Bény 1110.4872; Evenbly et al. 1307.0831, Czech et al. 1505.05515
Integral geometry

One can introduce a partial order on the set of intervals for which we calculate EE

This motivates introducing the light-cone coordinates $u = \theta - \alpha$ and $v = \theta + \alpha$ and considering space with the volume form $\omega = \partial_u \partial_v S \, du \wedge dv$

SSA guarantees $\partial_u \partial_v S \geq 0$. For the vacuum we obtain

$$\omega = \frac{c}{12 \sin^2(\alpha)} \, du \wedge dv$$

Unique conformally-invariant metric compatible with these is

$$ds^2 = \frac{c}{12 \sin^2(\alpha)} \, du \, dv = \frac{c}{12 \sin^2(\alpha)} \left( -d\alpha^2 + d\theta^2 \right)$$

$\text{de Sitter}_2$
Question behind this work

Is there a setup in which:

1) scale appears as an emergent time-like direction
   and

2) local DOFs in the emergent spacetime can be identified?
Entanglement first law

Consider small perturbation of some reference density matrix $\rho = \rho_0 + \delta \rho$

The change in the entropy is equal to the change in $\langle$the modular Hamiltonian$\rangle$

$$\delta S = -\text{tr} (\rho \log \rho) - S_0 = \delta \langle H_{mod} \rangle$$

In general, we expect $H_{mod} \equiv \log \rho_0$ to be nonlocal, but for $\rho_0 = \text{tr}_V |0\rangle \langle 0|$ :

$$H_{mod} = c' + 2\pi \int_{|\vec{x} - \vec{x}'|^2 \leq R^2} d^{d-1}x' \frac{R^2 - |\vec{x} - \vec{x}'|^2}{2R} T_{tt}(x')$$

$t = 0$ of $\mathbb{R}^{1,d}$

$V = B^{d-1}$

$|0\rangle$ is the vacuum$_{\text{CFTd}}$

Casini, Huerta & Myers 1102.0440
**Propagation on de Sitter**

As a result, the change in the entanglement entropy for small perturbations of \( |0\rangle \) is

\[
\delta S_B = 2\pi \int_{|\overline{x} - \overline{\overline{x}}'|^2 \leq R^2} d^{d-1}x' \frac{R^2 - |\overline{x} - \overline{\overline{x}}'|^2}{2R} \langle T_{tt}(x') \rangle
\]

see e.g. Xiao 1402.7080

This is the bulk-to-boundary propagator in \( \text{dS}_d \): \[
 ds^2 = -\frac{L^2}{R^2} dR^2 + \frac{L^2}{R^2} d\overline{x}^2
\]

This implies that \( \delta S \) is a local field in \( \text{dS}_d \) and obeys the Klein-Gordon equation:

\[
\nabla_a \nabla^a |_{\text{dS}_d} \delta S_B - m^2 \delta S_B = 0 \quad \text{with} \quad m^2 L^2 = -d
\]

Note that the scale \( R \) appears here as an emergent time-like coordinate.
How does it work?

$R = 0$ corresponds to one of the timelike boundaries in $dS_d$ (say to the future one).

$$\delta S \xrightarrow{R \to 0} F(x)/R + f(x) R^d + \ldots$$

with

$$F(x) = 0 \quad \text{and} \quad f(x) = \frac{\pi^{\frac{d+1}{2}}}{\Gamma\left(\frac{d+3}{2}\right)} \langle T_{tt}(x) \rangle$$

Explicit example in CFT$_2$: $\delta \rho = \eta (|0\rangle\langle \phi | + |\phi \rangle\langle 0|)$ with $|\phi \rangle = T_{tt}(t_0 + i \tau, x_0)|0\rangle$
Comments

\[ \nabla_a \nabla^a \big|_{dS_d} \delta S_B - m^2 \delta S_B = 0 \text{ in any CFT}_d \] (large c / strong coupling not needed)

It surfaced before in the studies of HEE & the Einstein equations

Takayanagi et al. 1304.7100 and 1308.3792

It relies only on the applicability of the first law for all values of \( R \)

As it is now, it concerns constant time slice configurations in CFT\(_d\)

\[ \nabla_a \nabla^a \big|_{dS_d} \delta S_B - m^2 \delta S_B = 0 \] is covariant and applies in any coords in dS\(_d\)

Our analysis, as it is now, does not fix the curvature scale of dS\(_d\)
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1-to-1 mapping between spheres and $dS_d$

$$\delta S_B = 2\pi \int_{|\vec{x}'|^2 \leq R^2} d^{d-1}x' \frac{R^2 - |\vec{x} - \vec{x}'|^2}{2R} \langle T_{tt}(x') \rangle$$

sphere B in $\mathbb{R}^{d-1}$ maps to a point in $dS_d$:

Time slice $\iff I^+ \equiv \{ x \mid R = 0 \}$

Causal relations between points in $dS_d$ $\iff$ partial order between B's on $t=0$:

- timelike
- null
- spacelike

generalizes Czech et al. 1505.05515
1-to-1 mapping between spheres and $dS_d$

$$\delta S_B = 2\pi \int_{|\bar{x}|^2 \leq R^2} d^{d-1}x' \frac{R^2 - |\bar{x} - \bar{x}'|^2}{2R} \langle T_{tt} \rangle(x')$$

sphere $B$ in $\mathbb{R}^{d-1}$ maps to a point in $dS_d$:

Time slice $\iff I^+ \equiv \{x | R = 0\}$

Causal relations between points in $dS_d$ $\iff$ partial order between $B$'s on $t=0$:

9/14 generalizes Czech et al. 1505.05515
Elliptic $dS_d$

If $\delta S_B = \delta S_B$, the field propagates on elliptic $dS_d$: $\delta \rho = \eta (|0\rangle\langle\phi| + |\phi\rangle\langle0|)$

If $\delta S_B \neq \delta S_B$, this is not the case, e.g. $\delta \rho = ge^{-\beta E_1}|E_1\rangle\langle E_1|$

Herzog [407.1358]
More dynamical scalar fields on $dS_d$

\[ H_{mod} = c' + \int_B dB^\mu J^{(2)}_\mu \text{ with } J^{(2)}_\mu \equiv T_{\mu \nu} K^\nu \]

\[ Q^{(s)} = \int_B dB^\mu J^{(s)}_\mu \text{ with } J^{(s)}_\mu = T_{\mu \nu_1 \ldots \nu_{s-1}} K^{\nu_1} \ldots K^{\nu_{s-1}} \]

see Belin et al. 1310.4180 for $s=1$

see Hijano & Kraus 1406.1804 for $d=2$ & $s>2$

\[ \delta S^{(s)}_B = (2\pi)^{s-1} \int_B d^{d-1}x' \left( \frac{R^2 - |\vec{x} - \vec{x}'|^2}{2R} \right)^{s-1} T_{tt\ldots t}(x') \]

It is clear now that all $\delta S^{(s)}_B$ will be local scalar fields in $dS_d$ and will obey

\[ \nabla_a \nabla^a |_{dS_d} \delta S^{(s)}_B - m^2 \delta S^{(s)}_B = 0 \quad \text{with} \quad m^2 L^2 = -(s-1)(d+s-2) \]
Summary

Entanglement in excited states is organized in a Lorentzian holographic way:

\[ \nabla_a \nabla^a \big|_{dS_d} \delta S_B - m^2 \delta S_B = 0 \]

with \( m^2 L_{dS_d}^2 = -d \)

This statement applies to any CFT in any \( d \) provided the first law holds.

The statement concerns constant time slices in a CFT.

For theories with conserved charges: one dynamical field in \( dS_d \) for each charge.
Some open problems

Can we describe full CFT in terms of local fields interacting in dS_d (novel dS/CFT)?

Geometry encapsulating time evolution between 2 constant time slices in a CFT?

Does dS_d play the role of the kinematic space / what fixes $L_{dS_d}$?

Link with MERA / cMERA?

Does the emergent local Lorentzian propagation persists if conformal symmetry?